FMT Modulation: Receiver Filter Bank Definition for the derivation of an Efficient Implementation

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Abstract-- **In this paper, we present the basic theory and advantages of Filtered Multitone Modulation (FMT), a multicarrier modulation technique introduced in [1] for VDSL. FMT systems consisting of *M* subcarriers are based on *M*-branch filters that are frequency shifted versions of a low pass prototype filter that provide higher spectral containment than conventional OFDM. This system would not be feasible if we could not derive an efficient implementation. We provide the time domain definition for the direct implementation of the receiver filter bank that will lead us to derive an efficient implementation based on the Fast Fourier Transform (FFT) and a network of *M* polyphase components of the prototype filter working at a rate *M* times more slowly than the direct approach. We will also show the relationship between the two implementations and how this can be utilised to compute the coefficients of an equalizer.**

I. INTRODUCTION

High data rate communications are often limited by the Intersymbol Interference (ISI) owing to the multipath propagation. The effects of the ISI are negligible as long as the delay spread is significantly shorter than the duration of one transmitted symbol. This implies that the symbol rate is limited by the channel memory. Multicarrier modulation is an approach to overcome this limitation. Here, a set of subcarriers is used to transmit the information symbols in parallel in so called subchannels. This allows a higher data rate to be transmitted by ensuring that the subchannel symbol duration exceed that of the channel memory.

There are several approaches to multicarrier transmission. The spectral partitioning can generally be realized in the form of overlapping or non-overlapping subbands. The multicarrier techniques used in today's standards (Digital Audio Broadcast, ADSL, HIPERLAN/2, Terrestrial Digital Video Broadcasting, etc [2]) are based on sinc(*f*) overlapping methods due to its low implementation complexity. These methods are known as Discrete Multitone Modulation (DMT) or Orthogonal Frequency Division Multiplexing (OFDM) [2]. On the other hand, we have modulation techniques such as FMT, initially introduced in [1] for VDSL, based on non-overlapping methods.

The remaining sections of this document are organized as follows. In section 2 we introduce the main problems associated with OFDM that FMT is aiming to solve. In Section 3 we introduce the FMT transmitter and show how to derive an efficient implementation. In Section 4 we show how the receiving filters may be defined in order to lead to an efficient implementation while in Section 5 we will demonstrate how the definition of the filters may be used to equalize the overall FMT system.

II. REASONS TO INTRODUCE FMT

The baseband representation of the OFDM signal consisting of *M* subcarriers is given by [1]:

$$x(t) = \frac{1}{\sqrt{M}} \sum_{k=-\infty}^{\infty} \sum_{i=0}^{M-1} A^{(i)}(k) h(t-kT) \cdot e^{ji\frac{2\pi}{T}t} \tag{1}$$

where *h(t)* is rectangular pulse of duration *T*, $A^{(i)}(k)$ are QAM or QPSK symbols and *T* is the OFDM symbol duration. In the previous representation, each of the *M* subcarriers is centered at frequency $f_i = i/T$, with $i=0,1,...,M-1$. The spectrum of these subcarriers will be the convolution of the Fourier transform of a single exponential at frequency $f_i=i/T$ (i.e. a frequency domain dirac delta function at the subcarrier frequency) with the Fourier transform of a rectangular pulse of duration *T* (i.e. a frequency domain sinc function). In this way, the spectrum of each of the subcarriers will be a sinc function centered at frequency $f_i = i/T$, with $i=0,1,\ldots,M-1$ as shown in Fig. 1.

Although these subcarriers have overlapping spectra, the resulting sinc(*f*) type spectrum yields zero ISI as well as zero intersubchannel interference (ICI) provided the adjacent carriers are at the nulls of the sinc(*f*) function. These orthogonality properties however do not hold at the output of a dispersive transmission channel.

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[1] In this paper, we use the superscript notation in brackets for subchannel number and subscript in brackets for polyphase component number.

