Characterisation of the Performance of Cooperative Networks in Ricean Fading Channels

Jaime Adeane, Miguel R.D. Rodrigues, and Ian J. Wassell
Laboratory for Communication Engineering
Department of Engineering University of Cambridge
15 JJ Thomson Avenue, Cambridge CB3 0FD, United Kingdom
{ja316, mrdr3, ijw24}@cam.ac.uk

***Abstract*— We consider cooperative networks where two single-antenna users assist each other in their transmissions by relaying their partner's messages. Both users achieve space diversity from sharing their partner's antenna. The target applications of cooperative networks are systems in which multiple antennas cannot be supported due to size limitation and wireless systems operating in very high frequency bands where line of sight (LOS) propagation dominates and little diversity is available from conventional antenna arrays. Most present work concerning cooperative networks has assumed that the propagation channel exhibits Rayleigh fading (e.g., [1], [2], [3], [4], [5]). However, in situations where an LOS component exists the Ricean fading model is more appropriate. Consequently, this paper presents a characterisation of the performance of cooperative networks in Ricean fading channels. It will be shown that in contrast to the situation in Rayleigh fading channels, in Ricean fading channels there is a region where cooperation is not advantageous, even though the inter-user SNRs are reasonably high. This is of interest particularly in high-frequency applications where LOS propagation often predominates. Also, in Ricean channels having low K-factor values, the gain from cooperating is found to be greater than that available in Rayleigh channels.**

Keywords: cooperative networks, Ricean K-factor.

I. Introduction

The major challenges in future wireless communication system are increased spectral efficiency and improved link reliability. The radio channel constitutes a hostile propagation medium, which suffers from fading and interference from other users. The use of multiple antennas at both ends of a wireless link introduces extra spatial degrees of freedom into a wireless communication system, promising significant improvements in terms of spectral efficiency and/or link reliability. With multiple antennas one can increase the data rate without the use of additional bandwidth by transmitting different data streams simultaneously over the various spatial sub-channels which are available in a rich scattering environment (i.e., spatial multiplexing) [6] and/or decrease the bit error rate (BER) without using extra power by employing space-time coding [7].

However, it is not always possible to deploy MIMO systems. In fact, MIMO capacity and diversity gains rely on the independence of multiple paths between the transmit and receive antennas, for which an antenna spacing of several wavelengths is often required. At a frequency of, say 2 GHz, this amounts to several multiples of 15 cm, which is often not practical for a mobile handset. At even higher frequencies (above 5 GHz), the propagation paths are even more correlated due to the presence of a line of sight (LOS) component, and therefore employing traditional multiple antenna arrays is not effective in this situation.

Future broadband wireless communication systems are expected to operate beyond 2 GHz, for example Wireless Local Area Networks (WLANs) at 5.470-5.725 GHz (Hiperlan) or at 2.400 - 2.4835 GHz (ISM bands). In this case, alternative techniques (other than MIMO) have to be exploited in order to guarantee capacity and diversity gains.

Recently, a new diversity method known as cooperative networks has been proposed [1], [2], [3], [4], [5]. In this approach, single-antenna mobiles in a multi-user environment share their antennas in order to generate virtual multiple-antenna arrays that allows them to exploit spatial diversity. Cooperative networks are potentially the key to providing spectral efficiency and/or link reliability in future communication systems for the following reasons: First, limitations in size or hardware complexity of wireless devices sometimes do not permit more than one antenna to be employed [8], and second, the lack of scattering inherent in environments exhibiting Ricean fading (e.g., in frequency bands in excess of 5 GHz where future communication systems are likely to operate [9]) will limit available performance gains.

Although the Ricean channel model has been considered in the study of distributed relay systems [10], most of the related work in cooperative networks to date has only considered the Rayleigh channel model [1], [2], [3], [4], [5]. To better understand the performance of cooperative networks in applications where an LOS path is likely to exist, the work to be presented here assumes the presence of Ricean fading channels. The system performance measures considered in this work are the bit error rate (BER) and the BER gain over direct transmission.

