Packet Error Probability for Decode-and-Forward Cooperative Networks of Selfish Users

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***Abstract*—In this paper, we consider cooperative networks of users that implement a selfish protocol to decode and forward packets of their partners. We use a threshold-based model to derive an analytical expression for the end-to-end packet error probability and we validate our approach comparing theoretical to simulation results. Furthermore, we explore the interplay and impact of network parameters, such as the number of users, the transmission scheme and the quality of the interconnecting channels, on the overall performance.**

1. Introduction

Space diversity, in the form of multiple transmit and receive antennas, combined with time and frequency diversity can be used to mitigate multipath fading in wireless communication networks. An alternative means of diversity is *user cooperation* [1], according to which each user "overhears" its partners and relays their data to the destination. The fundamental idea behind cooperative transmission can be traced back to the work of Cover and El Gamal [2]. Sendonaris *et al.* [1] presented a practical implementation of a user cooperation protocol employing code-division multiple access (CDMA), whilst Laneman and Wornell [3], [4] proposed several cooperative transmission schemes such as the amplify-and-forward (AF) and decode-and-forward (DF) protocols.

In DF protocols, a user relays the data of a partner only when they have been successfully decoded; otherwise, the user notifies its partner and the later re-transmits its own data to the destination. This approach is also known as *selection* DF in contrast to *fixed* DF, in which no feedback information is exchanged between cooperating users [4]. In AF protocols, a user simply amplifies and forwards the packets of a partner to the destination, at all times. Both selection DF and AF protocols achieve full diversity [4]. However, AF transmission requires that the destination has knowledge of the channel conditions between cooperating users, which is not possible in many practical scenarios, as explained in [5].

In this paper we consider selection DF cooperation in wireless networks over quasi-static fading channels. We note that the quasi-static fading model characterises various practical slow fading environments, in which neither temporal diversity nor frequency diversity can be exploited, for example fixed wireless access systems. Analysis of the error rate performance of DF cooperative networks on quasi-static fading channels was carried out in [6] and [7]. In particular, Sadek *et al.* provide an approximate expression for the symbol and packet error probabilities of a multi-user network that is accurate at high signal-to-noise ratio (SNR) values [6]. Souryal and Vojcic, on the other hand, use an SNR threshold-based model and propose an approximation for the packet error probability (PEP), which is accurate for networks of two cooperating users that employ turbo codes [7].

The motivation for this paper is to accurately characterise the end-to-end PEP of DF cooperative networks on quasi-static fading channels. We use a threshold-based model but, in contrast to [7] who arbitrarily define the SNR threshold, we use the technique presented in [8] to determine the exact SNR threshold of the adopted transmission scheme. Therefore, our proposed framework closely describes the PEP of networks consisting of not only two but up to sixteen users, each employing either iterative (turbo) or non-iterative schemes.

The rest of the paper is organized as follows: Section 2 introduces the system model and describes the stages of selection DF cooperation in a network of selfish users. Sections 3 and 4 present a framework, based on which an accurate expression for the end-to-end packet error probability of a user can be obtained analytically. Section 5 compares theoretical to simulation results and section 6 concludes this paper with a summary of the main contributions of our work.

2. System Model

We consider a wireless network that consists of M users, denoted as $\mathrm{U}_1, \mathrm{U}_2, \ldots, \mathrm{U}_M$, that transmit to the same destination D. Channels between users and the destination are usually referred to as *uplink channels*, whilst channels that link different users are known as *inter-user channels*. All channels are subject to frequency-flat Rayleigh fading and additive white Gaussian noise. Users transmit on orthogonal channels, which allows the destination to detect each user separately. Cooperation of the users occurs in two successive stages. Quasi-static fading is considered, hence each channel realization remains constant for the duration of the two-stage frame but changes independently from frame to frame.

