Optimum Power Allocation in Cooperative Networks

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***Abstract*— Cooperation among network users provides transmit diversity in cases where wireless transmitters, due to size and power limitation, cannot support multiple antennas. We consider cooperation between M users, where each user achieves space-time diversity by using other users' antennas as relays. Cooperation among users has been shown to achieve impressive gains as compared to a non-cooperative system while maintaining the same information rate, transmit power, and bandwidth [1], [2], [3], [4]. This paper formulates an optimum power allocation scheme appropriate for a cooperative network using transparent relaying. It will be shown that the proposed allocation scheme significantly outperforms the equal power allocation scheme, e.g., by up to 7 dB at a bit error rate of 10^{-3}. The results presented here may serve as bounds for the performance assessment of sub-optimal (but practical) algorithms.**

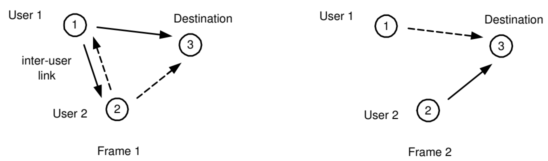

Fig. 1. Cooperation transmission scheme.

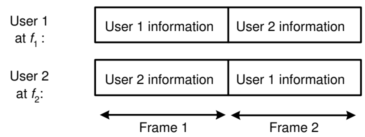

Fig. 2. Implementation of user cooperation using hybrid FDMA-TDMA.

I. Introduction

The relaying process in cooperative networks can be either transparent or regenerative. In transparent relaying, the signal stream is received in one frequency band and simply retransmitted in another band, whereas with regenerative relaying the signal is decoded, re-encoded, and retransmitted. Since a mobile terminal cannot relay information at the same time and in the same frequency band, practical multiple access methods are frequency division multiple access (FDMA), time division multiple access (TDMA), hybrid FDMA-TDMA, and space division multiple access (SDMA). In this paper, it is assumed that an orthogonal bandwidth (FDMA) and time slot (TDMA) (i.e., hybrid FDMA-TDMA) allocation with transparent relaying scheme is used.

The end-to-end performance of the system in terms of e.g., bit error rate (BER) and capacity, depends on the performance of each relaying node, which itself is dictated by the allocation of time slots, bandwidth, and transmission power. An optimum resource allocation system assigns different fractional bandwidth, time slots, and power for each transmission between two nodes. In [5], the authors consider a user cooperation structure where spatially adjacent mobiles are allowed to communicate with each other and are considered as one group.The problem formulation and solutions for power control in this network structure using a regenerative relaying scheme are presented in [5].

In this paper, the case of transparent relaying with an arbitrary inter-user channel signal-to-noise-ratio (SNR) will be examined. A method to optimise the end-to-end BER performance is proposed and solutions obtained using numerical optimisation at various average SNR values are presented. The performance of the proposed resource allocation strategy is found to be effective for an arbitrary number of cooperating users. Comparisons with an equal power/bandwith allocation strategy is provided in the numerical simulation section.

II. System Model

The cooperative scenario illustrated in Figures 1 and 2 shows an amplify-and-forward (AF) scheme between two users and one destination. The modulation is assumed to be binary phase shift keying (BPSK), and each receiver maintains channel state information and employs coherent detection. The channels between users (interuser channels) and from each user to the destination (uplink channels) are mutually independent and subject to flat fading. Each user is allocated different frequency bands (f_1 and f_2) and in each band a user transmits signals in two time frames, one frame is dedicated for its own bits and the other is for relaying the partner's bits. In the first time interval, user 1 transmits $b_1(n)$ and the received signal at user 2, $y_2(n)$ can be expressed as

$$y_2(n) = h_{12}\sqrt{E_b/2}\; b_1(n) + w_2(n), \qquad (1)$$

where b_1 is a BPSK signal with a unit energy, h_{ij} captures the effect of path loss and static fading on transmissions from radio i to radio j, and $w_j(n)$ models additive receiver noise and other forms of interference. Note that in the 2-user cooperation scheme, each of transmission time slots is divided into 2 non-overlapping slots, and therefore the transmission duration for each slot is half of that available for the direct transmission scheme. Consequently, to maintain the same total power consumption, the energy available per bit for the cooperative scheme is half of that for the direct transmission scheme.

