

# Cantor's Theorem in Naproche

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This is a formalization of *Cantor's Theorem* [1], i.e. of the assertion that any set  $x$  is strictly smaller than its powerset  $\mathcal{P}(x)$ :

**Theorem 1 (Cantor's Theorem).** Let  $x$  be a set. Then  $|x| < |\mathcal{P}(x)|$ .

*Proof.*

*Case  $x$  is empty.*  $\square$

*Case  $x$  is nonempty.* Assume the contrary. Then  $|x| \geq |\mathcal{P}(x)|$ . Hence there exists a surjective map from  $x$  onto  $\mathcal{P}(x)$  (by existence condition for surjections). Indeed  $\mathcal{P}(x)$  is a nonempty set. Contradiction (by Cantor (elementary version)).  $\square$



## References

- [1] Georg Cantor. “Über eine elementare Frage der Mannigfaltigkeitslehre”. In: *Jahresbericht der Deutschen Mathematiker-Vereinigung* 1 (1891), pp. 75–78.

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