Cooperative Amplify-and-Forward with Trellis Coded Modulation

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Abstract — **In a cooperative communication network, individual users are encouraged not only to transmit their own data, but also relay other user's data. This relaying transmission creates spatial diversity to combat the effect of individual severe fading and path loss. Since cooperative users utilize some degree of their transmission freedom for relaying other user's data, cooperative transmission results in lowering each user's transmission spectral efficiency. Therefore, a coding scheme with high spectral efficiency and optimized performance would be desirable for a cooperative network. This paper proposes the Trellis Coded Modulation (TCM) scheme to be incorporated with the cooperative Amplify-and-Forward (AF) systems. A criterion for designing good TCM codes for AF systems is also derived and two cooperative AF systems achieving 1 bits/sec/Hz and 1.5 bits/sec/Hz for each user are presented. Analyses in this paper show that cooperative TCM schemes can not only achieve high spectral efficiency, but also outperform convolutional codes with a high order modulation scheme.**

Keywords – Cooperative communications, Amplify-and-Forward, Trellis Coded Modulation

I. INTRODUCTION

In a practical communication system the effect of severe fading and path loss make it crucial to be able to create spatial diversity to improve the communication quality. One of the most popular diversity techniques is for each user to implement multiple transmit/receive antennas [1]. However, because of some practical restrictions, e.g. size of the mobile unit, multiple antennas may not be suitable for implementation. An alternative method to create spatial diversity is to encourage users with a single antenna to share their resources and help each other for transmission, generating a cooperative communication network. In a cooperative network, one user can 'overhear' the other user's transmission and 'relay' it to the intended destination. As a result, one user's signal can be transmitted through independent paths, creating a virtual multiple transmit antennas effect.

The earliest work of signal relaying was presented by Cover and El Gamal in [2]. Until recently, practical cooperative protocols were proposed by Laneman and Wornell in [3, 4]. They are: Amplify-and-Forward (AF) and Decode-and-Forward (DF). In the AF scheme, one user simply amplifies the 'overheard' signal from its partner and relays it to its partner's intended destination. While in the DF scheme, one user decodes and re-encodes the 'overheard' signal from its partner and relays it to its partner's intended destination. Stefanov and Erkip [5] as well as Hunter and Nosratinia [6] later proposed another cooperative scheme, called Coded Cooperation (CC). In the CC scheme, message re-encoding is achieved by using a different encoder, which incorporates signal relaying with channel coding design. The authors have carried out a comparative investigation of the existing three schemes in [7], showing the CC scheme produces the best diversity gain, but requires the highest system complexity. So far, incorporating the CC scheme with other coding techniques rather than the convolutional codes is yet to be developed. In this paper, the AF scheme is adopted as the cooperative model in this paper, as it achieves good spatial diversity and has a lower complexity.

Since in a cooperative network, each user will specify some degree of their transmission freedom to relay other user's signal, it results in lowering each user's spectral efficiency. In the literature [5, 6], cooperation results in a low spectral efficiency of 0.25 bits/sec/Hz for each user. This problem was first addressed by Chatzigeorgiou et al [8], in which channel coding with a high order modulation scheme is employed. However, considering channel coding and modulation scheme separately might not provide the optimized performance. Ungerboeck [9] showed that treating channel coding and modulation as a single entity, Trellis Coded Modulation (TCM), can achieve high spectral efficiency and optimize performance. As a result, TCM is a neutral coding technique to be applied to cooperative systems. This paper incorporates TCM codes with the cooperative AF scheme, showing significant diversity gain can be achieved over the noncooperation scenario. With respect to the design of good TCM code for the cooperative AF scheme, suitable mathematical model is yet to be developed. Therefore, our analysis is derived from optimizing the code for a slow Rayleigh fading channel. It is shown that the chosen TCM code shall maximize the code's square free distance. Based on the presented code design criterion, two TCM cooperative AF systems achieving 1 bits/sec/Hz and 1.5 bits/sec/Hz are proposed and simulated. Our results show that based on achieving the same spectral efficiency, TCM codes can not only achieve a diversity gain over the non-cooperative scenario, but also significantly

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outperform convolutional codes using a high order modulation scheme in a cooperative scenario.

This structure of the paper is organized as follows: in Section II, the cooperative AF signal model will be presented with analysis on each user's spectral efficiency. In Section III, code design criterion for the cooperative AF scheme will be presented and two suitable TCM codes will be proposed. Section IV will present our simulation results with discussions. The conclusion of the paper will be presented in Section V.