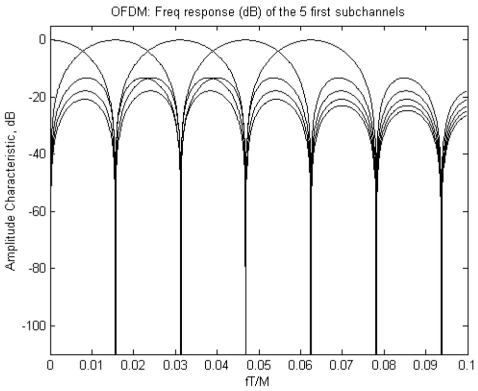

Fig. 1 Conventional OFDM spectrum with 64 subchannels: 5 first subchannels

To maintain orthogonality between subchannels in the presence of multipath, OFDM uses a time-domain cyclic extension of each symbol known as the cyclic prefix. The cyclic prefix allows time for multipath signals from the previous symbol to die away before the information from the current symbol is gathered. When the symbols are longer than the maximum delay spread we can consider that frequency flat fading for each subcarrier which can be easily equalized by a single tap filter.

Based on this overview we will highlight the main problems in OFDM that FMT will address.

Cyclic Prefix: The cyclic extension, although an elegant solution, leads to a loss in transmission efficiency. For instance, HIPERLAN/2 uses 16 samples out of 80 for the cyclic prefix, which leads to a loss in bandwidth efficiency of 20% [2].

Virtual carriers: Since the sidelobes of the OFDM signal (see Fig. 1) are high (the first sidelobe of a sinc signal is only 13dB lower than the main lobe, independent of the number of subcarriers), there is significant power leaking into adjacent bands. In order to reduce interference into adjacent bands, a number of subcarriers at the edge of the multiplex are not used. These virtual carriers are also introduced because the low pass filter following the Digital to Analogue Conversor will distort the subcarriers close to the pass band edges. In HIPERLAN/2, 12 out of 64 subcarriers are virtual carriers [2] which is 18.75% of the total number of subcarriers. Due to the high spectral containment of FMT, it needs fewer virtual carriers which will provide a higher data throughput.

Although FMT modulation goes someway to addressing the highlighted problems, it comes at the expense of higher computational complexity.

III. FMT TRANSMITTER

With FMT, we designate a particular case of a uniform filter bank consisting of frequency shifted versions of a low pass prototype filter. This filter is chosen to achieve a high degree of spectral containment, thus giving negligible ICI compared to the level of other noise signals. In [1], it is proposed that the prototype filter is not required to satisfy the perfect reconstruction condition [3] because this constraint is only assured when the ideal transmission channel does not introduce signal distortion. So when a channel is used that introduces amplitude and phase distortion, the objective of high spectral containment (the main purpose of FMT) is more easily achieved if the perfect reconstruction constraint is relaxed although we will need to use equalization to remove the ISI.

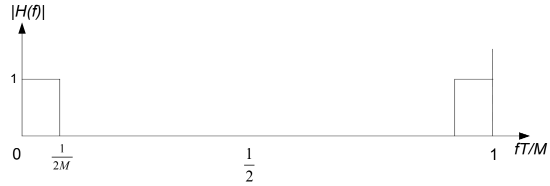

Fig. 2 Ideal Frequency Response of the low pass prototype

We can use any low pass filter design method [4] to design the low pass prototype filter *h(n)* with the objective of obtaining a symmetric FIR filter with real coefficients that would approximate the ideal frequency response *H*(*f*) in Fig. 2.

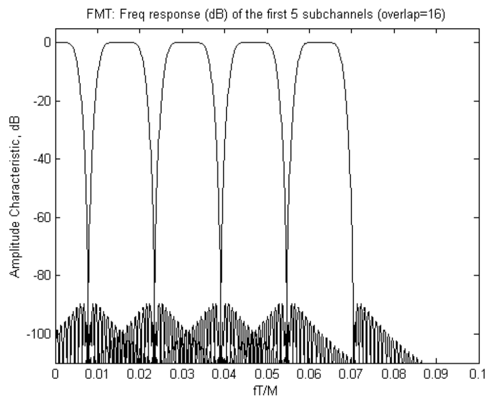

Fig. 3 FMT spectrum with 64 subchannels: 5 first subchannels

With FMT, orthogonality between subchannels is ensured by using non-overlapping spectral characteristics instead of overlapping sinc(*f*) type spectra. Since the linear transmission medium does not destroy orthogonality achieved in this manner, cyclic prefixing is not needed.

