Energy Efficient Signal Acquisition via Compressive Sensing in Wireless Sensor Networks

Wei Chen and Ian J. Wassell
Digital Technology Group (DTG), Computer Laboratory, University of Cambridge, UK
Email: {wc253,ijw24}@cam.ac.uk

***Abstract*—This paper presents a novel approach based on the compressive sensing (CS) framework to monitor 1-D environmental information using a wireless sensor network (WSN). The proposed method exploits the compressibility of the signal to reduce the number of samples required to recover the sampled signal at the fusion center (FC) and so reduce the energy consumption of the sensors. An innovative feature of our approach is a new random sampling scheme that considers the causality of sampling, hardware limitations and the trade-off between the randomization scheme and computational complexity. In addition, a sampling rate indicator (SRI) feedback scheme is proposed to enable the sensor to adjust its sampling rate to maintain an acceptable reconstruction performance while minimizing the energy consumption. A significant reduction in the number of samples required to achieve acceptable reconstruction error is demonstrated using real data gathered by a WSN located in the Hessle Anchorage of the Humber Bridge.**

I. INTRUDUCTION

Wireless sensor networks (WSN) provide the ability to monitor various physical characteristics of the real world, such as sound, temperature, humidity, etc., by distributing a large number of inexpensive small devices in the detected environment. The main constraints of WSNs are owing to the limited energy storage and low computational capability of each node. Those constrains are due to size and cost limitations, as most applications require the use of small and inexpensive sensor nodes. Although the sensor nodes have limited computational capability and energy, the fusion center (FC) (or any back-end processor) usually has a comparatively high computational capability [1].

The traditional approach to measure 1-D environmental information, e.g., temperature and humidity, is to uniformly sample and then report data to a FC. The sampling period could range from milliseconds to minutes depending on the particular application. Actually, such signals are generally compressible by transforming to some suitable basis. The traditional sampling approach is not energy efficient since the transmitted data contains a large portion of redundant information. An alternative method [2] is compressing and then transmitting. Although power consumption of the transmission is reduced, compression requires additional energy and makes computational demands of the sensor nodes. Furthermore, this approach is not suitable for real-time applications owing to the latency in gathering the data and the computation to execute the compression algorithm at the sensor node.

Compressive sensing (CS), also called compressed sensing and Sub-Nyquist sampling, has a surprising property that one can recover sparse signals from far fewer samples than is predicted by the Nyquist-Shannon sampling theorem [3]–[6]. Samples made via CS contain a little redundancy in the information level, and the sampling process accomplishes two functions, i.e., detection and compression. CS trades off an increase in the computational complexity of post-processing against the convenience of a smaller quantity of data acquisition and lower demands on the computational capability of the sensor node. CS directly acquires the compressed version while sampling, and so no explicit compression process is required.

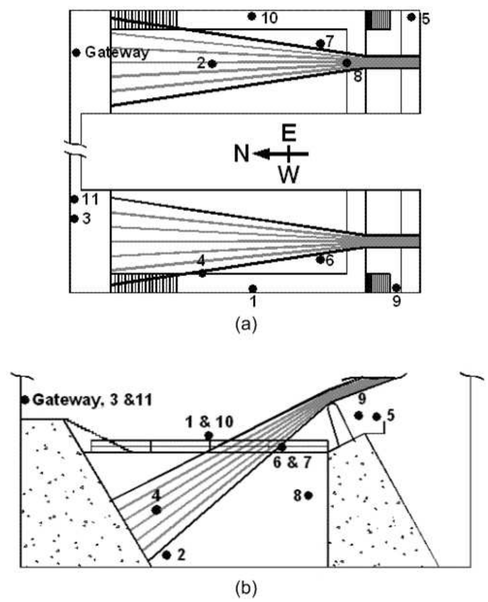

Fig. 1. Layout of the WSN at the hessle anchorage of the Humber Bridge: (a) Plan view of the WSN; (b) Elevation view of the WSN.

Motivated by the asymmetrical structure of WSNs, we propose a novel approach based on CS techniques that aims to reduce the energy consumption of a real WSN shown in fig 1. This WSN is located in the Hessle Anchorage of the Humber Bridge [7] for monitoring 1-D environmental information, i.e., temperature and humidity. As the total power consumption is approximately proportional to the number of samples, the power consumption reduction via CS results from sparser sampling and reduced transmission. Compared with the other two approaches mentioned previously, CS overcomes all the disadvantages with no penalty at the sensor, although the FC has to make a significant effort for signal recovery.