In the second time interval user 2 amplifies the signal by the relay gain α_2 and transmits:

$$d_2(n+1) = \alpha_2 \; y_2(n) \tag{2}$$

The destination, i.e., user 3 receives:

$$\begin{aligned} y_3(n) &= h_{13}\sqrt{E_b/2}\; b_1(n) + w_3(n) \qquad (3)\\ y_3(n+1) &= h_{23}\;\alpha_2\left(h_{12}\sqrt{E_b/2}\; b_1(n)\right.\\ &\quad \left. + w_2(n)\right) + w_3(n+1). \qquad (4)\end{aligned}$$

One choice for the relay gain, which amplifies the received signal (to a power level similar to that of a user's signal power level) before relaying it to the destination, is given in [3] as

$$\alpha_2^2 = \frac{1}{h_{12}^2 + (N_0/(E_b/2))}. \tag{5}$$

Note that this approach may be extended to the situation where more than two nodes cooperate.

III. Power Allocation Strategy

The performance of cooperative networks can be characterised in terms of BER performance, ergodic capacity, and outage capacity. In this paper we focus on BER performance as the metric to be optimised. Utilising the approximate expression for BER in [6], we can find an optimum power allocation such that the end-to-end average BER performance is minimised.

Statistically, we model the fading coefficients h_{ij} as zero-mean, mutually independent complex jointly Gaussian random variables with variances $\sigma^2_{h_{i,j}}$, and we model the additive noise $w_j(n)$ as zero-mean, mutually independent, white complex jointly Gaussian sequences with variance N_0. For a 2-cooperating user system, the equivalent received SNR at the destination, node 3 at time $n+1$ due to the 2-hop path, ρ_{eq} can be expressed as

$$\rho_{eq} = \frac{[h_{12}\alpha_2 h_{23}]^2\frac{E_b}{2}}{[(h_{23}\alpha_2)^2+1]N_0} = \frac{\frac{h_{12}^2}{N_0}\frac{h_{23}^2}{N_0}\frac{E_b}{2}}{\frac{h_{23}^2}{N_0}+\frac{1}{\alpha_2^2 N_0}}. \tag{6}$$

For cooperative scenario, let us define

$$\rho_{ij} = |h_{ij}|^2\frac{E_b}{2N_0}. \tag{7}$$

Substituting (5) into (6) and making use of the definition in (7) leads to ρ_{eq} given by:

$$\rho_{eq} = \frac{\rho_{12}\rho_{23}}{\rho_{12}+\rho_{23}+1} = f(\rho_{12},\rho_{23}) \tag{8}$$

Combining the received signal for 2 consecutive timeslots using the Maximum Ratio Combining (MRC) method, the conditional SNR of the combined signal given the channel fading coefficients $h_{ij}, i\in\{1,2\},\ j\in\{2,3\}$ is

$$\rho_{MRC|\rho_{13},\rho_{12},\rho_{23}} = \rho_{13} + f(\rho_{12},\rho_{23}) \tag{9}$$

and the probability of error conditional on the SNRs of the combined signal can be expressed as

$$P_e = \mathcal{Q}\sqrt{2(\rho_{13}+f(\rho_{12},\rho_{23}))} \tag{10}$$

where $\mathcal{Q}(.)$ is the standard Gaussian error function. At high SNR values, the unity term in the denominator of (8) is negligible, and thus (8) can be approximated as

$$\rho_{eq} = \frac{\rho_{12}\rho_{23}}{\rho_{12}+\rho_{23}}. \tag{11}$$

Using the approximate expression for SNR in (11), Ribeiro and Giannakis [6] derive the approximate BER expression for the high SNR region by looking at the pdf of the SNR around zero, i.e., $p_{\rho_{MRC}}(0)$. The authors argue that since in fading channels the probability of error is dominated by the probability of having deep fades, or equivalently, the probability that the channel coefficient is very small, the probability of error can be approximated by observing the behavior of the pdf of the SNR around zero. The resulting asymptotic average BER for user 1 is [6]

$$P_e^{(1)} \to \frac{3}{4k^2}\;[\,p_{\rho_{12}}(0)+p_{\rho_{23}}(0)]\;p_{\rho_{13}}(0) \tag{12}$$

where k is a constant depending on the type of modulation, e.g., for BPSK, $k=2$. In the case of Rayleigh fading,

$$P_e^{(1)} \to \frac{3}{4k^2}\left(\frac{1}{\overline{\rho}_{12}}+\frac{1}{\overline{\rho}_{23}}\right)\frac{1}{\overline{\rho}_{13}}. \tag{13}$$