In a critically sampled filter bank [3], the frequency separation of the pass bands will be $1/T$ with a total of M bands. In this way, each of the transmitter pass band filters will be frequency-shifted versions of the low pass filter:

$$h^{(i)}(n) = \frac{1}{\sqrt{M}} h(n) \cdot e^{j2\pi \frac{i}{M} n}, \qquad (2)$$

with n=0,1,...$M\gamma$ and i=0,1,...,M-1.

The length of the prototype $M\gamma$ is a multiple of the number of subchannels M. Parameter γ is called the overlap [3] since it is the number of blocks (each of M samples) to which the prototype is expanded. In Fig. 3 we show the frequency response of the first 5 subchannels of a 64 subchannel system using a prototoype with γ=16.

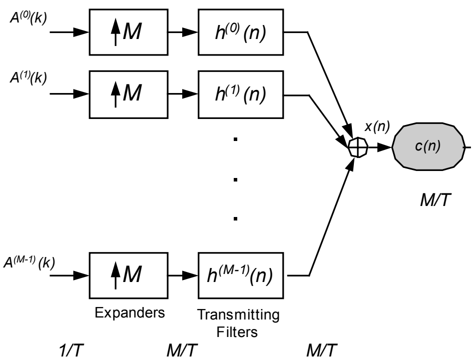

Fig. 4 FMT Transmitter: direct implementation

The direct implementation of the FMT filter bank is shown in Fig 4. The inputs $A^{(i)}(k)$ are QAM or QPSK symbols not necessarily from the same constellation. After upsampling by a factor of M (see [3]), each modulation symbol $A^{(i)}(k)$ is filtered at rate M/T, where T is the FMT symbol period, by the subchannel filter defined in Eq. (2) at a frequency f_i=i/T. The transmit signal $x(n)$ is obtained at the transmission rate of M/T by adding the M filter output signals that have been appropriately frequency shifted.

In the notation and figures, we have denoted k as the index for samples with sampling period equal to T and n for the samples with sampling period equal to T/M.

The implementation in Fig. 4 would not be practical if we could not derive an efficient implementation since all the filtering operations are performed in parallel and at a rate M/T. We will now see how to derive from Fig. 4, an efficient implementation that makes use of the Inverse Discrete Fourier Transform (IDFT).

When analysing multirate signal processing systems we usually arrive at the situation where filter responses are better described in terms of their polyphase components [3].

If we take the prototype $h(n)$ with Z transform

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} \qquad (3)$$

we can always partition the index n into M phases, where each phase is characterized by choosing indices which are identical modulo M. Then for any integer M, we can decompose $H(z)$ as:

$$\begin{aligned} H(z) = & \sum_{n=-\infty}^{\infty} h(nM) z^{-nM} + z^{-1} \sum_{n=-\infty}^{\infty} h(nM+1) z^{-nM} + \\ & \ldots + z^{-(M-1)} \sum_{n=-\infty}^{\infty} h(nM+M-1) z^{-nM} \end{aligned} \qquad (4)$$

Thus, the m-th phase of $h(n)$ is defined by:

$$h_{(m)}(k) \quad = h(kM+m) \qquad (5)$$

Using the filter definition from Eq. (2), the signal at the channel input in Fig. 4 is given by:

$$\begin{aligned} x(n) &= \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} \sum_{k=-\infty}^{\infty} A^{(i)}(k) h(n-kM) e^{j2\pi i(n-kM)/M} \\ &= \sum_{k=-\infty}^{\infty} h(n-kM) \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} A^{(i)}(k) e^{j2\pi in/M} \end{aligned} \qquad (6)$$

