Fast Start-up Equalizer for Radio ATM

M.P.Sellars*, J.Porter §, S.D.Greaves §, A.Hopper ††, W.J.Fitzgerald *
* Signal Processing Group, and † Laboratory for Communications Engineering, Dept. of Engineering, University of Cambridge, Trumpington St., Cambridge CB2 1PZ
§ ORL, 24A Trumpington St., Cambridge CB2 1QA, England
Fax: +44-1223-313542 Email: mps30@eng.cam.ac.uk http://www.orl.co.uk

Abstract—High speed TDMA wireless networks (such as radio ATM networks), require high data rate efficiency and fast turnaround times. In dispersive multipath channels, an equalizer is needed to remove ISI from the signal. The equalizer may be trained more quickly if its taps are preloaded with initial values. This paper describes a power ratio approximation method which produces estimates for the initial tap values. Simulation results demonstrate that these estimates are sufficiently accurate to considerably reduce the equalizer settling time. The performance and complexity of the method is compared with existing methods for calculating the initial tap values, and shown to offer advantages for high speed systems.

I. INTRODUCTION

Intersymbol interference (ISI) due to multipath is a major obstacle to be overcome in high speed wireless networks, especially in the indoor environment [1]. In TDMA systems, a decision feedback equalizer (DFE) may be used to remove ISI. Typically, each data packet will include a training sequence (preamble). The length of the training sequence represents an overhead in terms of data throughput, and the training time represents an overhead in terms of packet turnaround time. Possible algorithms for updating the equalizer taps are the least mean squares (LMS) algorithm and the recursive least squares (RLS) algorithm [2]. The LMS algorithm is attractive due to its low computational complexity, but it requires a longer training sequence and more iterations to train than the RLS algorithm [3].

One method to reduce the training sequence length and speed up the training time is to preload the DFE taps with initial values. Provided the initial values are reasonably accurate, the equalizer taps will be almost converged before equalization begins [1]. In this paper we describe a power ratio approximation method for calculating the initial DFE tap values, which has very low complexity.

II. EXISTING METHODS FOR CALCULATING INITIAL TAP VALUES

The most commonly-used solution for the initial equalizer taps is the minimum mean square error (MMSE) solution [4], [5]. The MMSE values may be found using matrix decomposition methods [6], [7], or else by calculating the inverse filter in the frequency domain using periodic training sequences [8], [9]. The disadvantage of these methods for calculating initial tap values is that they require a certain amount of computational effort. This makes these methods unattractive for high speed networks with tightly-restricted packet delays, for example wireless ATM.

III. SYSTEM MODEL

A data signal $x(t)$ is transmitted over a multipath channel to the receiver. The channel impulse response is characterised by the main pulse (cursor), leading echoes (precursors) and following echoes (postcursors), as shown in Fig. 1.

![Fig. 1. Channel Impulse Response showing main pulse, precursors and postcursors](image)

The sampled received signal is the convolution of the transmitted symbols $x$ with the sampled channel impulse response $h = [h_1, \ldots, h_L]^T$ plus noise $n$, where $L$ is the length of the sampled channel impulse response. The sampled received signal at time $t = kT$ (where $T$ is the symbol interval):

$$y_k = \sum_{i=0}^{L-1} x_{k-i} h_i + n_k$$

At the receiver, the data is sampled and passed into a correlator. The correlator computes the complex
correlation product of the sampled data sequence with a stored copy of the known preamble sequence of length \( P \). The correlator output at time \( t = kT \) is [10]:

\[
A(t) = A_k = R_{xy} R_{xy}^* \tag{2}
\]

where: \( R_{xy} = \sum_{i=1}^{P} x_i y_i^* \)

The value \( R_{xy}^* \) provides an estimate of the channel response \( h \). The peak value of the correlator output \( A(t_1) \) is used to extract frame synchronisation and symbol timing for the data packet. The data is then passed into a decision feedback equalizer (DFE), see Fig.2. The DFE has a feedforward (FF) filter of length \( M \) taps, and a feedback (FB) filter of length \( B \) taps.