For M-user cooperative diversity, the approximate BER expression for user l communicating with destination node d is [6]

$$P_e^{(l)} \approx \frac{C(M)}{k^{M+1}}\frac{1}{\beta_l^{(l)}\overline{\rho}_{ld}}\prod_{i=1}^{M}\left(\frac{1}{\beta_l^{(l)}\overline{\rho}_{li}}+\frac{1}{\beta_i^{(l)}\overline{\rho}_{id}}\right). \tag{14}$$

where $\beta_n^{(m)}$ is the power allocation factor for user n to transmit the data of user m, M is the number of cooperating mobiles between the user l and destination d and $C(M) = \frac{\prod_{k=1}^{M+1}(2k-1)}{2(M+1)!k^{(M+1)}}$ is a constant depending on the number of cooperating mobiles. Note that

$$0 \le \beta_n^{(m)} \le 1 \tag{15}$$

for $n, m = 1,\ldots,M$

$$\sum_{m=1}^{M}\beta_n^{(m)} = 1. \tag{16}$$

The problem of finding a set of power allocation factors, $\beta_n^{(m)}$ that minimises the total BER can be posed as follows.
$Problem\ \ (P_1)$: Find a set of power allocation factors $\beta_n^{(m)}$ which solves the following constrained optimisation problem

$$\min_{n,m=1,\ldots,M}\frac{1}{M}\sum_{l=1}^{M}P_e^{(l)}(\beta_1^{(1)},\ldots,\beta_M^{(M)}) \tag{17}$$

subject to the linear constraints (15) and (16). Note that the direct and relayed transmit powers are set to the same level prior to applying the power allocation factors.

To analytically illustrate the optimisation method for $n = 1,\ldots,M$, let us consider the 2-user highly symmetrical scenario where the uplinks and inter-user link SNRs are similar, i.e., $\overline{\rho}_{13} = \overline{\rho}_{12} = \overline{\rho}_{23}$. Note that in this highly symmetrical case the power allocation factors, $\beta_1^{(1)} = \beta_2^{(2)} = \beta$ and

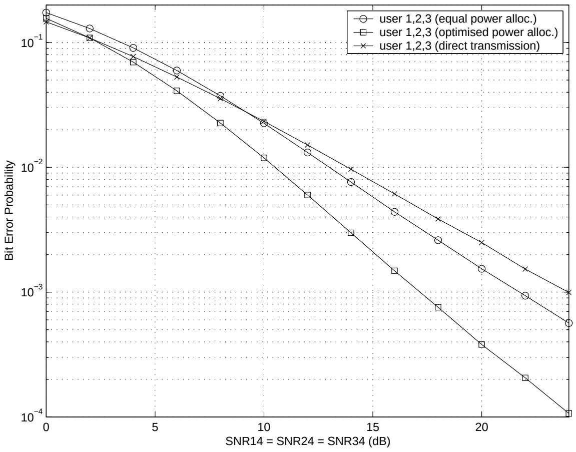

Fig. 3. Three user cooperation with symmetrical uplink channels, $\overline{\rho}_{14} = \overline{\rho}_{24} = \overline{\rho}_{34}$ and inter-user channel SNR, $\overline{\rho}_{14} = \overline{\rho}_{24} = \overline{\rho}_{34} = 5$ dB.

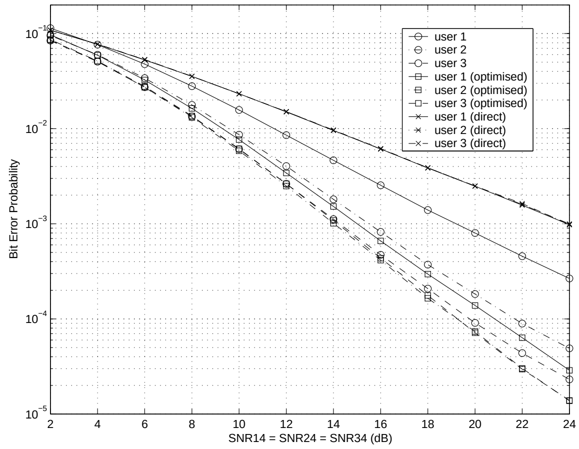

Fig. 4. Three user cooperation with symmetrical uplink channels, $\overline{\rho}_{14} = \overline{\rho}_{24} = \overline{\rho}_{34}$ and different inter-user SNRs, $\overline{\rho}_{12} = 5$ dB, $\overline{\rho}_{13} = 10$ dB, $\overline{\rho}_{23} = 20$ dB.