During the first stage of cooperation, each user broadcasts a packet of length N coded bits to the other users and the destination. At the end of the first stage, users also

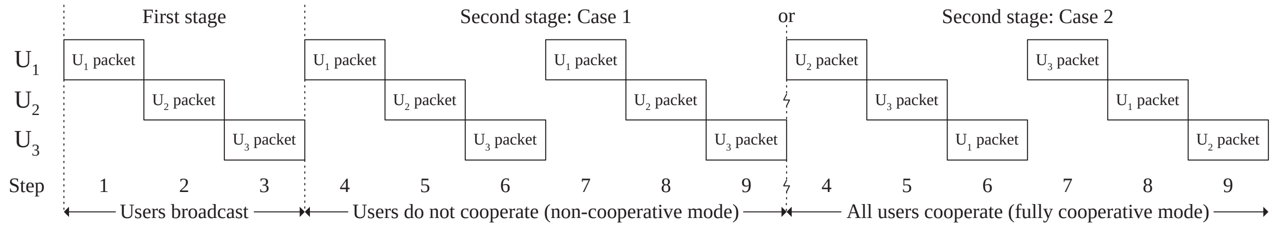

Fig. 1. Selfish cooperation in a network of $M=3$ users employing TDMA. Case 1 in the second stage occurs when at least one user cannot decode a packet from a partner; on the other hand, case 2 occurs when all users successfully decoded one another's packets.

broadcast a short message, whose payload is a single bit, to acknowledge successful decoding of all their partners' packets. If one or more users fail to decode packets of their partners and thus cannot assist them in the following stage, then all users act selfishly and opt not to cooperate; during the second stage, each user simply transmits $M-1$ copies of its own packet to the destination. However, if all users successfully decode the packets of their partners, cooperation is enabled; hence, during the second stage, each user re-encodes its partners' data and transmits $M-1$ packets, one for each partner.

An illustration of the cooperation protocol for time division multiple access (TDMA) and $M = 3$ users is presented in Fig. 1. In general, the first and second stages span M and $M(M-1)$ time slots, respectively. Therefore, the two-stage cooperation frame is completed after M^2 time steps. However, if each user is assigned a different frequency (FDMA/TDMA), the duration of the cooperation frame reduces to M time steps.

The quality of a channel in our system model is characterized by the corresponding average receive SNR. In particular, we assume that the fading coefficient of an uplink channel between user U_i, $i=1,\dots,M$, and the destination D during the n-th cooperation frame is described by $h_n(i,\mathrm{D})$. Each channel is also impaired by additive white Gaussian noise of variance N_0. Consequently, the instantaneous receive SNR at the destination is given by

$$\gamma_n(i,\mathrm{D}) = |h_n(i,\mathrm{D})|^2 \frac{E_\mathrm{s}(i)}{N_0}, \qquad (1)$$

where $E_\mathrm{s}(i)$ is the energy per symbol that is allocated by user U_i. Note that the fading coefficients $h_n(1,\mathrm{D}),\dots,h_n(M,\mathrm{D})$ of the uplink channels have been modelled as zero-mean, independent and identically distributed complex Gaussian random variables with variance σ_D^2, which implies that all users are located at a similar distance from the destination. Therefore, the average SNR, which is defined as $\overline{\gamma}(i,\mathrm{D}) \triangleq \mathbb{E}\big[\gamma_n(i,\mathrm{D})\big]$, can be calculated using

$$\begin{aligned}\overline{\gamma}(i,\mathrm{D}) &= \mathbb{E}\Big[\big|h_n(i,\mathrm{D})\big|^2\Big]\frac{E_\mathrm{s}(i)}{N_0} \\ &= \sigma_\mathrm{D}^2 \frac{E_\mathrm{s}(i)}{N_0},\end{aligned} \qquad (2)$$

where $\mathbb{E}[.]$ denotes the expectation operator. The average receive SNR, $\overline{\gamma}(i,j)$ for $i\neq j$, that characterises the quality of the inter-user channels can be obtained in a similar manner.

Throughout the paper, we shall assume the following:

- The same channel code is employed by all users in both stages of the cooperation frame.
- Channel state information is available to all users and the destination, therefore coherent detection is possible.
- All users allocate the same energy per symbol, hence $E_\mathrm{s}(i)=E_\mathrm{s}$ and, consequently, the uplink channels are statistically similar, that is $\overline{\gamma}(i,\mathrm{D})=\overline{\gamma}_\mathrm{D}$ for all values of i.
- The inter-user channels are also statistically similar, hence $\overline{\gamma}(i,j)=\overline{\gamma}_\mathrm{R}$ for all $i\neq j$.
- Realisations for each direction of an inter-user channel are mutually independent, that is $\gamma_n(i,j)$ is not necessarily equal to $\gamma_n(j,i)$. This is a valid assumption for FDMA/TDMA systems but represents the worst case scenario for TDMA systems.