![Block diagram of radio channel and receiver with DFE](image)

The combined impulse response of the channel and FF filter is

\[
d = h \otimes c \tag{3}
\]

where \( \otimes \) indicates convolution. The MMSE solution for the FF taps is of the form [4]:

\[
c(t) = \sum_{i=-\infty}^{0} g_i h^*(iT - t) \tag{4}
\]

\[
g_o = \frac{1}{N_o} (1 - U_o) \tag{5}
\]

\[
g_i = \frac{-U_i}{N_o} \tag{6}
\]

\[
U_i = \int_{-\infty}^{\infty} c(\tau) h(iT - \tau) d\tau \tag{7}
\]

where

\[
\sigma^2 = \frac{N_o}{L} \int_{-\infty}^{\infty} c^2(t) dt
\]

\[
\sigma_x^2 = \text{data variance}
\]

\[
N_o = \text{noise power spectral density}
\]

\[
N'_o = \frac{N_o}{L}
\]

The structure of the optimum receive filter may therefore be thought of as a matched filter with impulse response \( h^*(-t) \) followed by a one-sided (anti-causal) tapped delay line \( g \) with weights equal to \( g_i \) [4].

IV. POWER RATIO APPROXIMATION METHOD

Firstly, we classify the channel into one of two categories: line-of-sight (LOS) or non-LOS. A LOS channel will usually have a dominant main channel followed by some postcursor echoes. Initial values for the LOS channel are easily found by setting the reference tap of the equalizer \( c(0) = h^*(0) \), as suggested by Qureshi [11]. We concern ourselves with the more difficult task of finding initial values for a non-LOS channel which has significant power in leading echoes (precursors). Bidirectional or time-reversing equalizers [12] may be used for non-LOS channels, but these add extra complexity, and so will not be considered here.

An assumption often used in digital radio channel modelling is that the individual channel taps are independent random processes [2], [13, Chp.3]. This is also known as ‘uncorrelated scattering’. Using this assumption, it is possible to make a simple power ratio approximation for the MMSE initial tap values. The method is similar in spirit to Liu’s Tap-Variable Step Size LMS algorithm for fast fading channels [14] because both methods accelerate the convergence of those FF filter taps having the largest power.

A. Theory for the Power Ratio Approximation

Without loss of generality, we set the channel length \( L = 2n + 1 \), with the strongest pulse at \( h(0) \). The FB filter of the DFE only cancels ISI due to following echoes, and therefore we do not want to have any FF filter taps before the main tap active, since the effect of these taps cannot be cancelled. The power ratio method approximates the MMSE solution by using an individual scaling factor \( \alpha \) as in equation (8), applied to each coefficient of the matched filter. The expectations of the individual taps \( E \{ c(k) \} \) are approximated by the matched filter values \( h^*(-k) \) multiplied by a scalar constant \( \alpha_k \) for \( k = 0, 1, 2, \ldots, n \). Therefore:

\[
E \{ c(k) \} = \begin{cases} 
\alpha_k h^*(-k) & k \geq 0 \\
0 & k < 0 
\end{cases} \tag{8}
\]

Assuming that the individual components of \( h \) are independent (uncorrelated scattering), it is pos-
possible to show that:
\[
\alpha_0 = \frac{g_0}{1 + \left| h(-1)h^*(-1) + \cdots + h(-n)h^*(-n) \right|} \\
\alpha_{-1} = \frac{g_0}{1 + \left| h(-2)h^*(-2) + \cdots + h(-n)h^*(-n) \right|} \\
\vdots \\
\alpha_{-n+1} = \frac{g_0}{1 + \left| h(-n)h^*(-n) \right|} \\
\alpha_{-n} = g_0
\]  

(9)

Equation (9) is a stochastic solution for the feed-forward filter taps. If the channel is non-LOS with significant power in the precursors, this is a case of severe amplitude and phase distortion [15, Chp.7] and the values of \(\alpha_{-k} \) will be as shown in equation (9) above. If the channel is non-LOS, but the main pulse is significantly larger than any of the precursors, then further simplification is possible:

\[
g_0 \approx \frac{1}{1 + h^*(0)h(0)} \\
\alpha_{-k} \approx g_0 \text{ for } k = 0, 1, 2, \ldots, n
\]

(10)

Therefore, the expected value \(E\{c(k)\}\) is proportional to \(h^*(-k)\) for \(k = 0, 1, 2, \ldots, n\) with the constant of proportionality as given in the stochastic solution of equation (9). This can be simplified to the power ratio approximation of equation (10) for the case of a non-LOS channel with a dominant main pulse.