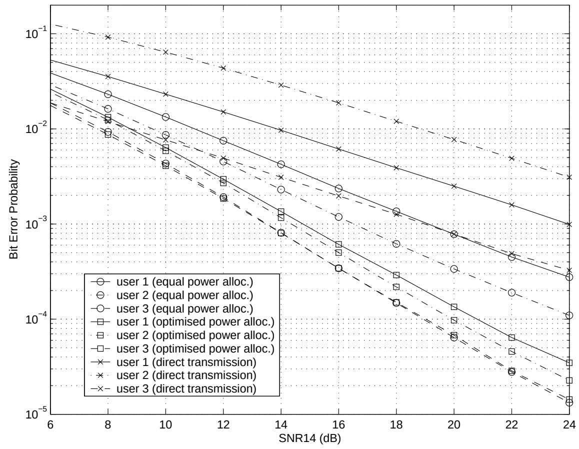

Fig. 5. Three user cooperation with asymmetrical uplink channels, $\overline{\rho}_{24} = \overline{\rho}_{14} - 5$ dB, $\overline{\rho}_{34} = \overline{\rho}_{14} + 5$ dB, and different inter-user SNRs, $\overline{\rho}_{12} = 5$ dB, $\overline{\rho}_{13} = 10$ dB, $\overline{\rho}_{23} = 20$ dB.

$\beta_1^{(2)} = \beta_2^{(1)} = 1 - \beta$. Therefore Equation (**??**) can be written as

$$P_e \rightarrow \frac{1}{2} \times \frac{3}{2k^2\overline{\rho}^2} \left(\frac{1}{\beta} + \frac{1}{(1-\beta)} \right) \frac{1}{\beta} \tag{18}$$

We can obtain the optimum power allocation factor β by differentiating P_e with respect to β and setting the resulting expression to zero, i.e., $\frac{dP_e}{d\beta} = 0$. The resulting optimum cooperation level in this case is $\beta = 2/3$, which is independent of the uplink SNRs. On the contrary, in the existing literature the value of $\beta = 1/2$ is often used (e.g., [3]).

In practice, the result of the optimisation in (17) may serve as a lower bound for the achievable BER. For the future, the intention is to develop sub-optimum (but practical) allocation algorithms and to assess their performance in terms of the lower bound BER performances established previously.

IV. Numerical Simulation

To test the effectiveness of the proposed power allocation strategy, numerical simulations have been performed. The channel is assumed to obey flat Rayleigh fading. For the situation where the terminals are mobile, the power allocation needs to be updated dynamically to ensure an optimum end-to-end performance. However, if the mobiles are of low mobility there will be enough time to properly update the power allocation. Note that for high mobility applications operating in a fast fading environment, there are other alternatives to exploit diversity, for example coding.

A. Performance of the proposed power allocation scheme

In this section, the performance of the proposed power allocation scheme will be investigated. The allocation scheme is optimum in the sense of minimising the total average probability of error as defined in (17).

Figure 3 shows that in 3 user cooperation schemes with symmetrical uplinks and inter-user channel SNRs of 5 dB, the proposed power control strategy achieves a gain of 4.7 dB, over equal power allocation at a BER of 10^{-3} and outperforms direct transmission by 6.9 dB. The proposed power control technique achieves more significant gains as the number of users (and therefore, the number of parameters to be optimised) increases.

The results in Figure 5 for the situation with highly asymmetrical uplinks and inter-user links show that the gains over equal power allocation for user 1, 2, and 3 at a BER of 10^{-3} are 4.6, 2.3, and 0 dB, respectively. The gain over direct transmission for user 1, 2, and 3 at a BER of 10^{-3} are 9.1, 14.9, and 5.5 dB, respectively. It can be seen that optimised power control is very important especially if there are some links with poor SNR.

B. Optimum power allocation factors

In this section, the power allocation factors that minimise the total probability of error are discussed. Most of the power allocation factor scenarios investigated in this section correspond to those considered previously in Section IV-A.

