Error Probability Analysis of Unselfish Cooperation over Quasi-static Fading Channels

Ioannis Chatzigeorgiou[1], Weisi Guo[1], Ian J. Wassell[1] and Rolando Carrasco[2]

[1] University of Cambridge, Computer Laboratory, United Kingdom – Email: { ic231, wg209, ijw24 }@cam.ac.uk

[2] Newcastle University, School of Electrical and Electronic Engineering, United Kingdom – Email: r.carrasco@newcastle.ac.uk

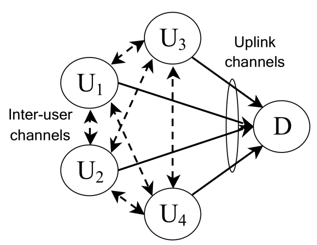
Introduction

- We have considered *decode-and-forward* (DF) cooperation in wireless networks over quasi-static fading channels.
- We have focused on *unselfish* cooperation, according to which a network user will be assisted by other users who have successfully decoded its data, even though cooperation might not be reciprocal.
- Accurate approximations for the packet error probability (PEP) of a two-user DF cooperative network have been derived by Souryal and Vojcic [1].
- Upper bounds on the PEP of a multi-user network have been presented by Sadek *et al.* in [2]. However, the authors assume that users who do not cooperate, remain idle for the duration of a cooperation frame.

Motivation

To derive an accurate closed-form expression for the PEP of multi-user networks that employ unselfish DF cooperation. In contrast to [2], we consider networks in which users who cannot assist a partner, *do not stay idle but retransmit their own data.*

System Model



Example for a network of M=4 cooperating users

- We consider a network of M cooperating users that transmit to the same destination.
- Both *uplink* and *inter-user* channels are subject to frequency-flat Rayleigh fading and AWGN.
- User cooperation occurs in two successive stages:
  1. Each user broadcasts its own packet of coded bits to the other users and the destination.
  2. A user who failed to decode m packets from m partners, relays the successfully decoded M-1-m packets and retransmits m copies of its own packet to the destination.
- Each channel realization remains constant for the duration of a two-stage cooperation frame but changes independently from frame to frame (quasi-static fading).
- Uplink channels are statistically similar and inter-user channels are also statistically similar.
- The average signal-to-noise ratio (SNR) of an uplink channel between a user and the destination is
  - $A\bar{\gamma}$: when the user broadcasts a packet (first stage of coop.)
  - $B\bar{\gamma}$: when the user forwards the packet of a partner to the destination (second stage of coop.)
  - $C\bar{\gamma}$: when the user retransmits its own packet (second stage of coop.)
- The average SNR of an inter-user channel is given by $A\bar{\gamma}'$
- For per-packet equal power allocation, we have $A = B = C = 1$

Conditional Packet Error Probability

At the end of the two-stage cooperation frame, the destination will collect and combine the following packets that carry information from a specific user U:

- one packet that was broadcast from user U during the first stage
- ℓ packets that were relayed by ℓ partners of U
- m copies that were retransmitted by user U during the second stage

The outage probability conditioned on m+ℓ+1 packets of user U being received by the destination at the end of the cooperation frame, can be expressed as

$$P(\bar{\gamma};m,\ell)=\begin{cases}P_{\mathrm{I}}(\bar{\gamma}), & \text{for } A+mC=B\\ P_{\mathrm{NI}}(\bar{\gamma}), & \text{otherwise}\end{cases}$$

where

$$P_{\mathrm{I}}(\bar{\gamma})=1-e^{-\frac{\gamma_o}{B\bar{\gamma}}}\sum_{k=0}^{\ell}\frac{1}{k!}\left(\frac{\gamma_o}{B\bar{\gamma}}\right)^k$$

corresponds to the the outage probability when the destination combines independent and identically distributed channels and

$$P_{\mathrm{NI}}(\bar{\gamma})=1-e^{-\frac{\gamma_o}{(A+mC)\,\bar{\gamma}}}\left(\frac{A+mC}{A+mC-B}\right)^{\ell}+e^{-\frac{\gamma_o}{B\bar{\gamma}}}\sum_{k=0}^{\ell-1}\frac{1}{k!}\left(\frac{\gamma_o}{B\bar{\gamma}}\right)^k\left[\left(\frac{A+mC}{A+mC-1}\right)^{\ell-k}-1\right]$$

refers to the case when the direct channel is **not** identically distributed to the ℓ indirect channels.