3. Packet Error Probability Expressions

Derivation of exact expressions for the end-to-end PEP of a network can prove difficult, depending upon the transmission scheme under consideration. In this section we invoke a tight approximation to the PEP and adapt it to the case of cooperative networks of selfish users.

3.1. Preliminaries: PEP Approximation

Let us first consider the general case of a receiver that combines the output of K independent but statistically similar quasi-static fading channels. If $\overline{\gamma}$ is the average receive SNR for each channel, the instantaneous output SNR of the combiner, denoted as γ_Σ, follows a chi-squared distribution with $2K$ degrees of freedom [9], [10]; its probability density function is given by

$$p(\gamma_\Sigma) = \frac{\gamma_\Sigma^{K-1}}{\overline{\gamma}^K (K-1)!} \exp\left(-\frac{\gamma_\Sigma}{\overline{\gamma}}\right). \qquad (3)$$

El Gamal and Hammons derived a simple yet close approximation for the average error probability of iteratively decoded schemes over single-input single-output (SISO) quasi-static fading channels [11], while Rodrigues *et al.* extended the approximation to multiple-input multiple-output (MIMO) channels [10]. In particular, the average PEP of a receiver that employs iterative decoding can be approximated as follows

$$P_{\gamma_o}(\overline{\gamma}, K) \simeq \int_0^{\gamma_o} p(\gamma_\Sigma)\, d\gamma_\Sigma = 1 - \exp\left(-\frac{\gamma_o}{\overline{\gamma}}\right) \sum_{k=0}^{K-1} \frac{1}{k!} \left(\frac{\gamma_o}{\overline{\gamma}}\right)^k . \tag{4}$$

Here, γ_o is an SNR threshold, based on which the instantaneous PEP of the receiver is either one or zero depending on whether the instantaneous SNR γ_Σ is less than or greater than γ_o, respectively. For systems employing turbo codes, γ_o coincides with the convergence threshold of the iterative decoder [10], [11]. It important to notice that (4) is accurate for low to moderate values of K (for example, $K \leq 16$); for large K, the MIMO channel effectively collapses into an additive white Gaussian noise (AWGN) channel, for which the framework for threshold-based PEP analysis does not apply.

Chatzigeorgiou *et al.* demonstrated that not only iterative but non-iterative and even uncoded schemes over SISO [12] and MIMO [8] quasi-static fading channels can also be characterised by an SNR threshold, therefore (4) can still be used to obtain a tight approximation to their average PEP. The generalised expression for γ_o assumes the form

$$\gamma_o = \left(\int_0^{\infty} \frac{P_s^G(\gamma)}{\gamma^2}\, d\gamma\right)^{-1}, \tag{5}$$

where $P_s^G(\gamma)$ is the probability of successful packet decoding in AWGN. A practical methodology for the evaluation of γ_o is discussed in [12].

3.2. PEP Adaptation to Selfish Cooperation

Let us now consider the proposed network configuration of M users that selfishly cooperate in an effort to convey information to the destination. When users choose to fully cooperate, M independently faded copies of a packet associated with user U_i are coherently combined at the destination. Remember that one packet is transmitted directly by user U_i to the destination during the first stage, whilst $M-1$ packets are transmitted indirectly through the partners during the second stage. The average PEP for the fully cooperative (FC) mode, denoted as P_e^{FC} can thus be computed using (4) as follows

$$P_e^{FC} = P_{\gamma_o}(\overline{\gamma}_D, M). \tag{6}$$

On the other hand, when users opt not to cooperate, user U_i transmits M copies of its packet to the destination through the same direct channel, whose output average SNR is $\overline{\gamma}_D$. It can be shown that this is equivalent to transmitting a single copy of the packet at an SNR value of $M\overline{\gamma}_D$. Consequently, the average PEP for the non-cooperative (NC) mode assumes the form

$$P_e^{NC} = P_{\gamma_o}(M\overline{\gamma}_D, 1). \tag{7}$$

4. End-to-End Packet Error Probability

In this section we determine the probability of the proposed selfish protocol being in either of the two available modes (FC or NC) and we subsequently use it to obtain an analytical expression for the average end-to-end PEP.