A change of notation n=lM+m allows us to introduce the polyphase components of $h(n)$. With the notations $x(lM+m)$ = $x_{(m)}(l)$ and $h(lM+m)$ = $h_{(m)}(l)$ for m=0, 1, ..., M-1, we obtain:

$$x_{(m)}(l) = \sum_{k=-\infty}^{\infty} h_{(m)}(l-k) \left\{ \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} A^{(i)}(k) e^{j2\pi im/M} \right\}$$

$$x_{(m)}(l) = \sum_{k=-\infty}^{\infty} h_{(m)}(l-k) a^{(m)}(k) \qquad (7)$$

where $a^{(m)}(k)$, $0 \le m \le M$-1, is the IDFT of $A^{(i)}(k)$ that may be efficiently implemented with the Inverse Fast Fourier Transform (IFFT). The m-th output of the IFFT is filtered by the m-th polyphase component of $h(n)$ and this filtering operation is performed at rate $1/T$. From Eq. (7) we can derive the efficient implementation shown in Fig. 5.

We can see in Fig. 5 that the filtering operation is performed at rate $1/T$ instead of M/T. At each instant, only the output of one polyphase filter needs to be and not the entire M samples as required in Fig. 4.

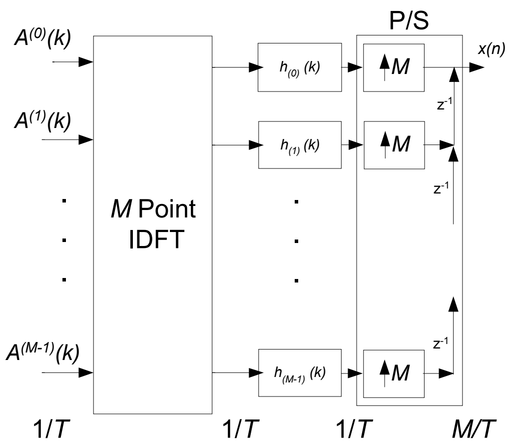

Fig. 5 FMT transmitter: Efficient Implementation

IV. FMT Receiver

In the receiver filter bank architecture (shown in Fig. 6) the receiving filters $\{ g^{(i)}(n) \}$ are designed to be matched to the corresponding ones in the transmitter, i.e. from Eq. (2): $G^{(i)}(f)=(H^{(i)}(f))^*$.

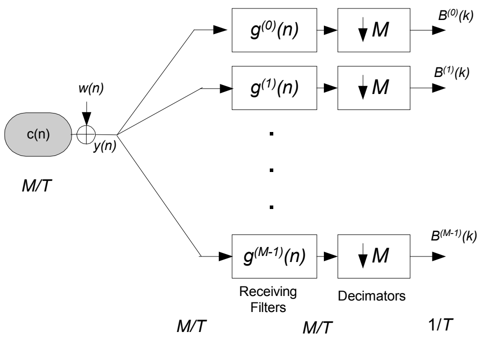

Fig. 6 FMT Receiver: direct implementation

Using the result that the inverse Fourier Transform of $(H^{(i)}(f))^*$ is $h^{(i)}(-n)$ we obtain:

$$g'^{(i)}(n) = (h^{(i)}(-n))^* \tag{8}$$

therefore, using Eq (2):

$$g'^{(i)}(n) = \frac{1}{\sqrt{M}} h(-n) e^{-j\frac{2\pi}{M}(-n)i}, n = -M\gamma+1,...,0 \tag{9}$$

However, this filter is anticausal. Since $g'(n)$ is defined for $n=-M\gamma+1,\dots,-1,0$ we need to apply a minumum delay of $M\gamma$-1 samples to make it causal. However, differently to some other publications [6], we will apply a delay which is a multiple of the block size M. Specifically, we delay it $M\gamma$ samples and we call this response $g^{(i)}(n)$. This sample delay difference compared with other publications is what will allow us to define the efficient implementation. We should note that since we are using multirate blocks, this difference of one sample makes a change into the overall response of the filter. In the efficient implementation, it will also allow us to take blocks of M samples in a different way, otherwise, there will be an offset in the way we take the blocks of samples in the transmitter and in the receiver.

Applying a delay of $M\gamma$ samples to Eq. (9), the matched filter will maximize the SNR at that specific instant [5]. However, since the prototype was not design with the perfect reconstruction constraint, we cannot say that the output of the filter bank is $A(k-\gamma)$.