II. System Model

As illustrated in Figures 1 and 2, the system under consideration consists of two users both transmitting to a single destination. The modulation is assumed to be binary phase shift keying (BPSK), and each receiver maintains channel state information and employs coherent detection. The channels between users (interuser channels) and from each user to the destination (uplink channels) are mutually independent and subject to flat fading. Each user is allocated different frequency bands (f_1 and f_2) and in each band a user transmits signals in

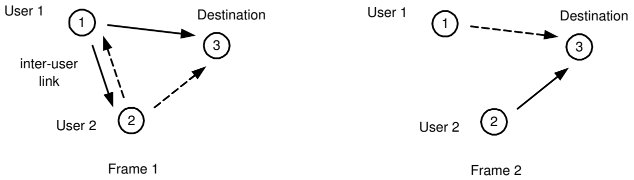

Fig. 1. Cooperation transmission scheme.

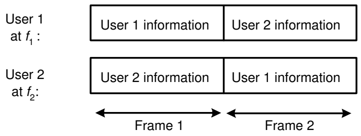

Fig. 2. Implementation of user cooperation using FDMA-TDMA.

two time frames, one frame is dedicated for its own bits and another frame for relaying its partner's bits. We model the baseband-equivalent discrete-time signal transmitted by user $i \in \{1, 2\}$ as

$$d_i(n) = \sqrt{E_b}\ b_i(n) \tag{1}$$

where E_b is the transmitted energy per bit and $b_i(n) \in \{-1, +1\}$ is the BPSK-modulated code bit at time n. The corresponding signal received by user $j \in \{1, 2, 3\}$ ($j \neq i$, and $j = 3$ denotes the final destination) is

$$\begin{aligned} y_{ij}(n) &= h_{ij}(n)d_i(n) + w_j(n) \\ &= h_{ij}(n)\sqrt{E_b}\ b_i(n) + w_j(n) \end{aligned} \tag{2}$$

where h_{ij} is the fading coefficient magnitude between users i and j at time n, which includes the effect of path loss, shadowing, and small-scale fading, and $w_j(n)$ is the noise observed at the receiver, Gaussian distributed with a one sided power spectral density of N_0. For slow (quasi-static) fading, the fading coefficients remain constant over the transmission of each source block ($h_{ij}(n) = h_{ij}$ for $n = 1, \ldots, N$), while for fast fading, they are i.i.d. for each transmitted symbol. We assume that the interuser channels are reciprocal, so that $h_{ij}(n) = h_{ji}(n)$.

A. Non-Cooperative (NC) Scenario

For direct transmission, the signal-to-noise-ratio,

$$\rho_{ij}^{(NC)} = |h_{ij}|^2 \frac{E_b}{N_0} \tag{3}$$

and the probability of error for a given SNR is

$$P_e^{(NC)}|\rho_{ij} = \mathcal{Q}\left(\sqrt{2\rho_{ij}^{(NC)}}\right) = \mathcal{Q}\left(\sqrt{\frac{2|h_{ij}|^2 E_b}{N_0}}\right) \tag{4}$$

where $\mathcal{Q}(.)$ is the standard Gaussian error function. Note that averaging over ρ_{ij} yields the average probability of error.

B. Cooperative Scenario

The relaying process in cooperative networks can be either transparent or regenerative. In transparent relaying, the signal stream is received on one frequency band and simply retransmitted instantaneously (on another frequency band) without being buffered whereas with regenerative relaying, the signal is decoded, re-encoded, and retransmitted. This paper focuses on the so called amplify-and-forward (AF) scheme between two users and one destination, which is an example of a transparent relaying system. The performance of the AF scheme will then be compared to that of a regenerative relaying scheme, specifically the decode-and-forward (DF) scheme.

1) Amplify-and-Forward: For the AF scheme, in the first time frame, user 1 transmits $d_1(n)$ and the received signal at user 2, $y_2(n)$ can be expressed as

$$y_2(n) = h_{12}\sqrt{E_b/2}\ b_1(n) + w_2(n). \tag{5}$$

Note that in the 2-user cooperation scheme, each of transmission time slots is divided into 2 non-overlapping slots, and therefore the transmission duration for each slot is half of that available for the direct transmission scheme. Consequently, to maintain the same total power consumption, the energy available per bit for the cooperative scheme is half of that for the direct transmission scheme.