4.1. Probability of Cooperation

Let us consider the instance when a user U_i broadcasts a coded packet and a user U_j receives it through the corresponding inter-user channel, whose average SNR is $\overline{\gamma}_R$. The probability that U_j will successfully decode the packet of U_i is given by

$$\mathbb{P}\{U_j \text{ decodes } U_i\} = 1 - P_{\gamma_o}(\overline{\gamma}_R, 1). \tag{8}$$

Taking into account that the inter-user channels are mutually independent, we obtain the joint probability that U_j will successfully decode the packets of all its M–1 partners as follows

$$\mathbb{P}\{U_j \text{ decodes all partners}\} = \prod_{\substack{i=1\\ i\neq j}}^{M} \mathbb{P}\{U_j \text{ decodes } U_i\} = (1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M-1} . \tag{9}$$

The probability that the protocol operates in the FC mode is equal to the joint probability of all users having successfully decoded their partners, that is[1]

$$\mathbb{P}\{\text{FC mode}\} = \prod_{j=1}^{M} \mathbb{P}\{U_j \text{ decodes all partners}\} = (1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M(M-1)} . \tag{10}$$

The probability that the NC mode is instead enabled, can be simply computed by

$$\mathbb{P}\{\text{NC mode}\} = 1 - \mathbb{P}\{\text{FC mode}\} = 1 - (1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M(M-1)} . \tag{11}$$

4.2. An Analytical Expression for the End-to-End PEP

The average end-to-end PEP for a particular user in the network, irrespectively of the mode that the cooperation protocol operates in, can be decomposed as follows

$$\begin{aligned} P_e &= \mathbb{P}\{\text{packet error}\} \\ &= \mathbb{P}\{\text{packet error}|\text{FC mode}\}\cdot\mathbb{P}\{\text{FC mode}\} \\ &\quad + \mathbb{P}\{\text{packet error}|\text{NC mode}\}\cdot\mathbb{P}\{\text{NC mode}\} , \end{aligned} \tag{12}$$

invoking the sum rule. The two conditional probabilities on the right hand side of (12) correspond to P_e^{FC} and P_e^{NC} respectively, which have been derived in Section 3.2. Substituting all terms in (12), using (6), (7), (10) and (11), yields

$$\begin{aligned} P_e &= P_{\gamma_o}(\overline{\gamma}_D, M)\,(1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M(M-1)} \\ &\quad + P_{\gamma_o}(M\overline{\gamma}_D, 1)\Big(1 - (1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M(M-1)}\Big) \end{aligned} \tag{13}$$

which can be further expanded to

$$\begin{aligned} P_e &= 1 - \exp\!\left(-\frac{\gamma_o}{M\overline{\gamma}_D}\right)\left(1 - \exp\!\left(-\frac{\gamma_o}{\overline{\gamma}_R} M(M-1)\right)\right) \\ &\quad - \exp\!\left(-\frac{\gamma_o}{\overline{\gamma}_R} M(M-1) - \frac{\gamma_o}{\overline{\gamma}_D}\right) \sum_{k=0}^{M-1} \frac{1}{k!} \left(\frac{\gamma_o}{\overline{\gamma}_D}\right)^k \end{aligned} \tag{14}$$

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[1] Note that if the two realisations of each inter-user channel were not mutually independent but reciprocal (for example, TDMA systems in slow fading environments), the probability of occurrence of the FC mode would be given by $\mathbb{P}\{\text{FC mode}\} = (1 - P_{\gamma_o}(\overline{\gamma}_R, 1))^{M-1}$.

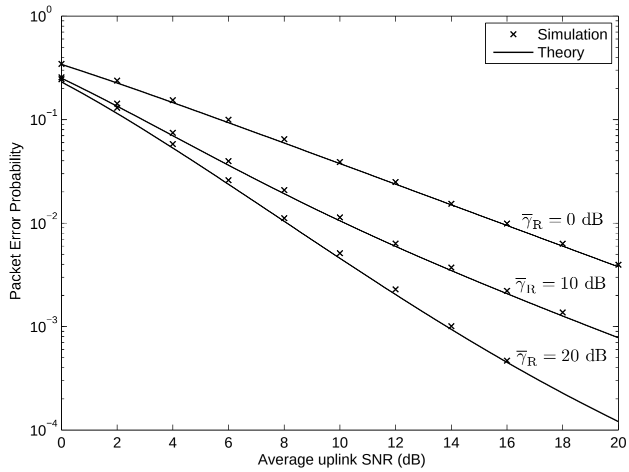

Fig. 2. Comparison of analytical values to simulation results for a network of $M=2$ users, employing the rate 1/2 NRNSC(15, 17) code.