The main contributions of this paper are summarized as follows. Firstly, we propose a practical random sampling generator considering the causality of sampling and hardware limitations, where the parameters of the generator can be adjusted to trade-off the randomness of sampling against fast reconstruction. Secondly, the proposed approach does not need any prior knowledge about the monitored signal to determine a suitable sampling rate, and sensors are able to adjust their sampling rates to make the reconstruction reliable for signals with time-varying sparsity levels. Simulations show that under a tolerable level of reconstruction error, the power consumption owing to both data acquisition and transmission can be significantly reduced by the proposed approach.

II. Compressive Sensing Overview

According to the Shannon sampling theorem, the sampling rate should be no less than twice the maximum frequency in the signal. Actually, the twice oversampling rate is a worst case bound. Most natural signals can be transformed to another space, where a small number of the coefficients represent most of the power of the signals, e.g., audio signals can be transformed into the frequency domain, images can be represented by a discrete cosine transform (DCT) or transformed into the wavelet domain.

CS is an alternative sampling theory, which asserts that certain signals can be recovered from far fewer samples than Shannon sampling uses. The idea of CS is that a signal $\mathbf{f} \in \mathbb{R}^N$ can be recovered from a small set of M ($M \ll N$) non-adaptive, linear measurements $\mathbf{y} \in \mathbb{R}^M$ if the signal can be represented as a sparse objective $\mathbf{x} \in \mathbb{R}^N$ in some orthonormal basis $\mathbf{\Psi} \in \mathbb{R}^{N\times N}$. The sampled signal via CS can be presented as

$$\mathbf{y} = \mathbf{\Phi}\mathbf{f} + \mathbf{z} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{x} + \mathbf{z}, \tag{1}$$

where $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ represents a sensing matrix and $\mathbf{z}$ is an unknown noise term.

The success of CS relies on two objective conditions, i.e., sparsity and incoherence. Sparsity makes it possible to abstract the signal with less samples than the Shannon sampling theory requires. We say the signal $\mathbf{f}$ is S sparse if $\mathbf{x} \in \mathbb{R}^N$ has only S nonzero elements. CS can also be used to approximately reconstruct a nearly sparse signal with power-law distributions, i.e., the ith largest entry of the transformed representation satisfies

$$|x_i| \le C \cdot i^{-p} \tag{2}$$

for each $1 \le i \le N$, where C is a constant and $p \ge 1$. In addition, incoherence between the sensing matrix $\mathbf{\Phi}$ and the transform system $\mathbf{\Psi}$ is also of crucial importance for CS. Random matrices are largely incoherent with any fixed basis [5], which makes CS a general strategy for sampling.

Since it is a linear program (LP) [4], ℓ_1 minimization is widely used for CS signal reconstruction, while ℓ_0 minimization is computationally intractable. One form of reconstruction using the ℓ_1 minimization is basis pursuit de-noise (BPDN) [8], which can be written as

$$\begin{aligned} \min_{\hat{\mathbf{x}}} \quad & \|\hat{\mathbf{x}}\|_{\ell_1} \\ \text{s.t.} \quad & \|\mathbf{\Phi}\mathbf{\Psi}\hat{\mathbf{x}} - \mathbf{y}\|_{\ell_2} \le \epsilon, \end{aligned} \tag{3}$$

where ϵ is an estimate of the noise level. In [9], Candès shows that CS is robust to the effect of noise since the solution $\mathbf{x}^*$ of (3) obeys

$$\|\mathbf{x}^* - \mathbf{x}\|_{\ell_2} \le C_0 S^{-1/2}\|\mathbf{x} - \mathbf{x}_S\|_{\ell_1} + C_1\epsilon, \tag{4}$$

where $C_0 = \frac{2+(2\sqrt{2}-2)\delta_{2S}}{1-(\sqrt{2}+1)\delta_{2S}}$, $C_1 = \frac{4\sqrt{1+\delta_{2S}}}{1-(\sqrt{2}+1)\delta_{2S}}$, $\mathbf{x}_S$ is an approximation of $\mathbf{x}$ with all but the S-largest entries set to zero, and δ_{2S} is the restricted isometry constant (RIC) [9] of matrix $\mathbf{\Phi}\mathbf{\Psi}$.