From the 2-user cooperation results shown in Figure 6, it can be seen that the quality of inter-user link limits the benefit of cooperation. As a user's own uplink SNR improves, the benefit

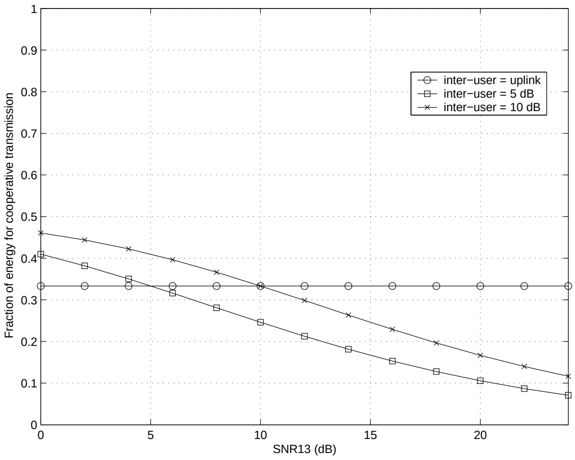

Fig. 6. Two user cooperation with symmetrical uplink channels, $\overline{\rho}_{13} = \overline{\rho}_{23}$.

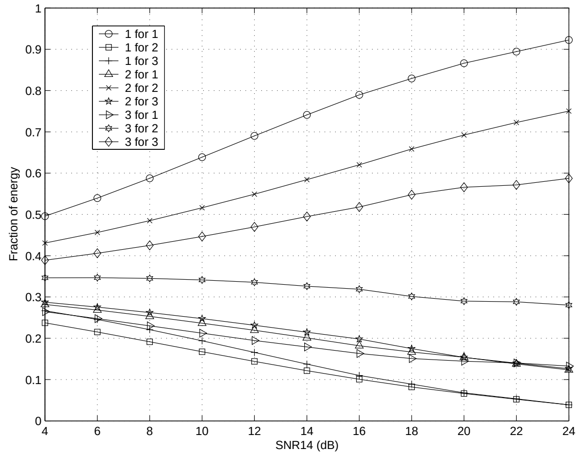

Fig. 9. Three user cooperation with asymmetrical uplink channels, $\overline{\rho}_{24} = \overline{\rho}_{14} - 5$ dB, $\overline{\rho}_{34} = \overline{\rho}_{14} + 5$, and different inter-user SNRs, $\overline{\rho}_{12} = 5$ dB, $\overline{\rho}_{13} = 10$ dB, and $\rho_{23} = 20$ dB.

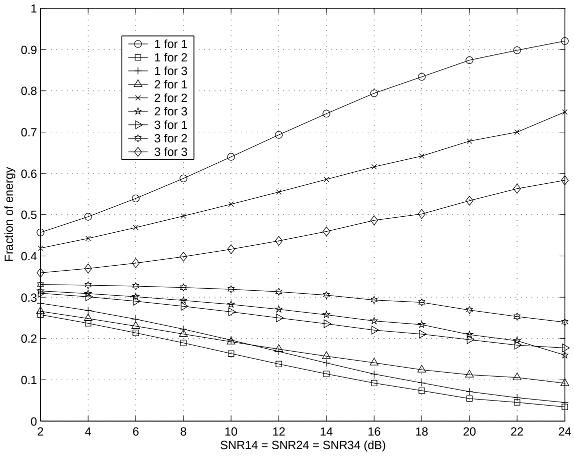

Fig. 7. Three user cooperation with symmetrical uplink channels, $\overline{\rho}_{14} = \overline{\rho}_{24} = \overline{\rho}_{34}$ and different inter-user SNRs, $\overline{\rho}_{12} = 5$ dB, $\overline{\rho}_{13} = 10$ dB, $\overline{\rho}_{23} = 20$ dB.

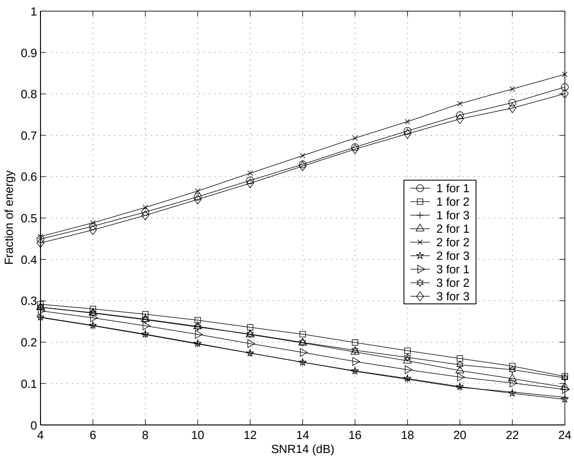

Fig. 8. Three user cooperation with asymmetrical uplink channels, $\overline{\rho}_{24} = \overline{\rho}_{14} - 5$ dB, $\overline{\rho}_{34} = \overline{\rho}_{14} + 5$ dB and similar inter-user SNRs, $\overline{\rho}_{12} = \overline{\rho}_{13} = \overline{\rho}_{23} = 10$ dB.