II. COOPERATIVE AF SIGNAL MODEL AND SPECTRAL EFFICIENCY ANALYSIS

This section will present the cooperative AF signal model and the spectral efficiency analysis for each user in a cooperative scenario.

A. *Cooperative AF Signal Model*

The classical cooperative network can be generalized as having two users: source (s) and relay (r). Both of them communicate with a common destination (d). In this paper, it is assumed that both source and relay transmit through orthogonal frequency channels in a full-duplex system so that simultaneous transmission/detection is achievable between both users. The cooperative transmission can be depicted by the two-segment process as shown by Fig. 1. To clarify the description of the cooperative network, the channels between source to destination and relay to destination are defined as *source uplink channel* and *relay uplink channel* respectively, while the channel between source and relay defined as the *interuser channel* which is assumed to be *reciprocal*.

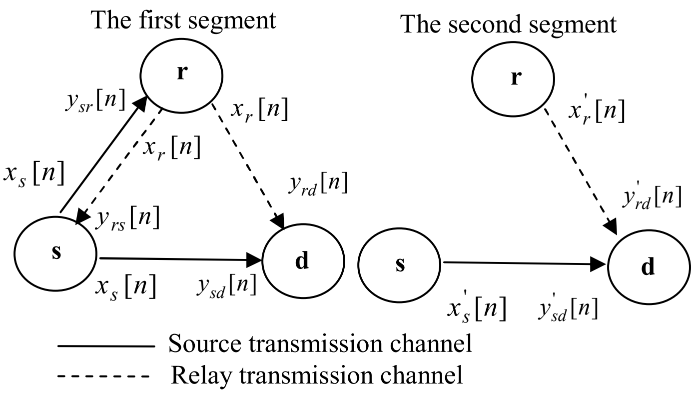


Figure 1 The cooperation process

In the cooperative AF system, symmetric transmission exists between source and relay. Therefore, in this paper, signal model analysis will be focused on the source transmitted signal $x_s[n]$, while the same process can be applied to the relay transmitted signal $x_r[n]$. As shown by Fig. 1, in the first segment, the source transmits its own signal $x_s[n]$ to the destination and it is overheard by the relay as:

$$y_{sd}[n] = \sqrt{\varepsilon}\, a_{sd}\, x_s[n] + z_{sd}[n], \tag{1}$$

$$y_{sr}[n] = \sqrt{\varepsilon}\, a_{sr}\, x_s[n] + z_{sr}[n], \tag{2}$$

where $n = 1, 2, \ldots, N_1$ and N_1 indicates the length of the first segment signal. In the second segment, the relay amplifies its overheard signal $y_{sr}[n]$ as [3]:

$$x_r'[n] = \beta\, y_{sr}[n], \tag{3}$$

and

$$\beta = \sqrt{\frac{1}{|a_{sr}|^2\, \varepsilon + N_{0,sr}}}\,. \tag{4}$$

$x_r'[n]$ will be transmitted to the destination through relay's uplink channel as:

$$y_{rd}'[n] = \sqrt{\varepsilon}\, a_{rd}\, x_r'[n] + z_{rd}'[n], \tag{5}$$

where $n = 1, 2, \ldots, N_2$ and N_2 indicates the length of the second segment signal and $N_2 = N_1$. After two segments, the destination will combine the received signal $y_{sd}[n]$ and $y_{rd}'[n]$ using maximum likelihood detection [3] as:

$$r_{sd}[n] = \frac{a_{sd}^{*}\sqrt{\varepsilon}}{N_{0,sd}}\, y_{sd}[n] + \frac{a_{rd}^{*}\beta a_{sr}^{*}\sqrt{\varepsilon}}{|a_{rd}|^2\, \beta^2 N_{0,sr} + N_{0,rd}}\, y_{rd}'[n], \tag{6}$$

$r_{sd}[n]$ is further passed to a decoder in order to retrieve source information.