Another form of reconstruction in the presence of noise is known as the least absolute shrinkage and selection operator (LASSO) [10], which instead minimizes the energy of detection error with an ℓ_1 constraint:

$$\begin{aligned} \min_{\hat{\mathbf{x}}} \quad & \|\mathbf{\Phi}\mathbf{\Psi}\hat{\mathbf{x}} - \mathbf{y}\|_{\ell_2}^2 \\ \text{s.t.} \quad & \|\hat{\mathbf{x}}\|_{\ell_1} \le \eta, \end{aligned} \tag{5}$$

where $\eta \ge 0$. Both BPDN and LASSO can be written as an unconstrained optimization problem for some $\tau \ge 0$ for any $\eta \ge 0$ and $\epsilon \ge 0$:

$$\min_{\hat{\mathbf{x}}} \quad \frac{1}{2}\|\mathbf{\Phi}\mathbf{\Psi}\hat{\mathbf{x}} - \mathbf{y}\|_{\ell_2}^2 + \tau\|\hat{\mathbf{x}}\|_{\ell_1}. \tag{6}$$

III. The Proposed Compressive Sensing Approach

In this section, we present a novel CS approach for a WSN to monitor 1-D environmental information. The proposed approach includes three main process, i.e., random sampling at the sensor, CS reconstruction at the FC and sampling rate indicator (SRI) feedback, as shown in fig 2.

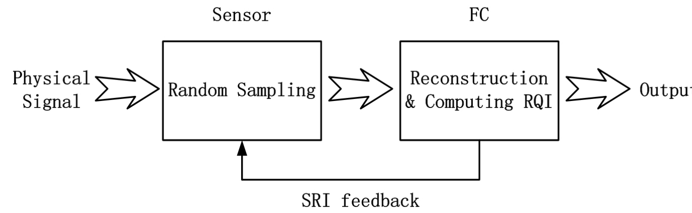

Fig. 2. The proposed CS approach.

A. Random Sampling

The technique to be used here is known as random sampling, which was successfully applied in [11], [12]. Sampling at uniformly distributed random time points satisfies the restricted isometry property (RIP) [9] when the sparse basis $\mathbf{\Psi}$ is orthogonal [13]. For random sampling 1-D signals, the entries of the sensing matrix $\mathbf{\Phi}$ are all zeros except for M entries in M different columns and rows. To maintain the causality of the sampling process, the order of the M unity entries in different rows should be sorted by the column number, e.g.,

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ & & \vdots & & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$

However, randomized sampling cannot be applied directly into a real WSN since two sampling times may be too close to be handled by the hardware. To overcome this, Dang, et al. generate random indexes in a short time period and scale them to a large time scale [12]. To increase the sampling time randomness, they embed a normally distributed time jitter to the result. The sampling time $\mathbf{t} \in \mathbb{R}^{M\times 1}$ in [12] can be expressed by the following equation:

$$\begin{aligned} \mathbf{t} =& \alpha \times \texttt{randsample}\left(\frac{N}{\tau}, M\right) \\ &+ \beta \times \texttt{round}(\texttt{randn}(M, 1)), \end{aligned} \tag{7}$$

where α is the down sampling factor, N is the number of the maximum samples limited by the hardware, M the number of samples we actually take, β is scaling factor of the jitter, function $\texttt{randsample}(\frac{N}{\tau}, M)$ is randomly picking M numbers from 1 to $\frac{N}{\tau}$ and function $\texttt{randn}(M, 1)$ is generating an $M \times 1$ matrix with i.i.d. normally distributed entries satisfying $\mathcal{N}(0, 1)$. Another simpler approach to solve this issue is using the additive random sampling process [14]. In this case, the sampling time is

$$t_i = t_{i-1} + \alpha_i, \tag{8}$$

where $i \in [1, M]$, $t_0 = 0$, α_i is an i.i.d. Gaussian random variable$\sim \mathcal{N}(\frac{N}{M}, \frac{r^2N^2}{M^2})$ where the constant r determines the speed of convergence. The authors of [14] use $r = 0.25$ in their implementation. Although this randomized sampling approach is simple, it inhibits the use of fast Fourier transform (FFT) in recovery algorithms, since FFT requires the randomized sampling intervals to be equal to one or several fixed time units.