**B. Reshaping Profiles**

The scale factors for the FF filter taps are grouped into a scale factor vector

\[
A = [\alpha_{-n}, \ldots, \alpha_0, \ldots, \alpha_n]
\]

(11)

There are 2 types of scale factor vectors:

1. Scale factor vector for a LOS channel \(A_1\)
2. Scale factor vector for a non-LOS channel \(A_2\)

The exact values chosen for the scale factor vectors \(A_1\) and \(A_2\) may vary slightly according to the particular equalizer configuration, but once chosen they remain constant. Examples of scale factor vectors \(A_1\) and \(A_2\) for an equalizer with 9 symbol-spaced feedforward taps are given below:

**LOS channel**

\(A_1 = [0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.2, 0.2, 0.2, 0.2]\)

**non-LOS channel**

\(A_2 = [0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.8, 1.8, 1.8, 1.8]\)

Simulation results have shown that the scale factor vectors \(A_1\) and \(A_2\) as listed above, give good performance for a range of different radio channels. The scale factor vectors may be thought of as **reshaping profiles** for the matched filter (see Fig.3).

The profiles have been normalised so that the scale factor \(\alpha_0\) corresponding to the reference tap has a magnitude of unity.

**C. Channel Classification**

For reshaping, we need to start with the matched filter response \(R_{xy}(t) = h^*(-t)\). The correlator output \(R_{xy}\) gives reliable estimates of \(h^*\) provided that the autocorrelation function of the training sequence is flat in the region of interest around the central peak [16]. The next task is to classify the channel into one of two groups: LOS or non-LOS. This is done by calculating the power in each matched filter tap coefficient value where:

\[
\text{Power}_i = |c_i|^2 \text{ for } i = 1, \ldots, M
\]

(12)

Note that the values of \(|c_i|^2\) are immediately available since they are equal to the correlator output values \(A_k\), and so the values \(|c_i|^2\) need not be calculated. Denote the largest tap (reference tap) as \(c_m\), where \(1 < m < M\).

Power in leading echoes \(P_{leading} = \sum_{i=1}^{m-1} |c_i|^2\)

Power in following echoes \(P_{following} = \sum_{i=m+1}^{M} |c_i|^2\)

If \(P_{leading}\) is greater than \(P_{following}\), a non-LOS channel is detected.

**D. Preloading the FB filter taps**

The FB filter taps should converge to \(b = h \odot c\) which is the convolution of the channel impulse response with the FF filter [2]. We now propose an efficient method for performing this convolution: preload the FF filter tap values \([c_1, \ldots, c_M]\) and then run the set of correlator output values \(R_{xy} = [r_1^*, \ldots, r_M^*]\) (which is the same as the estimated channel impulse response \(h\)) as data samples through the FF filter. The output values of the FF filter will then be estimates of the FB filter tap coefficients \([b_1, \ldots, b_M]\). This procedure is realised very simply by attaching the values \([r_1^*, \ldots, r_M^*]\) at the front of the data packet and running them through the FF filter taps. These estimates for the FB taps are then used to preload the FB filter tap vector \(b = [b_1, \ldots, b_M]\) before equalization begins.

**V. Computational Complexity**

Here we list the number of arithmetic operations required to calculate the initial values for the FF taps only, where \(M\) is the number of FF taps. For purposes of comparison, it is assumed that \(M\) is equal to the length of the channel impulse response \(L\). Operation counts are given for the Levinson and back-substitution methods in [7], and for the Fourier Transform method in [17]. In the power
ratio approximation method, $M$ additions are required to sum the power in the leading and following echoes, and $M$ multiplies are required to multiply each matched filter coefficient value by its associated constant scaling factor (reshaping profile value). Furthermore, the $M$ multiplies may be simplified into shift registers and adder trees because the scale factor vectors $A$ consist of constant, real numbers. From Table I, it can be seen that the power ratio approximation method requires much fewer arithmetic operations than the matrix inversion (Levinson method), the back substitution method and the Fourier Transform method.