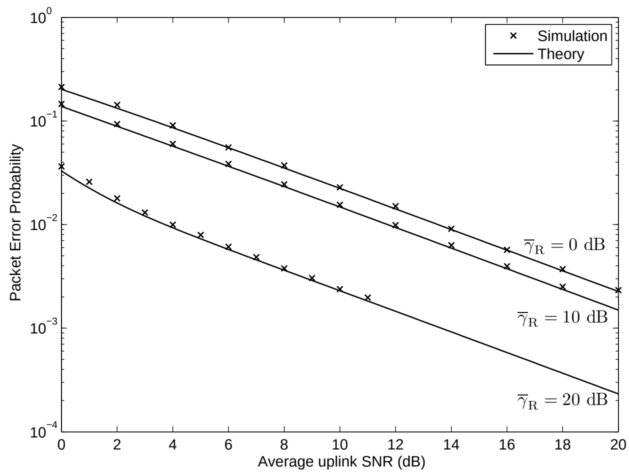

Fig. 3. Comparison of analytical values to simulation results for a network of $M=4$ users, employing the rate 1/2 NRNSC(15, 17) code.

using (4). The derived analytical expression for the average end-to-end PEP will be validated in the following section. The impact of each network parameter, i.e., M, γ_{o}, $\overline{\gamma}_{\mathrm{D}}$ and $\overline{\gamma}_{\mathrm{R}}$, on the PEP will also be investigated.

5. Results

Let us consider a network of M users, each using a rate 1/2 non-recursive non-systematic convolutional (NRNSC) code with octal generator polynomials (15, 17) to encode packets of $N=512$ information bits; the generated packets of coded bits are then modulated using binary phase shift keying (BPSK). Invoking (5), we found that the SNR threshold for the convolutional code under consideration is $\gamma_{\mathrm{o}}=-0.441$ dB.

Curves that were obtained using (14) are compared to simulations in Fig. 2 and Fig. 3 for networks of $M=2$ and $M=4$ users, respectively. Observe that the simulation results closely follow the theoretical predictions, confirming the validity of our analysis. Furthermore, it is important to notice that when the quality of both the inter-user and the uplink channels is good, the average end-to-end PEP is better for $M=2$ than for $M=4$. This indicates that if the FC mode is often enabled when the uplink SNR is high, the network instantly benefits from the additional diversity. Owing to the fact that the likelihood of the protocol operating in the FC mode reduces as the number of users increases, the average PEP in a two-user network should indeed be lower than that in a four-user network. On the other hand, we observe that when the quality of the uplink channel is poor, an increase in the number of cooperating users from two to four improves the average PEP. Thus, the additional diversity offered by the rare but plausible event of full cooperation in the four-user network, markedly improves the average PEP at low uplink SNR values.

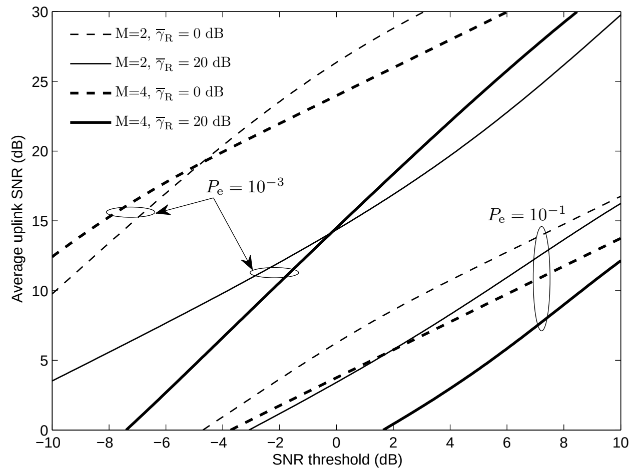

Fig. 4. Effect of the network parameters on the average uplink SNR for 10^{-1} and 10^{-3} packet error probabilities.

Having observed a good match between simulations and theoretical results, we now use our theoretical model to explore the impact of the various network parameters on the end-to-end PEP. An example is given in Fig. 4 for $M=2$ and $M=4$. Here, the SNR threshold γ_{o}, which relates to the modulation and coding scheme employed by the users, is shown on the horizontal axis; the average uplink SNR $\overline{\gamma}_{\mathrm{D}}$, that is required to achieve a target PEP, is shown on the vertical axis. Two target PEP values have been selected: $P_{\mathrm{e}}=10^{-1}$ and $P_{\mathrm{e}}=10^{-3}$. We observe that, for a fixed number of users, powerful error-correcting codes that exhibit a low SNR threshold will reduce the required uplink SNR and, hence, provide a coding gain. As it is also expected, an increase in the average SNR level of the inter-user channel from $\overline{\gamma}_{\mathrm{R}}=0$ dB to $\overline{\gamma}_{\mathrm{R}}=20$ dB will yield a coding gain.