To trade-off the randomness of sampling against faster computation, the sampling intervals should be one or several multiples of a given unit. Let ε denote the minimum sampling interval that the hardware can use and μ be a fixed positive integer. A large value of μ trade-offs randomization for the convenience of the FC. Now assume $\frac{2N}{M}$ is a positive integer. We suggest a simpler approach for sampling that can be written as follows

$$t_i = t_{i-1} + \left\lceil \gamma_i \left(\frac{2N}{M} - 1\right)\right\rceil \mu\varepsilon, \tag{9}$$

where γ_i is an i.i.d. random variable$\sim \mathcal{U}(0, 1)$ and function $\lceil a \rceil$ gives the smallest integer no less than a. Note that the expected value of the sampling interval is approximate to $\frac{N}{M}\mu\varepsilon$ for this random sampling scheme.

We define $SRI = \frac{N}{M}$. The sensor does not require prior information about the sparsity level of the signal to determine the value of SRI. At the beginning, the sensor sends its pseudo-random generator seed to the FC. Then it samples the unknown signal at its highest rate and sends the samples to the FC until the SRI feedback is received. The SRI is determined from the reconstruction quality indicator (RQI) that is calculated at the FC, and the SRI is set to maintain the RQI within a specialized range. The calculation of RQI and the design of the range will be explained later in detail. When the RQI goes out of the range, the sensor will be notified to increase or decrease its random sampling rate via the SRI until the RQI again becomes acceptable. This scheme enables the sensor to sample any 1-D environmental signal blindly and then adaptively adjust its sampling rate when unanticipated changes of the sparsity level of the monitored signal occur.

B. CS On-line Reconstruction

CS algorithms can recover an off-line signal that has a sparse representation with relatively few samples. Employing the same approach to deal with an on-line signal such as the instantaneous temperature of the environment, the FC starts each reconstruction when enough new samples are gathered. Thus, the latency of this approach is the time taken to gather the data. However, many applications of interest for sensor networks require timely feedback based on the latest information concerning the environment. The off-line approach that has a long response delay is not suitable for these applications.

Ideally, for an on-line application, the FC should make the reconstruction and report the update when it receives each new sample. The response delay of this ideal method is given by the random sampling interval. To reconstruct a signal within one sampling interval, the FC must have the power of a super computer even using up to date reconstruction algorithms. To trade-off the response delay for lower computational requirements, one possibility is to perform a reconstruction after receiving several new samples. Both the new samples and a number of prior samples are then used to recover the signal. The period of reconstruction should be chosen to be less than the required delay sensitivity of the specific application, and longer than the time that one reconstruction process consumes.

C. SRI Feedback

The SRI feedback enables the sensor to adjust its sampling rate to keep the reconstruction quality in an acceptable range. However, it appears that we cannot compute the reconstruction quality directly without knowing explicitly the whole signal. Instead, we propose to use some additional samples to evaluate the reconstruction performance. The use of these additional samples can be viewed in a similar way to that of pilot symbols in a communication system.

At the beginning, the FC calculates the sparsity level of the received samples which are sampled at the highest rate of the sensor. If a sparse representation is found, the FC will send a SRI back to the sensor to let it adjust its sampling rate, where the value of SRI should result in an acceptable RQI. Then for the following received data that are randomly sampled, the FC reserve a small portion of the data for calculating the

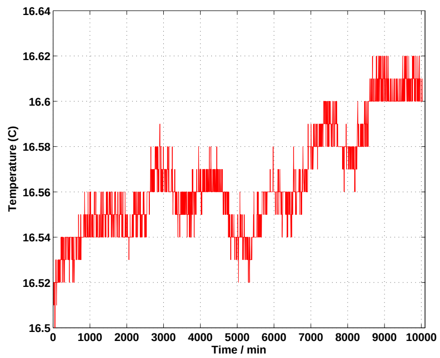

Fig. 3. The first segment of original temperature samples.

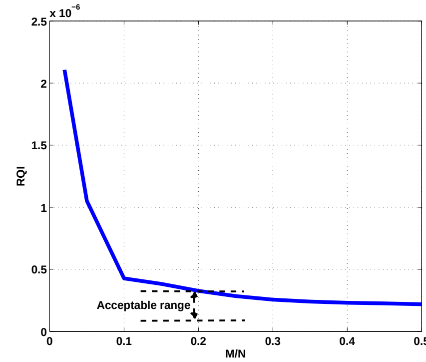

Fig. 4. The RQI trend of the first segment of original temperature samples.