**TABLE I**

<table>
<thead>
<tr>
<th>Method for finding initial tap values</th>
<th>Complex Multiplies</th>
<th>Complex Adds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix inversion</td>
<td>$4M^2 + 4M + 2$</td>
<td>$4M^2 + 5M + 2$</td>
</tr>
<tr>
<td>Back substitution</td>
<td>$M^2$</td>
<td>$M^2 - 2M + 1$</td>
</tr>
<tr>
<td>Fourier Transform</td>
<td>$M + M \log_2 M$</td>
<td>$2M(1 + \log_2 M)$</td>
</tr>
<tr>
<td>Power Ratio Approx.</td>
<td>$M$</td>
<td>$M$</td>
</tr>
</tbody>
</table>

VI. SIMULATION RESULTS

A set of indoor radio channels, ‘Channel 1’, consisting of 1000 different channel realizations, was generated using power delay profiles of typical indoor channels listed by Pahlavan [13, Chp.6]. ‘Channel 1’ uses the Joint Technical Committee (JTC) ‘Indoor Office Channel B’ power delay profile (RMS delay spread $D_{RMS} = 176\text{ns}$), the profile is shown in Fig.4.

**Fig. 3** Reshaping profiles for (a) LOS channel, (b) non-LOS channel

**Fig. 4** Channel Impulse Response Profile for Channel 1

The amplitudes of the individual paths are Rayleigh distributed, and the phases uniformly distributed over $[0, 2\pi)$. Each channel is assumed to be time-invariant for the duration of a packet.
of length 2000 QPSK symbols are transmitted over each channel at a rate of 25Mbits. The transmit and receive filters are square-root Raised Cosine (RC) filters with a roll-off factor of 0.5. The received signal is detected using a DFE with 15 FF taps (symbol-spaced) and 10 FB taps, using the LMS algorithm for training. The following fast start-up preloading methods are used: (1) Cold start (no preloading) and (2) Preload Power Ratio Approximation tap values. The performance of the RLS algorithm is also shown for comparison.

The graph of Fig.5 shows the output mean square error (MSE) averaged over 1000 realizations of ‘Channel 1’ at SNR = 20 dB. All equalizers operated continuously in training mode for this simulation. The dotted vertical line marks the minimum number of iterations required to achieve $MSE < 0.1$, which corresponds to a symbol error rate below $10^{-4}$ at $SNR = 20 dB$, (using the method described in [18]). These results demonstrate that for this particular channel, the cold start LMS equalizer requires 346 symbols to train, whilst the LMS equalizer preloaded with the power ratio approximation values requires only 96 symbols (which is a saving of about 70% in this case). Clearly, this allows a significant reduction in both training sequence length and training time. The RLS equalizer (no preloading) requires only 36 training symbols to reach $MSE < 0.1$, but has a far higher computational cost than the LMS equalizers.

The graph of Fig.6 shows the SER (symbol error rates) for LMS equalizers with the different preloading schemes and various lengths of training sequences. The SER results are obtained from 1000 runs of 2000 symbols each. Fig.6(a) shows that if the training sequence length (Ntrain) is kept constant across the different equalizer types, the cold start LMS equalizer gives a very poor SER. The performance of the LMS (Power Ratio Approximation) algorithm is closer to that of RLS when using the same length training sequence. Fig.6(b) shows that in order to achieve similar SER performance to RLS, the LMS equalizers require longer training sequences. It is shown that preloading the LMS equalizer with power ratio approximation tap values achieves good performance with a reasonable length training sequence.

VII. CONCLUSIONS

We have described a simple, power ratio approximation method for finding the initial values for preloading a DFE. Simulations show that the method allows an LMS equalizer to give good error rate performance, whilst operating with significantly shorter training sequences and training times. The low computational requirement makes the power ratio approximation method suitable for use in high speed wireless networks such as radio ATM.

VIII. ACKNOWLEDGEMENTS

The first author is grateful for financial support from Cambridge Commonwealth Trust and the University of Cape Town (South Africa), and for support and facilities provided by the Olivetti & Orade Research Laboratory (ORL).

REFERENCES

Mean Square Error (averaged over 1000 runs) at SNR = 20dB

Fig. 5. Convergence of MSE for Channel 1

(a) Constant length training sequences

(b) Different length training sequences

Fig. 6. Symbol Error Rates for various training sequence lengths (Ntrain)