EFFECTIVE CHANNEL CODING USING QUATERNARY ENCODERS AND CONTINUOUS PHASE MODULATION

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ABSTRACT

A generic CPM scheme can be decomposed into a continuous-phase encoder (CPE) and a memoryless modulator (MM). The CPE can be combined with a channel encoder (CE). This creates an extended CE, as the CPE is now an extension of the CE. In this paper, a ½ rate quaternary CE is combined with a quaternary CPE. The CE used here is a convolutional encoder, thus, the extended CE is also a convolutional encoder. The paper explains the design of an effective serially interleaved concatenated channel coded scheme, where the inner encoder is the extended CE and the outer encoder is a convolutional encoder. A BER of $2.8X10^{-4}$ *at 0.6dB SNR in an AWGN channel is obtainable.*

Keywords: Concatenated, Convolutional Encoders, CPE, CPM, extended CE, Viterbi Decoder

I. INTRODUCTION

Continuous phase modulation (CPM) schemes constitute a class of constant-envelope modulation schemes. The constant-envelope nature of the signals make these schemes suitable for transmission over non-linear and/or fading channels, such as satellite and mobile radio channels. Moreover, CPM signals have good spectral properties due to their phase continuity.

Besides providing spectral economy, CPM schemes exhibit a "coding gain" when compared to PSK modulation. This "coding gain" is due to the memory that is introduced by the phase-shaping filter and the decoder can exploit this. CPM modulation exhibits memory that resembles in many ways how a convolutionally encoded data sequence exhibits memory - in both cases, a "trellis" can be used to display the possible output signals.

The work of [1] had suggested that CPM could be decomposed into two parts: one a continuous phase encoder (CPE) with memory, and the other a memoryless modulator (MM). Such decomposition has two advantages [2]. Firstly, the "encoding" operation can be studied independently of the modulation. The second advantage is that the isolation of the MM would allow the cascade of the MM, the waveform channel (e.g. additive white Gaussian noise (AWGN)) and the demodulator to be modeled as a discrete memoryless channel.

In [2, 3, 4], Rimoldi derived a generic decomposition model of an *M*-level CPFSK, comprising a continuous-phase encoder (CPE) and a memoryless modulator (MM). He showed that the CPE is a linear (modulo some integer *M*) time-invariant encoder and the MM another time-invariant device. It is then of interest to optimally combine a convolutional coder with the CPE.

In [5] it is explained how a convolutional encoder can be combined with the CPE without the use of a mapper. This is provided, among other conditions, both the encoder and the CPE operate over the same algebra and the modulation index of the CPE is 1/*M*. This is unlike the usual approach discussed in [6, 7] where mappers are pertinent. We, thus, combine a 4-ary convolutional encoder, hereafter called a *channel encoder* (CE), with a 4-ary CPE. As the CE and the CPE now operate over the same algebra, no mapper is required. This also enables the state of the CPE to be fed back and be used by the CE. This allows the use of a CE with a shorter constraint length. Such a combination will be called an *extended CE* as the CPE is now an extension of the CE and it is effectively a convolutional encoder.

In this paper, a ½ rate CE over the ring of integers modulo 4 is combined with a 4-ary CPM scheme. More specifically, a 2 raised cosine (2*RC)* scheme with a modulation index of $h = ¼$ is considered. The study concentrates on designing an effective serially interleaved concatenated channel coding scheme, where the inner encoder is the extended CE and the outer encoder is another convolutional encoder. Hard decoding techniques based on the Viterbi algorithm are investigated.

The design of each component is described in Section II. Section III discusses the simulations executed and the results obtained. A conclusion is given in Section IV.

II. SYSTEM DESIGN

The structure of the coding scheme is shown in Figure 1:

a. The outer encoder is a quaternary convolutional encoder while the inner encoder is the extended CE. No mapper is required between the two encoders as both of them operate over the same algebra structure.

b. Both the inner and outer quaternary decoders perform hard decision decoding based on the Viterbi algorithm.

c. A 10 X10-block interleaver/ deinterleaver is used.

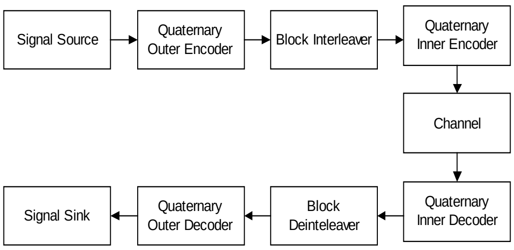

Figure 1. Block diagram of the serially interleaved concatenated coding system.

A. Design of the extended CE

Figure 2 shows the design of the extended CE. For the purpose of system evaluation, a 2*RC* (partial-response) scheme with h = ¼ ($M = 4$) was implemented using the baseband decomposed CPM model. The CPE has two memory (delay) cells. One cell stores the previous transmitted symbol while the other memory cell stores the sum (mod 4) of all previously transmitted symbols.