In the second time interval user 2 amplifies the signal by the relay gain α_2 and transmits:

$$d_2(n+1) \quad = \quad \alpha_2\ y_2(n) \tag{6}$$

The destination, i.e., user 3 receives:

$$y_3(n) = h_{13}\sqrt{E_b/2}\ b_1(n) + w_3(n) \tag{7}$$

$$y_3(n+1) = h_{23}\alpha_2(h_{12}\sqrt{E_b/2}\ b_1(n) + w_2(n)) + w_3(n+1). \tag{8}$$

To amplify the relayed signal to a level similar to that of a user's own power level, one option for the relay gain is [3]

$$\alpha_2^2 = \frac{1}{h_{12}^2 + (N_0/(E_b/2))}. \tag{9}$$

In this case, the received SNR at the destination due to the 2-hop path can be written as

$$\rho_{eq} = \frac{\rho_{12}\ \rho_{23}}{\rho_{12} + \rho_{23} + 1} = f(\rho_{12}, \rho_{23}) \tag{10}$$

where ρ_{ij} is defined as $\rho_{ij} = h_{ij}\ \frac{E_b}{2N_0}$. Combining the received signal from the direct and the 2-hop path for 2 time frames using the Maximum Ratio Combining (MRC) method, the conditional SNR of the combined signal given the SNR values of each link is

$$\rho_{MRC}|\rho_{13}, \rho_{12}, \rho_{23} = \rho_{13} + f(\rho_{12}, \rho_{23}) \tag{11}$$

and the conditional probability of error of the combined signal can be expressed as

$$P_e^{(AF)}|\rho_{12}, \rho_{13}, \rho_{23} = \mathcal{Q}\sqrt{2(\rho_{13} + f(\rho_{12}, \rho_{23}))}. \tag{12}$$

Note that averaging over ρ_{ij} yields the average probability of error.

2) Decode-and-Forward (DF): For the DF scheme, let us again consider the 2-user scenario. Firstly user 1 transmits its information $d_1(n)$ both to the destination and to user 2. Secondly, user 2 which is acting as a relay processes the received signal $y_2(n)$ by decoding and forming an estimate $\hat{d}_1(n)$ of user 1's data. Under a repetition coded scheme, user 2 relays the signal

$$d_2(n+1) = \hat{d}_1(n) \tag{13}$$

and the destination, i.e., user 3 receives

$$y_3(n) = h_{13}\ \sqrt{E_b/2}\ b_1(n) + w_3(n) \tag{14}$$

$$y_3(n+1) = h_{23}\ d_2(n+1) + w_3(n+1). \tag{15}$$

The destination combines the received signal from the direct transmission $y_3(n)$ and the 2-hop path, $y_3(n+1)$ using the Maximum Ratio Combining (MRC) method.

C. Alternative Expression for Probability of Error

For notational simplicity, let us define $\rho_{ij} = |h_{ij}|^2\rho$ and denote the received SNR of an n-hop link with path loss $h_{ij}, h_{i(j+1)}, \ldots, h_{i(j+n)}$ as $\rho_{i(j+1)\ldots(j+n)}$. Using the alternative representation for the Gaussian $\mathcal{Q}$ function [11],

$$\mathcal{Q}(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp(-\frac{x^2}{2\sin^2\theta})d\theta, \quad x \geq 0, \tag{16}$$

the total average probability of error for user 1 can be expressed as [12]

$$Pe_{13}^{(AF)} = \frac{1}{\pi}\int_0^{\pi/2} M_{\rho_{13}}\left(-\frac{a^2}{2\sin^2\theta}\right) M_{\rho_{123}}\left(-\frac{a^2}{2\sin^2\theta}\right) d\theta \tag{17}$$

where $M_X(.)$ is the moment generating function (MGF) of the random variable X, defined as $M_X(x) = \boldsymbol{E}\{e^{sX}\} = \int_0^\infty e^{sx} p_X(x) dy$. Note that for BPSK modulation, $a = \sqrt{2}$.