In equation (1) – (6), ε is the symbol energy which is normalized to unit energy as $\varepsilon = 1$. a_{ij} are the fading coefficients of the channel between nodes i and j ($i = s$ or r, $j = s$, r, or d; $i \neq j$). They are modeled as zero-mean mutually independent complex Gaussian random variables with variances $\sigma_{ij}^{\ 2}$. They change independently from (two-segment) process to process, yielding a slow fading channel. $z_{ij}[n]$ ($z_{ij}'[n]$) represents the additive noise which is modeled as a zero-mean, mutually independent complex Gaussian sequence with variance $N_{0,ij}$. For channel between nodes i and j, if the instantaneous channel received Signal-to-Noise Ratio (SNR) is defined as:

$$\gamma_{ij} = \frac{|a_{ij}|^2\, \varepsilon}{N_{0,ij}}, \tag{7}$$

the average channel received SNR between nodes i and j can be further defined as:

$$\overline{\gamma_{ij}} = \frac{E\left[\left|a_{ij}\right|^2\right]\varepsilon}{N_{0,ij}}. \tag{8}$$

In this paper, if source and relay have similar uplink channel quality as $\overline{\gamma_{sd}} = \overline{\gamma_{rd}}$, the system is defined as having *symmetric uplinks*. Otherwise if $\overline{\gamma_{sd}} \neq \overline{\gamma_{rd}}$, the system is defined as having *asymmetric uplinks*.

B. Spectral Efficiency Analysis

Based on the above description, it can be seen that each cooperative user utilizes half of their transmission freedom for transmitting its partner's signal, resulting in lowering each user's spectral efficiency. In a practical communication system employing channel coding and modulation, if R denotes the code rate and M denotes the modulation scheme order as M bits/symbol, the spectral efficiency for each user in a non-cooperative scenario ($I_{\text{non-coop}}$) and a cooperative scenario (I_{coop}) can be defined as:

$$I_{\text{non-coop}} = MR \text{ bits/sec/Hz, and } I_{\text{coop}} = \frac{MR}{2} \text{ bits/sec/Hz.} \quad (9)$$

In literature [5, 6], rate 1/2 convolutional code and binary phase shift keying (BPSK) were employed, resulting in $I_{\text{coop}} = 0.25$ bits/sec/Hz. While in literature [8], rate 1/2 convolutional codes and turbo codes were employed with Gray-coded 16 point quadrature amplitude modulation (16-QAM), resulting in $I_{\text{coop}} = 1$ bits/sec/Hz. However, it is realized that simply combining a coding scheme with a high order modulation scheme is at the expense of severe performance degradation. In order to achieve both a high spectral efficiency and optimized performance, TCM code is a suitable coding scheme.

III. TCM CODES FOR COOPERATIVE AF SCHEME

In this section, suitable TCM codes will be proposed to incorporate with the cooperative AF scheme. For the design of suitable TCM codes, it is yet to develop a suitable mathematical model for optimizing the TCM code in the cooperative AF system. However, since both the uplink channels and the interuser channel are slow Rayleigh fading channels, our analysis of designing suitable TCM code is derived from optimize the performance of the code over a slow fading channel. Following on, two TCM codes that can achieve 1 bits/sec/Hz and 1.5 bits/sec/Hz spectral efficiencies for each user will be proposed.

Trellis code design criterion for fading channel was presented by Divsalar and Simon [10], in which interleaving effect and Racian fading effect were both considered. Deriving the analysis of [10] to a slow Rayleigh fading channel without interleaving effect, the Bit Error Rate (BER) probability of a TCM code is approximately upper bounded by:

$$P_b \cong V(1 + d_{free}^2 \frac{\varepsilon}{4N_{0,ij}})^{-1}, \quad (10)$$

where V is a constant and d_{free}^2 is the square free distance of the code. With large $\varepsilon / N_{0,ij}$ ratio, equation (10) can be further approximated as:

$$P_b \cong 4V(d_{free}^2 \frac{\varepsilon}{N_{0,ij}})^{-1}. \quad (11)$$

Based on the above analysis, it can be seen that the TCM code's BER performance over a slow fading channel is inversely proportional to the square free distance d_{free}^2 of the code. Therefore, to optimize a TCM code's performance, d_{free}^2 needs to be maximized. In this paper, two TCM codes that can achieve the maximal d_{free}^2 are proposed. These two TCM codes can achieve spectral efficiency of 1 bits/sec/Hz and 1.5 bits/sec/Hz in a cooperative AF scheme. In order to evaluate the performance of TCM codes based on achieving the same spectral efficiency, the rate 1/2 convolutional code which achieves the maximal d_{free}^2 was chosen with high order modulation schemes. Both the TCM encoder and the convolutional encoder have the same constraint length and both codes are decoded by the Soft-decision Viterbi algorithm [12] with similar decoding complexity.