If the number of cooperating users can also vary, then Fig. 4 provides some interesting insights. For example, let us consider the case when the target PEP is 10^{-3}, the inter-user channel quality is $\overline{\gamma}_{\mathrm{R}}=20$ dB and networks of either two or four users can be formed. If users can afford powerful channel codes (i.e., low γ_{o}), a partnership between four users is markedly more beneficial, as can be seen in Fig. 4. If, however, users employ coding schemes with limited error correction capabilities (i.e., high γ_{o}), they should be clustered in pairs to minimise the required

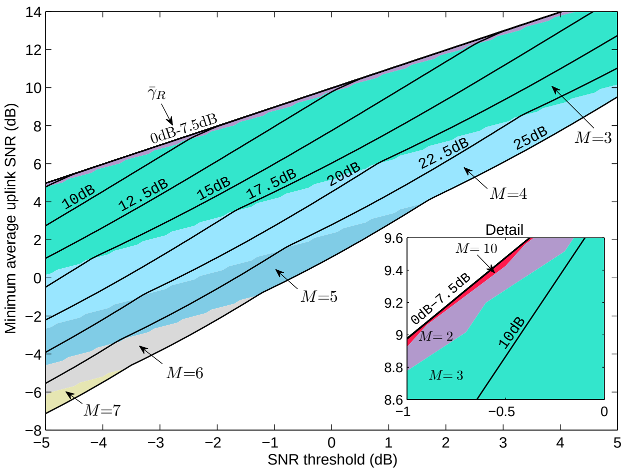

Fig. 5. Optimal network parameters for $P_e = 10^{-2}$. The number of users ranges from $M = 2$ to $M = 10$.

transmit power.

Using the proposed analysis, we can construct graphs that provide the optimal network parameters for which the transmit power is minimised. For instance, the optimal parameters for networks of $M = 2, 3, \ldots, 10$ users and a target PEP of $P_e = 10^{-2}$ can be determined using Fig. 5. Note that solid lines correspond to various inter-user SNR values, while coloured regions are associated with a fixed, optimal number of partners. We shall explain the functionality of the graph in Fig. 5 by means of an example. Let us assume that the average inter-user SNR is $\overline{\gamma}_R = 20$ dB and the SNR threshold for the selected transmission scheme is -1 dB. According to Fig. 5, the target PEP of $P_e = 10^{-2}$ can be achieved if the average uplink SNR is set to 2.73 dB, provided that $M = 4$ users are involved in the cooperation frame. If, however, uncoded transmission was used, the SNR threshold would increase, say to 2 dB for instance. This point would now be on that part of the $\overline{\gamma}_R = 20$ dB curve that crosses the $M = 3$ region. Consequently, the minimum uplink SNR of 7.76 dB could only be achieved if exactly three partners were cooperating.

In general, we observe in Fig. 5 that when the inter-user channel quality is good ($\overline{\gamma}_R \geqslant 7.5$ dB), the average uplink SNR can be reduced by increasing the number of partners, provided that the adopted transmission scheme exhibits a low SNR threshold. Interestingly, however, the occasional diversity offered by multi-user networks on poor inter-user channels, reduces the required uplink SNR for $P_e = 10^{-2}$; irrespective of the SNR threshold, a thin region associated with $M = 10$ users exists on top of all other regions accommodating low inter-user channel SNR values (see inset, Fig. 5). Nevertheless, we found that this region becomes thinner as the target PEP improves and vanishes for low PEP values (for example, $P_e = 10^{-3}$).

6. Conclusions

In this paper, we presented a closed-form expression that accurately predicts the end-to-end packet error probability of decode-and-forward wireless networks, in which users employ a strictly selfish cooperation protocol. Based on this expression, we demonstrated that the optimal number of partners depends on both the quality of the inter-user channels and the error correction capability of the adopted channel code.

An investigation of the packet error probability of cooperative networks that adopt either a selectively selfish profile, i.e., each user assists only those partners that have successfully decoded its own data, or an unselfish profile, i.e., each user assists all partners whose data can be successfully decoded, will be carried out in future work.

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