

Theorem 1 (Cantor's Theorem). Let x be a set. There exists no surjective map from x onto $\mathcal{P}(x)$.

Proof. Assume the contrary. Take a surjective map f from x onto $\mathcal{P}(x)$. Define $C := \{u \in x \mid u \notin f(u)\}$. Then $C \in \mathcal{P}(x)$. Hence we can take a $u \in x$ such that $f(u) = C$. Then $u \in C$ iff $u \in f(u)$ iff $u \notin C$. Contradiction. ■