The probability density function of a Ricean distributed random variable X is given by

$$p_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+A^2}{2\sigma^2}} I_0\left(\frac{Ax}{\sigma^2}\right), x \geq 0. \tag{18}$$

where A is the peak amplitude of the LOS component and $I_0(.)$ is the zero-th order modified Bessel function of the first kind. The Ricean channel is sometimes described using the K-factor, where $K = \frac{A^2}{2\sigma^2}$ is the ratio of the power of the LOS or specular component, to that of the scattered signals or Rayleigh component, and $\sigma > 0$ corresponds to the standard deviation of the constituent real and imaginary Gaussian variables of the Rayleigh component. Observe that when $K = 0$ the Ricean distribution becomes the Rayleigh distribution. The pdf of the SNR per symbol ρ can be expressed as

$$p_\rho(\rho) = \frac{(1+K)e^{-K}}{\overline{\rho}} \exp\left(-\frac{(1+K)\rho}{\overline{\rho}}\right) \times I_0\left[2\sqrt{\frac{K(1+K)\rho}{\overline{\rho}}}\right] \tag{19}$$

where $\overline{\rho}$ is the average SNR per symbol.

The MGF of the received SNR in Ricean fading can be expressed as

$$M_\rho(s) = \frac{(1+K)}{(1+K)-s\overline{\rho}} \exp\left[\frac{sK\overline{\rho}}{(1+K)-s\overline{\rho}}\right]. \tag{20}$$

For Ricean fading channel, let us consider the case where the interuser channel is perfect. Substituting (20) into (17), the average BER for user 1 in the AF scenario can be expressed as

$$Pe_{13}^{(AF)} = \frac{1}{\pi}\int_0^{\pi/2} \frac{(1+K)\sin^2\theta}{(1+K)\sin^2\theta + \overline{\rho}_{13}} \exp\left(-\frac{K\ \overline{\rho}_{13}}{(1+K\sin^2\theta + \overline{\rho}_{13})}\right) \frac{(1+K)\sin^2\theta}{(1+K)\sin^2\theta + \overline{\rho}_{23}} \exp\left(-\frac{K\ \overline{\rho}_{23}}{(1+K\sin^2\theta + \overline{\rho}_{23})}\right) d\theta. \tag{21}$$

As a comparison, the average BER of a direct transmission in a non cooperative case from user 1 to the destination (node 3) can be expressed as

$$Pe_{13}^{(NC)} = \frac{1}{\pi}\int_0^{\pi/2} \frac{(1+K)\sin^2\theta}{(1+K)\sin^2\theta + 2\overline{\rho}_{13}} \exp\left(-\frac{2K\overline{\rho}_{13}}{(1+K\sin^2\theta + 2\overline{\rho}_{13})}\right) d\theta. \tag{22}$$

In the following section, the benefit of cooperation is quantified in terms of the BER gain at a particular SNR

$$\text{BER gain} = \frac{Pe_{ij}^{(NC)}}{Pe_{ij}^{(C)}} \tag{23}$$

where $Pe_{ij}^{(NC)}$ is the probability of error for the direct transmission from user i to j and $Pe_{ij}^{(C)}$ is the probability of error for the cooperative scenario from user i to j. For the AF scenario investigated here, $Pe_{ij}^{(C)}$=$Pe_{ij}^{(AF)}$. Comparing (21) and (22), it can be seen that for the case where the inter-user channel is perfect the BER gain falls to one as $K \longrightarrow \infty$. Therefore, we can expect that for an imperfect inter-user channel, the BER gain decays to a value less than one as $K \longrightarrow \infty$.

