

Signature 1. Let n be a natural number. $n!$ is a natural number.

Axiom 2. $0! = 1$.

Axiom 3. Let n be a natural number. Then $n + 1! = n! \cdot (n + 1)$.

Proposition 4. Let n be a natural number. Then $n! \neq 0$.

Proof. Define $\Phi := \{n' \in \mathbb{N} \mid n'! \neq 0\}$.

(1) Φ contains 0. Indeed $0! = 1 \neq 0$.

(2) For all $n' \in \Phi$ we have $n' + 1 \in \Phi$.

Proof. Let $n' \in \Phi$. We have $n' + 1! = (n' + 1) \cdot n'!$. $n' + 1$ and $n'!$ are nonzero. Hence $n' + 1! \neq 0$. \square

Thus Φ contains every natural number (by induction). Therefore $n! \neq 0$.

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