Applying the $M\gamma$ delay to the receiver filters in Eq. (9) we obtain:

$$g^{(i)}(n) = g'^{(i)}(n - M\gamma) \tag{10}$$

which simplifies to:

$$g^{(i)}(n) = \frac{1}{\sqrt{M}} h(M\gamma - n) \cdot e^{j\frac{2\pi}{M}ni}, \quad n = 1,2,...,M\gamma \tag{11}$$

and since $h(n)$ is symmetric, then the receiver filter at the i-th subchannel is:

$$g^{(i)}(n) = \frac{1}{\sqrt{M}} h(n-1) \cdot e^{j\frac{2\pi}{M}ni}, \quad n = 1,2,...,M\gamma \tag{12}$$

Applying Eq. (12), at the output of the i-th subchannel in Fig. 6 we get:

$$\begin{aligned} B^{(i)}(k) &= \sum_{n=1}^{M\gamma} y(kM-n) g^{(i)}(n) = \\ &= \frac{1}{\sqrt{M}} \sum_{n=1}^{M\gamma} y(kM-n) h(n-1) e^{j\frac{2\pi}{M}ni} \end{aligned} \tag{13}$$

To introduce the polyphase components of $h(n)$ defined in Eq. (5) we decompose n as $n=lM+t$, $l=0,1,\dots,\gamma-1$ and $t=1,2,\dots,M$ to yield,

$$B^{(i)}(k) = \frac{1}{\sqrt{M}} \sum_{t=1}^{M} \sum_{l=0}^{\gamma-1} y(kM - lM - t) h(lM + t - 1) e^{j\frac{2\pi}{M}(lM+t)i}$$

$$= \frac{1}{\sqrt{M}} \sum_{t=1}^{M} \sum_{l=0}^{\gamma-1} y((k-l)M - t) h(lM + t - 1) e^{j\frac{2\pi}{M}ti} \tag{14}$$

If we make a change of variable $p=t-1$:

$$B^{(i)}(k) = \frac{1}{\sqrt{M}} \sum_{p=0}^{M-1} \sum_{l=0}^{\gamma-1} y_{(k-l)M-(p+1)} h(lM + p) e^{j\frac{2\pi}{M}(p+1)i} \tag{15}$$

and applying:

$$e^{j\frac{2\pi}{M}(p+1)i} = e^{-j\frac{2\pi}{M}(M-p-1)i} \tag{16}$$

we obtain:

$$B^{(i)}(k) = \frac{1}{\sqrt{M}} \sum_{p=0}^{M-1} \left\{ \sum_{l=0}^{\gamma-1} y_{(k-l)M-p-1} h(lM + p) \right\} e^{-j\frac{2\pi}{M}(M-p-1)i} \tag{17}$$

From Eq. (17) we are able to derive the efficient implementation in Fig. 7 where we apply the DFT operation (efficiently implemented with the FFT) to the M outputs of the M polyphase filters.

We can make some comments about Eq. (17):

(a) The first output in the receiver filter bank will be at k=1 (M samples at M/T rate) and not at k=0.

(b) The polyphase components of $h(n)$ are in reverse order with respect the DFT.

(c) The implementation in Fig. 7 is mirrored (matched) to the one in Fig. 5. Since the prototype is symmetric and has $M\gamma$ samples, for each of the polyphase components $h_{(i)}(n)=h(nM+i)$, the matched filter is actually $h_{(M-i-1)}(n)$. That is why they are in reverse order to the ones in Fig. 5, since the whole implementation is matched to that one.