III. Numerical Simulations

To illustrate the performance of the amplify-and-forward (AF) method, numerical simulations have been performed. Figs. 3 and 4 show the BER performance in slow fading Rayleigh and Ricean channels, respectively, for the case of statistically similar uplink channels (i.e., $\overline{\rho}_{13} = \overline{\rho}_{23}$). It can be seen that in the Rayleigh case, AF with a perfect inter-user channel (i.e., an SNR of 100 dB) achieves a diversity order of close to 2, whereas the non-cooperative case has a diversity order of 1. The diversity gain decreases as the inter-user channel SNR worsens, as can be seen from the slope of the curves in Fig. 3. For the Ricean case, shown in Fig. 4, the diversity order very much depends on K. As $K \longrightarrow \infty$ the channel will approximate an AWGN channel, in which case cooperation is not advantageous as it may add noise due to imperfect inter-user channel. From Figs. 3 and 4, it can be concluded that cooperation tends to increase the diversity order, especially when the inter-user channel is good. In other words, the inter-user channel limits the achievable diversity advantage over direct transmission.

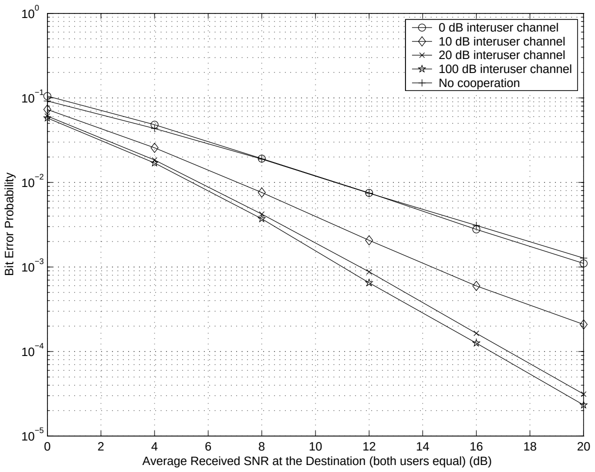

Fig. 3. BER performance of amplify-and-forward (AF) for symmetrical uplinks with a Rayleigh fading channel model.

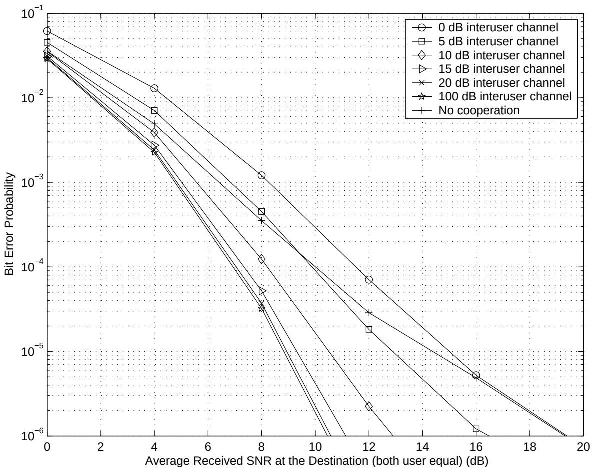

Fig. 4. BER performance of amplify-and-forward (AF) for symmetrical uplinks with a Ricean fading channel model ($K = 10$).

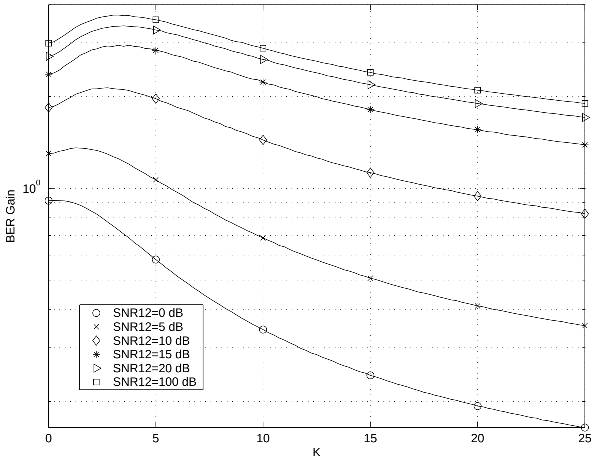

Fig. 5. BER gain of amplify-and-forward (AF) over direct transmission for symmetrical uplinks, $\overline{\rho}_{13} = \overline{\rho}_{23} = 5$ dB and various inter-user link SNRs.