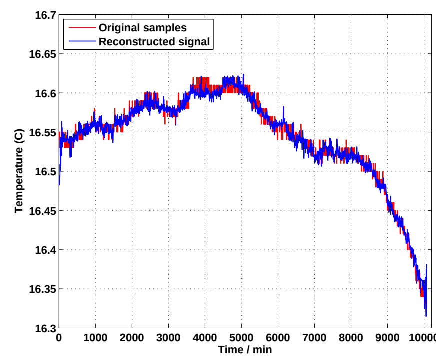

Fig. 5. The second segment of original temperature (red) and reconstructed temperature (blue).

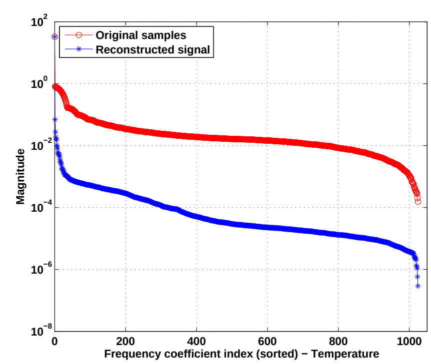

Fig. 6. Sorted Fourier transform coefficients of the second segment of original temperature (red) and reconstructed temperature (blue).

RQI, and uses all the other data for CS reconstruction. With the reconstructed signal, the FC calculates the RQI, which is defined as

$$RQI = \frac{\left\| \hat{\mathbf{f}}_{\mathcal{J}} - \mathbf{y}_{\mathcal{J}} \right\|_{\ell_2}^2}{\left\| \mathbf{y}_{\mathcal{J}} \right\|_{\ell_2}^2}, \tag{10}$$

where $\mathcal{J}$ is the index set of the reserved samples. If the RQI is below the acceptable reconstruction quality, the FC will send an SRI and the sensor will increase its sampling rate. Contrarily, if the RQI is above the upper threshold, the sensor will be notified to decrease the sampling rate to reduce energy consumption.

IV. Performance Evaluation

In this section, we provide multiple examples to show the performance of the proposed CS approach described in the previous section. All the data we use are gathered by the wireless environmental sensor network located in the Hessle Anchorage of the Humber Bridge [7] from 08/09/2007 06:25:00 to 15/10/2007 05:39:46. The temperature and relative humidity of the environment are sensed approximately every 5 minutes, and some of the samples are lost during the transmission. We demonstrate that the proposed approach can significantly reduce the number of samples needed for representing the environmental information under a required reconstruction quality.

In our evaluation, each sampling time is derived from equation (9). We use the interior point algorithm in ℓ_1-magic [15] for reconstruction. Fig 3 shows the first segment of original temperature signal, which has 2000 samples and is then randomly sampled at different rates. In this example, 90% of random samples are used for CS reconstruction, while the remaining 10% samples are reserved for evaluation of the reconstruction quality. We compute the RQI at different sampling rates and average it with 100 independent trials, giving the results shown in fig 4. We notice that the RQI decreases as the sampling rate is increased, i.e., the reconstruction quality is better for at higher rates. However, we also notice that the RQI trend has a floor effect due to the sampling noise in the samples reserved for comparison, which can not be computed or estimated.

For a given RQI requirement, the FC will notify the sensor of the SRI, which can later be changed if a different reconstruction performance is needed. In this example, we require that the RQI is less than 3×10^{-7} and higher than 1×10^{-7}. For the second segment of the temperature signal, the sensor reduces its random sampling rate to one fourth of its prior rate, i.e., 500 samples out of 2000 original samples, where 90%

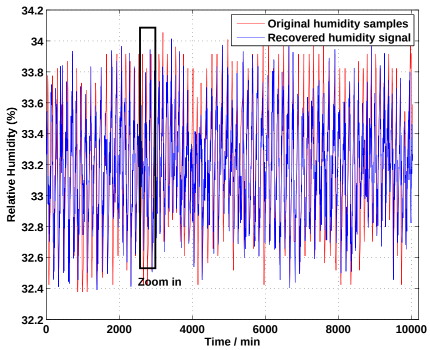

Fig. 7. Original relative humidity (red) and reconstructed relative humidity (blue) via the proposed approach.

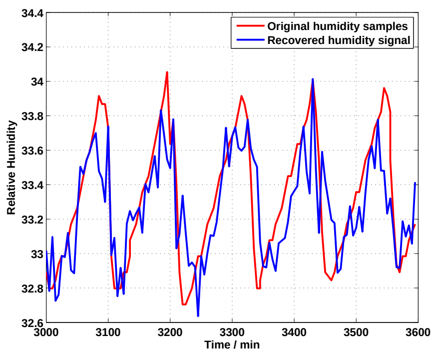

Fig. 8. Original relative humidity (red) and reconstructed relative humidity (blue) via the proposed approach.

are used for reconstruction and the other 10% are reserved for calculating the RQI.