The channel encoder (CE) is a quartenary convolutional encoder. It has 3 memory cells (memory cells D1 and D2 in Figure 2 form part of the CPE). The total number of states generated by the extended CE will be M^v where M is the alphabet size of the input symbol and v (3 in this case) is the constraint length of the extended CE. Hence, the coded quaternary scheme will need to process 4^3 = 64 states for each received channel symbol. As there are 4 branches/waveforms emanating from and arriving at each state and each "analog" waveform is made of 8 samples, a total of 8 X 4 X 64 = 2048 waveform samples would require processing for the extended CE at each iteration of the Viterbi algorithm.

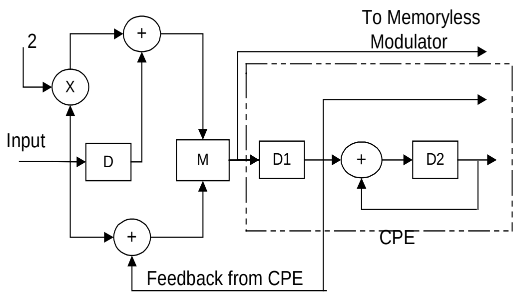

Figure 2. ½ rate extended CE with feedback from the CPE. D denotes a memory cell while M denotes a multiplexer. Additions are all modulo 4. D1 and D2 are the memory cells of the CPE.

B. Design of the Memoryless Modulator

The raised cosine (*RC*) phase shaping function is implemented using two finite impulse response (FIR) digital filters, namely FIR filter A and FIR filter B as shown in Figure 3. In this study, each output waveform is made up of eight samples.

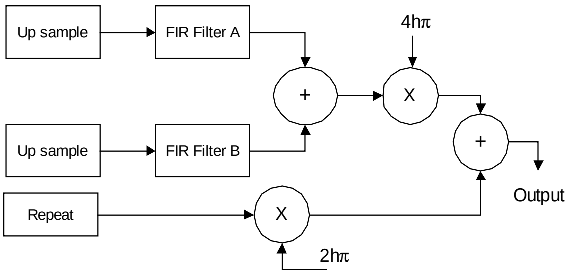

Figure 3. Block diagram of the memoryless modulator. The Up Sample and Repeat blocks increase the samples of the input signals by a factor of 8. h denotes the modulation index.

C. Design of the Outer Encoders

The outer encoder is a quaternary design as shown in Figure 4. It has a constraint length of 3 and a generator polynomial of $(6, 17)_{10}$.

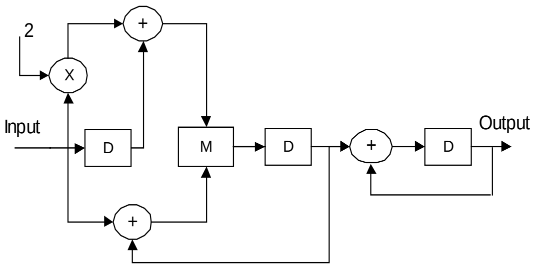

Figure 4. ½ rate quaternary outer encoder. The design is quite similar to the extended CE. Additions are all modulo 4. D and M denote the memory cells and multiplexer respectively.

D. Design of the Viterbi Decoder

A general over view of the Viterbi decoder is shown in Figure 5. Its design is based on the concept in [8]. The Viterbi algorithm has been employed to facilitate maximum-likelihood decoding of the convolutional trellis code. It works by assigning probabilities (or metrics) to the states and branches in the receiver trellis. These probabilities aim to match the path made by the received symbols in the receiver trellis to the path made by the transmitted symbols in the transmitter trellis. In the trellis, all the branches connecting the states correspond to waveforms (from a finite set of waveforms) that in-turn correspond to a given transmitted symbol.

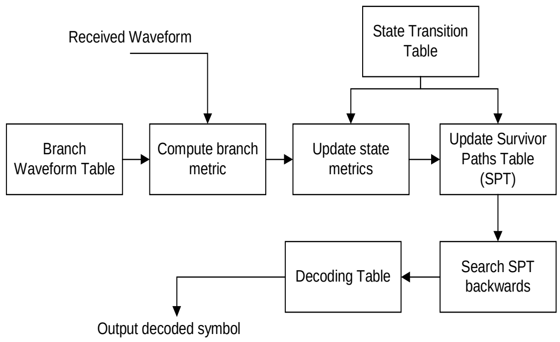

Figure 5. High-level diagram of the Viterbi Decoder

When a waveform is received, (corrupted by additive noise or fading), the Euclidean distances between it and all possible waveforms are calculated. These distances (or branch metrics) serve as a probability that a given waveform/ symbol was actually sent. However, these branch metrics are not used in isolation. They are used in conjunction with the probabilities (state metrics) associated with the states of the trellis that each of the branches departs from. In this way, the Euclidean distance between the entire received sequence and the most likely path through the trellis can be calculated on a symbol-by-symbol basis.

