The Effect of Building Height Variation on the Multiple Diffraction Loss Component of the Walfisch-Bertoni Model

David Crosby†, Steve Greaves†, Andy Hopper*
† Cambridge Broadband Ltd., Edinburgh House, Cowley Rd, Cambridge CB4 0DS, U.K.
email: dcrosby@cambridgebroadband.com http://www.cambridgebroadband.com
* LCE, Dept. of Engineering, University of Cambridge, Cambridge, U.K.

Abstract—

The Walfisch model predicts the average path loss from a conceptualised model of the propagation environment in which propagation is assumed to take place over rows of buildings of equal height and spacing. The propagation loss is then separated into that resulting from free space wavefront spreading, multiple forward-diffraction past the rows of buildings and diffraction over the final rooftop down to the receiver.

In this paper we use numerical simulations to examine the effect of random building height variations on the multiple diffraction loss component of the model. In particular, our results show that the average multiple diffraction loss component is increased by any building height variations. A simple equation, which agrees to within 1 dB of the simulation results, and that relates building height variations, wavelength and average building separation to the increase in average multiple diffraction loss, is presented.

I. Introduction

The Walfisch-Bertoni [1] model is used widely for predicting the average path loss for mobile systems in urban areas. The model assumes that the street grid in a typical city organises buildings into rows that are nearly parallel and that an idealised representation for the urban environment would therefore be as shown in Figure 1, where the precise heights and spacings of the buildings have been ignored and the profile is characterised by just two parameters: the mean building spacing d, and the mean building height h_b. The transmitting antenna is positioned at height H above rooftop level and the receiver antenna is positioned at distance $-h_r$ relative to rooftop height. With this simple representation, Walfisch *et al.* explicates the average path loss from the elevated transmitter to the receiver as a sum of free space wavefront spreading, multiple forward-diffraction past the rows of buildings and diffraction over the final rooftop down to the receiver.

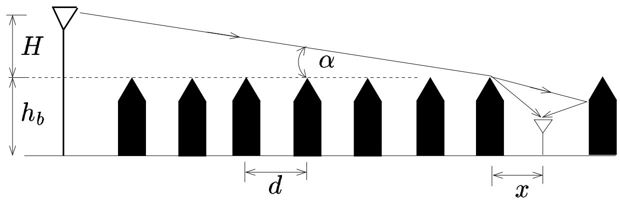

Fig. 1. Walfisch-Bertoni model

In decibels, the average excess loss $\langle XL \rangle$ (*i.e.* the loss above that of free space) predicted by the Walfisch-Bertoni model can therefore be written as the summation of two independent terms: the *multiple diffraction loss* as the field propagates past the rows of buildings L_{md}, and the *diffraction loss from the last rooftop-to-receiver* L_{rtr}:

$$\langle XL \rangle = L_{md} + L_{rtr} \tag{1}$$

The diffraction loss from the last rooftop-to-receiver is usually approximated by a Geometric Theory of Diffraction (GTD) solution for diffraction over an absorbing half-screen [1]. For Wireless Local Loop (WLL) systems, in which the receiver antenna is positioned at heights approaching or above average rooftop level, then other approximations based upon the height gain of ITU or the Okumura-Hata models offer better accuracy [2]. Indeed, when the receive antenna is positioned at average rooftop level, this latter term may be disregarded alltogether and any excess loss results only from the multiple diffraction loss component.

In order to evaluate the multiple diffraction loss component, all buildings are modelled as absorbing half-screens and the multiple diffraction loss component can be evaluated in a number of ways. The original analysis of Walfisch *et al.* [1] assumes the transmitter to receiver distance to be sufficiently large enough to approximate the spherical wave radiation originating from the elevated transmitter antenna as a localised place wave with angle of incidence $\alpha = \tan^{-1}\left(\frac{H}{R-x}\right) \approx H/R$, where R is the horizontal distance separating the transmitter and receiver antennas and $x \ll R$ is the horizontal distance of the receiver antenna from the last half-screen. The field in the aperture of adjacent half-screens can then be numerically evaluated by a simplified form of the Fresnel-Kirchoff integral. Walfisch *et al.* examined the amplitude of the settled field for a large number of simulation geometries and was able to show that a good approximation for the multiple diffraction loss component was:

$$L^W_{md} \approx -20\ log_{10}(Q_\infty(g_p)) \tag{2}$$

where $Q_\infty(g_p)$ is the value of the settled field, given by

$$Q_\infty(g_p) = 2.35\ g_p^{0.9} \quad , \quad 0.01 < g_p < 0.4 \tag{3}$$

and $g_p = \sqrt{\alpha^2 d/\lambda}$.

Maciel *et al.* [3] have since used a polynomial fit to the numerical results which has an improved accuracy of 0.5 dB and is valid over the extended range of $0.01 < g_p < 1$. The polynomial expression is:

$$Q_\infty(g_p) = 3.502 g_p - 3.327 g_p^2 + 0.962 g_p^3 \quad , \quad 0.01 < g_p < 1 \tag{4}$$

Other authors have since derived analytic solutions for the amplitude of the multiply-diffracted rooftop field [4] [5]. The derivation of these analytic solutions use a physical optics approximation of the field at rooftop height (either the Vogler or Fresnel-Kirchhoff integrals) and the resulting multi-dimensional integral is evaluated by a general method proposed by Boersma [6].