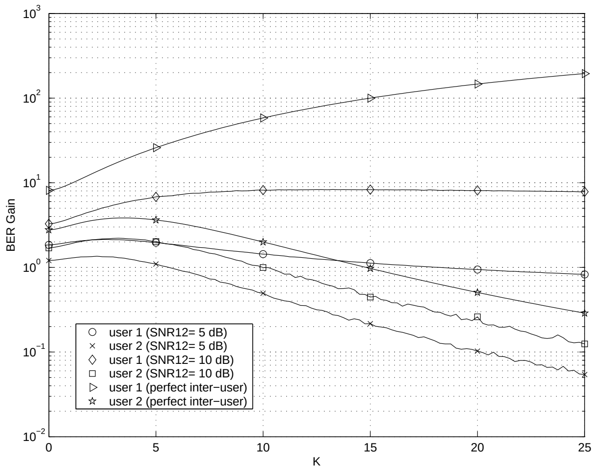

Fig. 6. BER gain of amplify-and-forward (AF) over direct transmission for asymmetrical uplinks, $\overline{\rho}_{13} = 5$ dB, $\overline{\rho}_{23} = 10$ dB and various inter-user link SNRs.

Fig. 5 shows the BER gain owing to user cooperation over direct transmission for symmetrical links as the Ricean K-factor is varied. From this figure, it can be seen that in a Rayleigh channel (i.e., the case when $K = 0$), there is always a large incentive to cooperate when the inter-user channel is reasonably good (and often some small incentive when the inter-user channel is poor). Fig. 5 also shows that in the Ricean channel, as the Ricean K-factor increases, the BER gain from cooperation first increases to a peak value and then decays as K is further increased towards ∞. Note that as $K \longrightarrow \infty$, the channel is effectively AWGN and thus cooperating is not advantageous since it only results in the addition of extra noise from the inter-user channel. For the Ricean channel it can be observed that the minimum inter-user link SNR required to make user cooperation advantageous is higher than is the case for the Rayleigh channel. For a perfect inter-user channel case the BER gain decays to one as $K \longrightarrow \infty$ as can be seen by comparing Eqs. (21) and (22). For an imperfect inter-user channel, the gain decays to a value less than one as $K \longrightarrow \infty$. Note that both Figs. 5 and 6 show that the maximum BER gain in the Ricean channel case is higher than that for the Rayleigh channel and that this normally occurs at low values of K.

The results for an asymmetrical case presented in Fig. 6, show similar trends and once again, cooperation is only advantageous at low to medium values of K. However, note than in the asymmetrical uplink case, the user with the weaker uplink enjoys most of the benefit of cooperation. It can be seen particularly at high K values, inter-user link quality plays a very significant role in determining the benefit/gain of cooperating (i.e., the spread of the curves at high K values increases).

For completeness of our study, we have also conducted simulations for the decode-and-forward (DF) protocol. As can be seen from Figs. 7 and 8, for the DF protocol, diversity gain over direct transmission at high SNR values can only be achieved when the inter-user link SNR is almost perfect (e.g., 100 dB), owing to the fact that the link capacity is limited by the minimum link SNR of the 2-hop link. Also, when the inter-user channel is poor, the BER performance of the DF scheme at high SNR can be worse than that due to direct

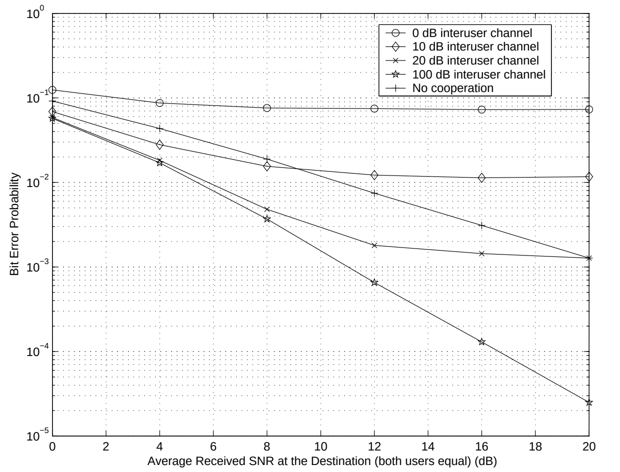

Fig. 7. BER performance of decode-and-forward (DF) for symmetrical uplinks with a Rayleigh fading channel model.