Fig 5 plots the recovered temperature and the original samples for comparison, where the RQI is 2.8×10^{-7}. Since the RQI is acceptable, no SRI feedback is given to the sensor so the sensor maintains this sampling rate. It shows that the recovered result closely approximates the original signal. Fig 6 plots the Fourier transform coefficients of the original temperature and the reconstructed temperature. The coefficients are sorted in the order of magnitude for enhanced visibility. We note that the original temperature coefficients are nearly sparse and the large coefficients are conserved in the CS approach. For this example, we do not give more results for the remaining temperature signal. We point out that for some scenario that the sparsity level of the signal may vary very little in time, consequently the FC only need to send the SRI occasionally when its value is out of the acceptable range.

Fig 7 show the recovered and the original relative humidity samples, while fig 8 zooms into a small range of fig 7 to aid visibility. Again, reasonable agreement between recovered result and original signal is observed.

V. Conclusion

In this paper, we propose a novel approach based on CS theory to reduce energy consumption of a real wireless sensor that is monitoring 1-D environmental information. Because of the smoothness and periodicity of 1-D signals of interest, they have a nearly sparse representation in the frequency domain, which makes compression possible. The proposed techniques include random sampling and the SRI feedback. In practical applications, the hardware limits the minimum sampling interval, while the fast Fourier transform (FFT) operation at the FC restricts the randomization since it requires all sampling intervals to be a multiple of one or several fixed time units. Our random sampling solution trades off the randomization and computational complexity. Further more, it is not necessary for the sensor to acquire any prior knowledge about the signal. The SRI feedback enables the sensor to have the ability to adjust its random sampling rate to maintain the reconstruction quality and reduce energy consumption. We evaluate the performance of the proposed approach using real temperature and relative humidity data gathered by sensors. By employing these techniques, energy consumptions in both data acquisition and transmission can be significantly reduced. The proposed approach can be applied in other WSN application monitoring 1-D environmental information, although the energy savings will vary for different scenarios and applications.

VI. Acknowledgments

The authors would like to thank Andrew Rice and Robert Harle for their valuable comments on this work.

References

[1] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Computer networks*, vol. 38, no. 4, pp. 393–422, 2002.

[2] T. Hamamoto, S. Nagao, and K. Aizawa, "Real-time objects tracking by using smart image sensor and FPGA," in *Image Processing. 2002. Proceedings. 2002 International Conference on*, vol. 3. IEEE, 2002.

[3] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[4] R. Baraniuk, "Compressive sensing," *IEEE Signal Processing Magazine*, vol. 24, no. 4, pp. 118–121, 2007.

[5] E. Candès and M. Wakin, "People hearing without listening: An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[6] J. Haupt, W. Bajwa, M. Rabbat, and R. Nowak, "Compressed sensing for networked data," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 92–101, 2008.

[7] "Humber bridge, http://www.bridgeforum.com/humber/hessle-anchorage-cal.php."

[8] S. Chen, D. Donoho, and M. Saunders, "Atomic decomposition by basis pursuit," *SIAM review*, vol. 43, no. 1, pp. 129–159, 2001.

[9] E. Candès, "The restricted isometry property and its implications for compressed sensing," *Comptes rendus-Mathématique*, vol. 346, no. 9-10, pp. 589–592, 2008.

[10] R. Tibshirani, "Regression shrinkage and selection via the lasso," *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 58, no. 1, pp. 267–288, 1996.

[11] F. Boyle, J. Haupt, G. Fudge, and C. Yeh, "Detecting signal structure from randomly-sampled data," pp. 326–330, 2007.

[12] T. Dang, N. Bulusu, and W. Hu, "Lightweight Acoustic Classification for Cane-Toad Monitoring," 2008.

[13] M. Rudelson and R. Vershynin, "Sparse reconstruction by convex relaxation: Fourier and Gaussian measurements," pp. 207–212, 2006.

[14] Z. Charbiwala, Y. Kim, S. Zahedi, J. Friedman, and M. Srivastava, "Energy efficient sampling for event detection in wireless sensor networks," pp. 419–424, 2009.

[15] E. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on information theory*, vol. 52, no. 2, pp. 489–509, 2006.