The Walfisch-Bertoni propagation model is widely used and has been verified with measurements by independent studies in Europe [7], [8], the USA [1] and Australasia [9]. The European Cooperation in the Field of Scientific and Technical Research (COST) program for GSM systems has also incorporated part of the model into the COST231 model [10], [11], [12].

The error between the average path loss predicted by the model and that observed in practice will be smallest when the propagation environment conforms closely with the models assumptions (*i.e.* those urban environments exhibiting minimal variation in the height and separation of buildings). In particular, any building height variations can be expected to cause a significant error in the model predictions and this has been theoretically investigated by Chung *et al.* [2], Chrysanthou *et al.* [13] and Saunders *et al.* [14].

The study by Chrysanthou *et al.* [13] also used the plane wave multiple edge technique of Walfisch *et al.* . The buildings (modelled as absorbing screens) were assigned heights that were uniformly distributed between $\langle h_b \rangle - \Delta_h/2$ and $\langle h_b \rangle + \Delta_h/2$, where $\langle h_b \rangle$ is the average building height and Δ_h is the maximum height deviation, which ranged from 1 to 9 metres. The field immediately above each screen is calculated from a modified form of the Fresnel-Kirchoff integral. The field at street level is then calculated by including the rooftop-to-receiver loss. For a limited number of configurations of average building heights and incidence angles, the statistics of the street level field were calculated.

As compared to the settled field solution, which applies for buildings with uniform height equal to $\langle h_b \rangle$, the average signal strength at street level was found to be reduced by approximately $-0.43\Delta_h$ dB and was almost independent of $\langle h_b \rangle$. This is an important result and suggests that the Walfisch-Bertoni model is likely to underestimate the average path loss values in those areas which have large deviations of building heights from the mean.

In the study by Saunders *et al.* [14], the excess loss is calculated directly at street level from a Monte-Carlo evaluation of the Vogler integral. The radiation emitted by the transmitter is approximated as a plane wave and only the last five buildings in the path are modelled. The building heights are drawn from a normal distribution with a standard deviation of 1 m. The building spacing is constant and set to 40 m. Only one angle of incidence is examined. At 900 MHz, the effect of the random building heights is found to increase the average path loss by 0.8 dB, thereby confirming the results of [13]. This also suggests that the mean difference is independent of the actual distribution of building heights, as would be expected from the central limit theorem.

Chung *et al.* [2] also used the plane wave multiple edge technique of Walfisch *et al.* to evaluate the field strength over a range of receiver antenna heights (both above and below average rooftop height). Building were modelled as attenuating phase or absorbing screens and the effect of ground reflections was considered. Only a single distribution of building heights was evaluated: that of a uniform distribution extending over the range of 6–14 metres (roughly corresponding to buildings with 2–4 storeys) and at a screen spacing of $d/\lambda = 50$. For absorbing screens the average of the multiple diffraction loss component was found to increase by approximately 3.7 dB. Moreover, the increase was observed to be relatively independent of the value g_p.

In this study we investigate the dependence on the average of the multiple diffraction loss component, $\langle L_{md} \rangle$, with building height variance in further detail. We consider a much larger number of different system configurations of building height variance, mean building separation, and wavelength than previous studies. We assume that propagation takes place over n equally spaced buildings which are modelled as absorbing screens (Figure 2). The height of the screens, denoted $h_1, h_2, \ldots, h_{n-1}$, are randomly distributed with mean $\langle h_b \rangle$ and variance ${\sigma_h}^2$. The height of the nth screen is constrained to the average building height $h_n = \langle h_b \rangle$.

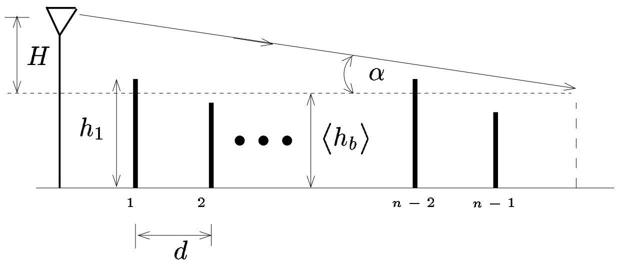

Fig. 2. Evaluating the multiple diffraction loss component with random screen heights

Using these simplifications, a cylindrical wave simulation technique is used to determine values for the multiple diffraction loss component $\langle L_{md} \rangle$, and hence the range dependence of the average path loss characteristic. For each system configuration, the evaluations were performed over 100 screens and from the analysis of the simulation results, an approximating expression for the average of the multiple diffraction loss component, which incorporates the building height variability, is presented.

In the next section we describe the simulation technique and in Section III we present the simulation results.