The first proposed TCM code has rate 2/3 and employs the 8PSK set partitioning [9]. It has a systematic encoder with constraint length 4 and its parity check polynomials can be written in the octal form as:

$$(H^0(D), H^1(D), H^2(D)) = (23, 04, 16)_8. \quad (12)$$

In a cooperative AF system, this TCM code can achieve a spectral efficiency of 1 bit/sec/Hz for each user. Based on achieving the same spectral efficiency and the code design criterion described above, the best rate 1/2 non-systematic non-recursive convolutional code with generator polynomial written in octal form as [11]:

$$(G^1(D), G^2(D)) = (15, 17)_8 \quad (13)$$

was chosen with Gray-coded 16-QAM modulation in a cooperative scenario and Gray-coded QPSK in a non-cooperative scenario. This comparison platform is shown in Table I.

The second proposed TCM code has rate 3/4 and employs 16-QAM set partitioning [9]. It is a systematic code having the same parity check polynomials defined by equation (12). This cooperative AF system achieves a spectral efficiency of 1.5 bits/sec/Hz for each user. Based on achieving the same spectral efficiency, the same convolutional code defined by equation (13) uses Gray-coded 64-QAM in a cooperative scenario and Gray-coded 8PSK in a non-cooperative scenario. The comparison platform is shown by Table II

Table I The first cooperative AF system with TCM code

Schemes	Code Rate (R)	Modulation scheme (M)	Spectral efficiency (I)
TCM, coop	2/3	8PSK ($M = 3$)	$I_{\text{coop}} = 1$ bits/sec/Hz
Conv, coop	1/2	16-QAM ($M = 4$)	$I_{\text{coop}} = 1$ bits/sec/Hz
Conv, non-coop	1/2	QPSK ($M = 2$)	$I_{\text{non-coop}} = 1$ bits/sec/Hz

Table II The second cooperative AF system with TCM code

Schemes	Code Rate (R)	Modulation scheme (M)	Spectral efficiency (I)
TCM, coop	3/4	16-QAM ($M = 4$)	$I_{coop} = 1.5$ bits/sec/Hz
Conv, coop	1/2	64-QAM ($M = 6$)	$I_{coop} = 1.5$ bits/sec/Hz
Conv, non-coop	1/2	8PSK ($M = 3$)	$I_{non\text{-}coop} = 1.5$ bits/sec/Hz

These two cooperative AF systems defined by Table I and II are further simulated by the authors. The simulation results and discussion with be presented in the next section.

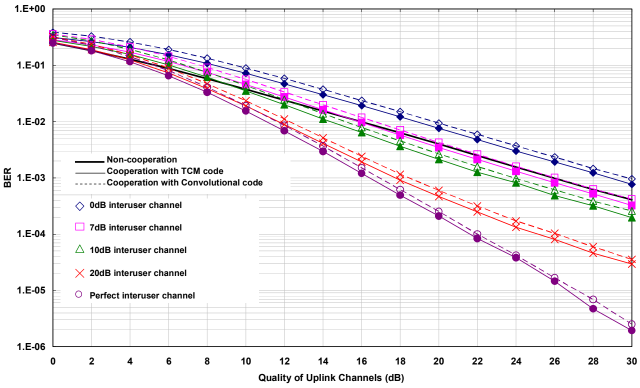
IV. SIMULATION RESULTS AND DISCUSSION

This section presents the authors' simulation results for the first and second cooperative AF system. Simulation results are achieved under both the symmetric uplink scenario and asymmetric uplink scenario.

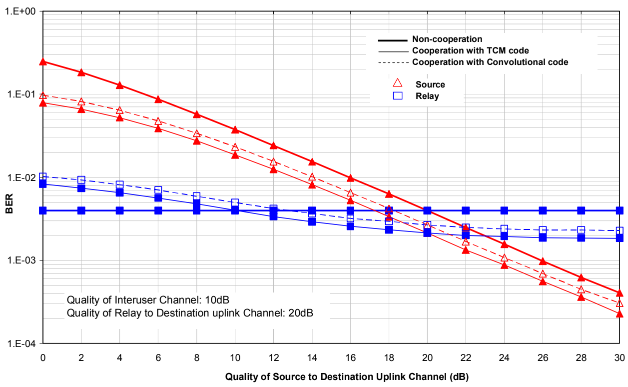
Fig. 2 (a) and (b) show the performance of the first cooperative system defined by Table I. Fig. 2 (a) and (b) show that during cooperation, the TCM code that employs 8PSK set partitioning outperforms the convolutional code with Gray-coded 16-QAM.

Fig. 2 (a) shows the performance under symmetric uplinks scenario. AF cooperation with TCM code starts to achieve a diversity gain as the quality of the interuser channel exceeds 7dB. While for the system incorporating a convolutional code to achieve a diversity gain, slightly higher average received SNR of the interuser channel is required. As the quality of the interuser channel improves, further diversity gains can be achieved. Taking the perfect interuser channel scenario as an example for comparison, at a BER of 10^{-5}, the TCM scheme has a 0.5dB coding gain over the convolutionally coded scheme.