To implement the demodulator, three tables are required. These were generated using the decomposed CPM modulator and are used by the demodulator to decode the transmitted symbol sequence. The three tables are:

a. State Transition Table - which takes as its inputs the current state and symbol and outputs the next state.

b. Waveform Table – which takes as inputs the current state and symbol input and outputs the output waveform.

c. Decoding Table – which takes as its inputs the current state and previous state and outputs the transmitted symbol.

In this design we assume there are no parallel transitions between states. The advantage of using these tables is that the Viterbi demodulator need not be redesigned for each new scheme to accommodate more or less states or a different symbol alphabet size.

III. SIMULATIONS AND RESULTS

The entire coding scheme (encoders, extended CE, Viterbi hard decision decoders) was built in software and tested using Monte Carlo based simulations. The results obtained were in terms of the bit error rate (BER) as a function of the signal to noise ratio (SNR).

The simulations were executed in two types of channels, namely, the additive white Gaussian noise (AWGN) channel and the Rician fading channel. For the simulations in the Rician fading channel, the channel was designed with a Rician parameter (K) of 10. This parameter is defined as:

$$K = \frac{power\ \ of\ \ dominant\ \ path}{power\ in\ scattered\ \ path} \qquad (1)$$

When K = 0, the channel is Rayleigh, and if K is infinite, the channel is Gaussian. The fades have a high probability of being very deep when K = 0 to being very shallow when K = 32 (approaching Gaussian) [9].

In all the simulations, a normalized bandwidth of BT_s = 1.2 was assumed where B is the bandwidth and T_s the symbol duration. For the quaternary 2*RC* scheme, this bandwidth contains approximately 99.97% of the total power. The scheme has 64 states and an overall code rate of ¼ (both the inner and outer encoders have a code rate of ½).

Results of the simulations are shown graphically in Figure 6. A BER of $2.8X10^{-4}$ at 0.6dB SNR is obtainable in an AWGN channel. The scheme clearly shows a superior error correcting capability as compared to other modulation schemes like QPSK. This could be due to:

a. Unlike CPM schemes, QPSK schemes do not exhibit coding gains.

b. The CPM scheme employed here uses the natural ring of integers modulo 4 between the two concatenated encoders. This does away with the use of mappers.

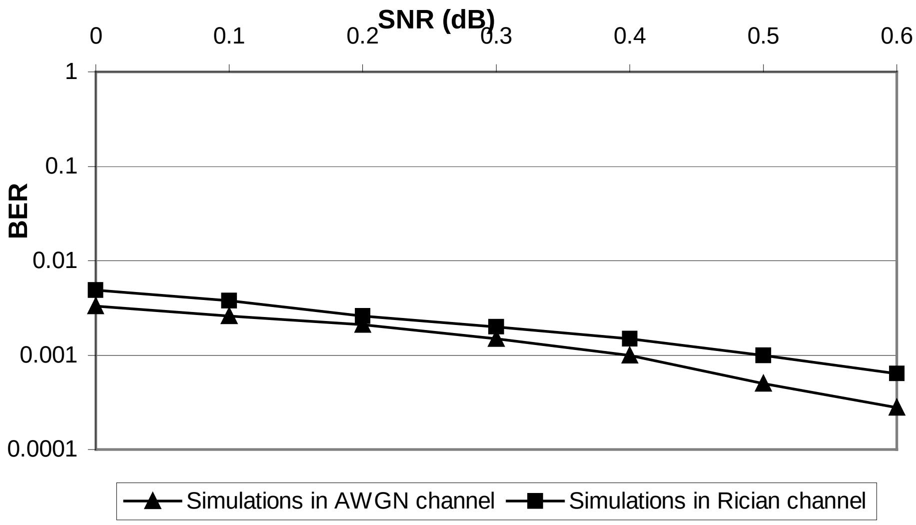

Figure 6. BER as a function of SNR (dB)

IV. CONCLUSION

A serial concatenated interleaved coding scheme has been studied. It consists of the cascade of an outer encoder, an interleaver permuting the outer codeword bits and an inner encoder whose input words are the permuted outer codewords. The scheme was tested using Monte Carlo based simulations over AWGN and Rician channels.

The scheme clearly shows good error correcting capabilities, compared to schemes employing QPSK modulation. The main potential area of application of the scheme is the improvement of already established standards, for example GSM and cellular digital packet data.

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