When γ_o is set equal to the SNR threshold that characterizes the adopted transmission scheme, **the outage probability provides an accurate approximation of the packet error probability** on quasi-static fading channels.

End-to-end Packet Error Probability

For **mutually independent** inter-user channels:

$$\bar{P}(\bar{\gamma},\bar{\gamma}')=\sum_{m=0}^{M-1}\sum_{\ell=0}^{M-1}\binom{M-1}{m}\binom{M-1}{\ell}p_{\bar{\gamma}'}^{M-1-m+\ell}(1-p_{\bar{\gamma}'})^{M-1+m-\ell}P(\bar{\gamma};m,\ell)$$

For **reciprocal** inter-user channels:

$$\bar{P}(\bar{\gamma},\bar{\gamma}')=\sum_{m=0}^{M-1}\binom{M-1}{m}p_{\bar{\gamma}'}^{\ell}(1-p_{\bar{\gamma}'})^{m}P(\bar{\gamma};m,M-1-m)$$

where

$$p_{\bar{\gamma}'}=1-P(\bar{\gamma}';0,0)=e^{-\frac{\gamma_o}{A\bar{\gamma}'}}$$

is the probability that a user will successfully decode the packet of a partner.

Future Work: Optimization of Power Allocation

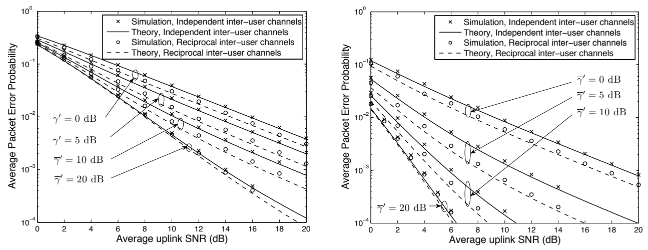
Let E_S be the total energy required by a user who employs per-packet **equal power allocation** to transmit M packets during a cooperation frame. Moreover, let E_k be the required transmit energy during the k-th cooperation frame when **power allocation** is used. Then:

$$E_k=A(E_S/M)+B(M-1-m)(E_S/M)+mC(E_S/M)$$

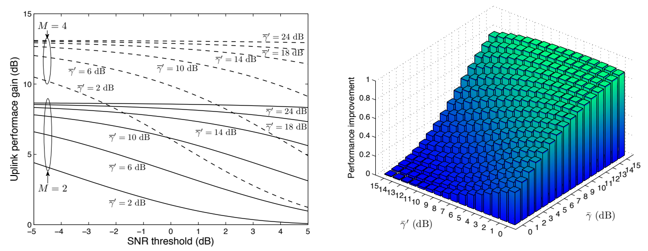
Optimization problem: Minimize the end-to-end PEP (hence, optimize A, B, and C) subject to an energy constraint, for example

$$\mathrm{E}\{E_k\}\le E_S\Rightarrow A+B\,p_{\bar{\gamma}'}(M-1)+C\,(1-p_{\bar{\gamma}'})\,(M-1)\le M$$

Results



Comparison between analytical values and simulation results when the rate ½ NRNSC(15, 17) code is used. The network size is M=2 (left-hand side) and M=4 (right-hand side).



(Left-hand side) Effect of the network parameters on the performance gain for a target PEP of 10^{-2}. (Right-hand side) Theoretical performance improvement that optimal power allocation can achieve over equal power allocation for a network of M=4 users.

Conclusions

- ✓ We presented closed-form expressions that accurately predict the average end-to-end packet error probability of networks that employ unselfish decode-and-forward cooperation.
- ✓ We demonstrated that the error correction capability of the adopted channel code has a significant impact on the performance gain when the quality of the inter-user channels is poor.
- ✓ For good inter-user channels the performance gain is mainly determined by the network size.
- ✓ Our framework can be used to investigate power control schemes subject to energy constraints.

References

[1] M. R. Souryal and B. R. Vojcic, "Performance of amplify-and-forward and decode and forward relaying in Rayleigh fading with turbo codes", in Proc. Int. Conf. Acoustics, Speech and Sig. Proc., Toulouse, France, May 2006

[2] A. K. Sadek, W. Su and K. J. R. Liu, "Clustered cooperative communications in wireless networks", in Proc. IEEE Conf. on Global Communications (GLOBECOM), St. Louis, MO, USA, Nov. 2005

Acknowledgements

UNIVERSITY OF CAMBRIDGE
Newcastle University
EPSRC Engineering and Physical Sciences Research Council