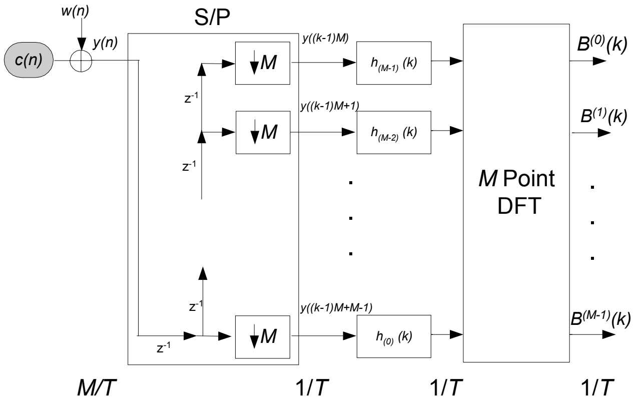

Fig. 7 FMT Receiver: Efficient implementation

V. OVERALL SYSTEM

We have shown, that the FMT system causes negligible ICI due to the high spectral containment. However, since the filters are not defined to accomplish the perfect reconstruction condition, the overall system will introduce ISI into each of the subchannels. This can be easily understood from Fig. 3 and from the Nyquist criterion for ISI free modulation [5]. We see that the Nyquist frequency (inverse symbol period) is exactly the same value as the frequency separation of the subchannels.

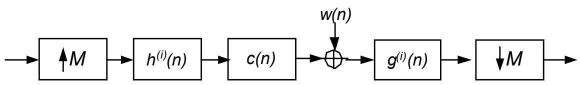

Fig. 8 Equivalent subchannel

Assuming that the subchannels are well separated in frequency, the overall response for each of the subchannels will be independent of the adjacent channels (no ICI) and it can be considered equivalent to the cascade of the *i*-th transmitter filter, the multipath channel, *c(n),* and the *i*-th receiver filter (Fig. 8). This response will need to be equalized by a per subchannel equalizer.

Different per subchannel DFE equalization techniques have been proposed in [6] for FMT.

We note here, that with our definition of the receiver filters, the overall response of the *i*th subchannel becomes:

$$h_{overall}^{(i)}(k) = \left(g^{(i)}(n) \otimes \sum_{p=0}^{L-1} c(p) h^{(i)}(n-p) \right)_{\downarrow M} \quad (18)$$

Using definitions from Eq. (2) and Eq. (12) we get:

$$h_{overall}^{(i)}(k) = \frac{1}{M} \sum_{n=1}^{M\gamma} h(n-1) \sum_{p=0}^{L-1} c(p) h(kM-n-p) e^{-j\frac{2\pi}{M}pi} \quad (19)$$

which is channel dependent and consequently we will need a different equalizer for each of the subchannels. To remove the ISI introduced by the overall response in Eq. (19), the coefficients of an equalizer can be computed based on the MMSE criterion.

As shown in [6], the equalizer can be designed as a cascade of a fixed equalizer (computed offline) that compensates for the ISI introduced by the prototype filter with a second equalizer that compensates for the effect of the transmission channel. This fixed equalizer will compensate for the overall response due to the transmitter and receiver filters only, i.e.,

$$h_{overall}^{(i)}(k) = \sum_{n=0}^{M\gamma-1} h^{(i)}(n) g^{(i)}(kM-n) \quad (20)$$

and using Eq (2) and Eq (12) we obtain:

$$h_{overall}^{(i)}(k) = \frac{1}{M} \sum_{n=0}^{M\gamma-1} h(n) e^{j\frac{2\pi}{M}ki} h(kM-n-1) e^{j\frac{2\pi}{M}(kM-n)i}$$

$$h_{overall}^{(i)}(k) = \frac{1}{M} \sum_{n=0}^{M\gamma-1} h(n) h(kM-n-1) \quad (21)$$

which is independent of the subchannel index *i*. Therefore, we can use the same coefficients to equalize each of the subchannels.

VI. CONCLUSIONS

In this paper we have presented the main ideas and advantages of FMT. From the definition of FMT we have derived an efficient implementation based on the IFFT/FFT and a network of polyphase filters operating *M* times slower than the direct filter bank implementation. We have also shown that to define an efficient implementation, each of the *M* matched filters at the receiver filter bank should be defined as:

$$g^{(i)}(n) = \frac{1}{\sqrt{M}} h(n-1) e^{j\frac{2\pi}{M}ni} \quad , n = 1,2,,\dots, M\gamma \quad (22)$$

This is the matched response to the transmitter filters defined in Eq. (2) after applying a delay of $M\gamma$ samples and not $M\gamma$-1 as used in previous publications.

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