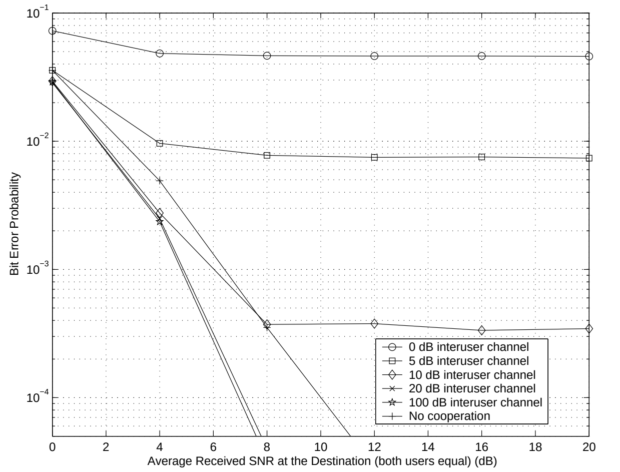

Fig. 8. BER performance of decode-and-forward (DF) for symmetrical uplinks with a Ricean fading channel model.

transmission. Comparing Figs. 4 and 8, it can be seen that at $K = 10$, for low to moderate quality inter-user link, the case for not cooperating is stronger in the DF scheme than in the AF scheme. From Figs. 5 and 9, similar patterns of BER gain in the AF and DF schemes can be observed, however the DF scheme is more sensitive to the inter-user channel quality than is the AF scheme. To achieve reasonable gain at low to medium SNR values, the DF scenario needs better inter-user quality than does AF.

IV. Conclusion

In this paper, the performance of cooperative networks in Ricean fading channels is evaluated. In some cases, for example in some existing fixed wireless systems, there is an LOS path from the basestation to the subscriber units. In particular, it is shown that cooperating is only advantageous at low to medium values of K. In Ricean channels having low K-factor values, the gain from cooperating is found to be greater than that available in Rayleigh channels. However, there is a high K value region where cooperation is not advantageous

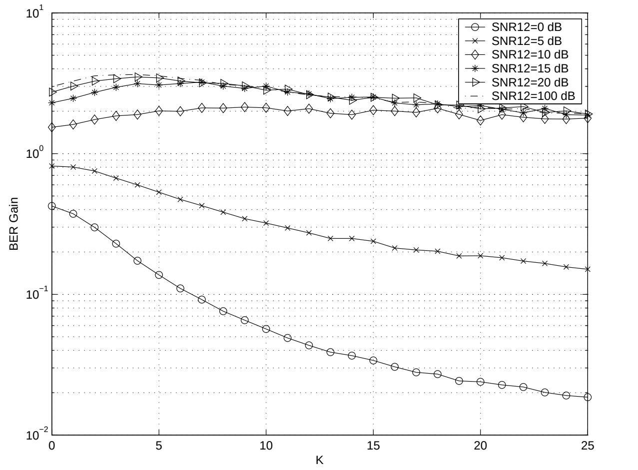

Fig. 9. BER gain of decode-and-forward (DF) over direct transmission for symmetrical uplinks, $\overline{\rho}_{13} = \overline{\rho}_{23} = 5$ dB and various inter-user link SNRs.

even though the interuser SNRs are reasonably high. The significance of the findings is that Ricean fading channels having high K factors are often encountered in high frequency wireless networks, which is considered a target application for cooperative networks. In this case, cooperation between nodes may not be as advantageous as may have been expected based on previous work conducted using Rayleigh channels.

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