Fig. 2 (b) shows performance under asymmetric uplinks scenario. It can be observed that when the source uplink channel is poor, the user (source) with weaker uplink channel quality will always benefit from cooperation, but not the user (relay) with a stronger uplink channel quality. Only when the source uplink channel is sufficiently good, both users will benefit from cooperation.

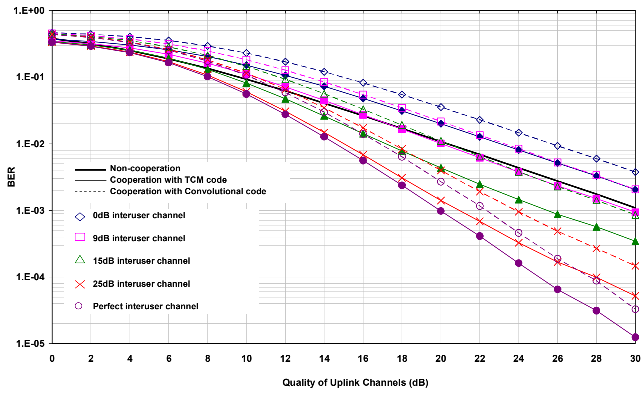


(a) Symmetric uplinks

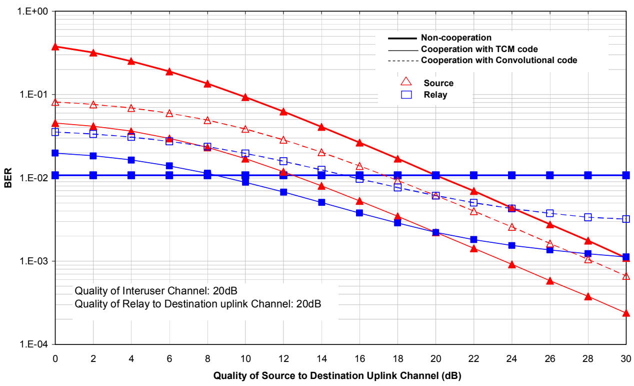


(b) Asymmetric uplinks

Figure 2. Performance of the first cooperative AF system



(a) Symmetric uplinks



(b) Asymmetric uplinks

Figure 3. Performance of the second cooperative AF system

Fig. 3 (a) and (b) show the performance of the second cooperative system defined by Table II. Fig. 3 shows that the TCM scheme achieves more significant improvements over the convolutionally coded scheme. In Fig. 3 (a), under the symmetric uplink scenario, the TCM coded scheme can achieve a diversity gain with interuser channel quality of 9dB. While for the convolutionally coded scheme, interuser channel quality of 15dB is required. Over a perfect interuser channel at BER of 10^{-4}, the TCM scheme can achieve a 2.5dB coding gain over the convolutionally coded scheme.

Similar phenomenon to Fig. 2 (b) can also be observed from Fig. 3 (b) in which the improvement from incorporating a TCM code over a convolutional code is more significant.

Based on comparing the presented results of this section and the results shown by literature [5-7], it is worthwhile to point out that by increasing the spectral efficiency, higher quality of interuser channel is required in order to achieve a diversity gain. In the cooperative scheme of [5-7], having a spectral efficiency of 0.25 bits/sec/Hz, a diversity gain can be achieved with 0dB interuser channel. While in this paper, for the first and second cooperative systems with a spectral efficiencies of 1 bits/sec/Hz and 1.5 bits/sec/Hz, at least 7dB and 9dB interuser channels are required to achieve diversity gain by using TCM codes.

V. CONCLUSION

This paper proposed the TCM codes to be incorporated in a cooperative AF system in order to achieve high spectral efficiency and optimized performance. Design of suitable TCM codes was drawn from analyzing the BER performance over a slow fading channel. Our analyses showed that the chosen TCM code shall have the maximal square free distance. Two suitable TCM codes were proposed to incorporate with the cooperative AF scheme, achieving spectral efficiencies of 1 bits/sec/Hz and 1.5 bits/sec/Hz respectively. Our simulation results show that TCM codes can outperform a convolutional code with a high order modulation scheme. The improvement is especially significant for a system with a high spectral efficiency. Building upon the current work, our future work will mainly focus on developing a performance analysis model of the cooperative AF scheme from which more accurate code design criterion can be derived.

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