of transmitting via an inter-user channel of a fixed quality diminishes, therefore it is more advantageous for the mobile to dedicate more energy to its direct link to the destination and allocate less energy for cooperation. This is not the case with the highly symmetrical scenario where the quality of inter-user channels improves together with that of the uplink channels. In this case, the optimum cooperation level stays at a level independent of SNR (i.e., 2/3 in this 2-user case, as was demonstrated analytically in Section III). It can be concluded that whereas equal power allocation might perform reasonably well when all the links are good, power control is absolutely necessary in non-ideal cases where there are some poor quality links. Also, Figure 6 shows that good inter-user link quality promotes a higher level of cooperation.

For 3 user cooperation with symmetrical uplinks and interuser links, Figure 7 shows that as the uplink SNRs increase, users 1, 2, and 3 dedicate more power for their direct transmissions. This is due to the non-ideal inter-user channels that connect the three users. In terms of the cooperation level, user 1 cooperates more with user 3 than it does with user 2 due to the better inter-user channel between user 1 and 3 as compared to the one between user 1 and 2. User 2 cooperates more with user 3 rather than user 1. Again, this is due to a better inter-user channel between user 2 and 3 as compared to that between user 2 and 1. To conclude, the users with the best inter-user channel SNRs, i.e., users 2 and 3, dedicate more energy to improve the BER performance of the weakest user, while the weakest user concentrates most of its energy for its direct transmission. With symmetrical uplinks and asymmetrical inter-user links, the power allocated for transmission via link 23 (having the best inter-user link) is always more than link 12 (having the poorest inter-user link).

The results for 3-user cooperation with asymmetrical uplink and similar inter-user link quality presented in Figure 8 show that as the uplink SNR increases, users 1, 2, and 3 devote most of their power to transmit their own information bits and cooperate less (as uplink SNRs $\rightarrow \infty, \beta \rightarrow 1$). This is due to the non-ideal inter-user links and the low SNR of their partners' uplinks. In general, user 1 together with user 3 dedicate more energy to help user 2. Note that in the case of similar inter-user channels, user 1 dedicates more energy to help user 2 (having the poorest uplink) rather than helping user 3 (having the best uplink). This promotes fairness. User 2 also allocates more energy to help user 1 (medium uplink) rather than user 3 (best uplink). Comparing Figures 7 and 8, it can be seen that the effect of inter-user link quality can be more significant than that of the uplink SNR as the curves in Figure 7 are more spread out than those in Figure 8.

There are several points worth mentioning with respect to the highly asymmetrical scenario in Figure 9:
User 1: Medium quality uplink with worst inter-user links. Having the poorest inter-user links, user 1 keeps increasing its power for transmitting its own bits. It also cooperates with users 2 and 3 almost equally. Why? Because it prefers to help a user with poor uplink and good inter-user links. But users 2 and 3 only have one of these properties each. So user 1 helps them almost equally.
User 2: Worst uplink with one good and one poor inter-user link. Limited by the inter-user channel quality, at low to medium SNR user 2 keeps increasing its power for the direct transmission. User 2 cooperates with user 1 (medium quality uplink with worst inter-user links) and 3 (best uplink with best inter-user links) almost equally.
User 3: Best uplink with best inter-user links. As own uplink quality improves, user 3 steadily increases its power for direct transmission. User 3 cooperates more with user 2 (worst uplink with medium quality inter-user links) rather than with user 1 (medium quality uplink with worst inter-user links). This shows the significance of inter-user link quality.

V. Conclusion

This paper presents a power allocation strategy employing transparent relaying in a cooperative network. The aim is to minimise the average end-to-end BER. As compared to the equal power allocation strategy, our proposed strategy is shown to yield an SNR gain of up to 4.7 dB over the equal power allocation strategy and 14.9 dB over direct transmission at a BER of 10^{-3}. In terms of the optimum level of cooperation required to minimise the overall BER, it is found that a user with a better uplink channel should cooperate more with other user(s) than should a user having a poorer uplink SNR quality, provided that there is a (are) good inter-user link(s) to convey the relayed information reliably. Simulation results show that the inter-user link quality plays a significant role in determining the optimum level of cooperation. Based upon knowledge of the lower bound error rate performances obtained in this work and on the insight obtained concerning optimum power allocation factors, it is hoped that practical but sub-optimum power allocation algorithms can be developed.

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