II. Cylindrical Wave Simulation Method

The simulation technique used is similar to the technique used by Piazzi [15] and is based upon repeated numerical evaluation of the Fresnel-Kirchhoff integral and the transmitter is modelled as a radiating line source. The technique is general enough to calculate the field amplitude, and hence the corresponding path loss value, at any height above or below the half-screen and uses aperture interpolation to minimise numerical roundoff errors

The geometry for the cylindrical wave simulation is shown in Figure 3. A series of N absorbing half-screens are equally separated by the distance d (corresponding to the mean build-

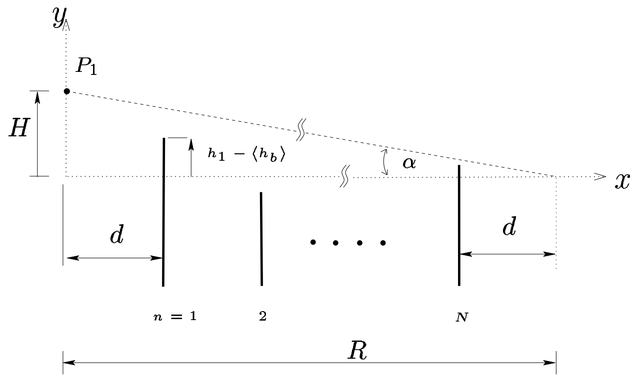

Fig. 3. Geometry for Fresnel-Kirchhoff evaluation of cylindrical wave diffraction by a series of absorbing half-screens.

ing spacing). For the purposes of this simulation the x-axis is assume to lie co-incident with the average screen height, the y-axis is through the plane of the $n = 0$ screen and the z-axis is directed perpendicularly into the page. The top of the nth screen has (x, y) co-ordinates of $(nd, h_n - \langle h_b \rangle)$, where h_n is the height of the nth screen.

A uniform magnetic line source parallel to the z axis is located at the position of the transmit antenna (the point P_1), with coordinates $(0, H)$. The radiated cylindrical wave has a z component only, and for $kr \gg 1$ the field in the aperture of the $(n = 1)$ half-screen is $H_0(y) \approx e^{ikr}/\sqrt{kr}$ where $r = \sqrt{d^2 + (y - H)^2}$.

The field on the $(n+1)$ half-screen at the points $y = p\ \Delta_{n+1}$, where p is an integer, is obtained from the field on the nth half-screen by a numerical evaluation of the Fresnel-Kirchhoff integral [15]:

$$
\begin{aligned}
H_{n+1}(p\Delta_{n+1}) = \Delta_n e^{-j\pi/4} & \sum_{m=\lfloor (h_n - \langle h_b \rangle)/\Delta_h \rfloor}^{\infty} \Bigg\{ \\
& \frac{1}{D_{p,m}} \Big[H_n(m\Delta_n + \Delta_n) f_{p,m+1} \\
& \cdot e^{-jkr_{p,m+1}} - H_n(m\Delta_n) f_{p,m} e^{-jkr_{p,m}} \Big] \\
& + \frac{j}{k {D_{p,m}}^2} \Big[H_n(m\Delta_n + \Delta_n) f_{p,m+1} \\
& \cdot e^{-jk(R_m - R_{m+1})} - H_n(m\Delta_n) f_{p,m} \Big] \\
& \cdot \Big[e^{-jk(R_{m+1} - R_m + r_{p,m+1})} - e^{-jkr_{p,m}} \Big] \Bigg\}
\end{aligned}
\tag{5}
$$

where Δ_n is the numerical step size in the aperture of the nth half-screen, h_n is the height of the nth screen, $\lfloor x \rfloor$ denotes the floor of x and the functions $r_{p,m}$, $f_{p,m}$, R_m and $D_{p,m}$ are given by:

$$
\begin{aligned}
r_{p,m} =& \sqrt{(p\Delta_{n+1} - m\Delta_n)^2 + d^2} \\
f_{p,m} =& \frac{nd/R_m + d/r_{p,m}}{2k\sqrt{\lambda r_{p,m}}} \\
R_m =& \sqrt{(nd)^2 + (H - m\Delta_n)^2} \\
D_{p,m} =& R_m - R_{m+1} + r_{p,m} - r_{p,m+1}
\end{aligned}
\tag{6}
$$

This implementation uses two different step sizes Δ_n and Δ_{n+1} on the nth and $(n + 1)$th half-screens respectively.

(5) is susceptible to round-off errors in some instances. This is through the $D_{p,m}$ term, which appears in the denominator of (5), and can depend on a small difference. This source of error can be avoided by ensuring that the minimum of the phase of the aperture field, which occurs at $y' = \frac{ny+H}{n+1}$, lies at one end of an integration step. This can be guaranteed by setting the values of Δ_n and Δ_{n+1} according to:

$$\Delta_n = \frac{H}{(n+1)q_n} \tag{7}$$

$$\Delta_{n+1} = \Delta_n \frac{n+1}{n} \tag{8}$$

where q_n is an integer. For propagation over N screens, one possible solution that satisfies these conditions is $\Delta_n = \frac{Hn}{p}$, $n = 1, 2, \ldots, N$, where p is an integer. This implies that Δ_1 should be chosen to be smaller than λ/N in order to restrict the step size on the Nth screen to a value less than λ. If propagation takes place over a large number of screens this quickly becomes computationally intensive. To avoid this complexity the following method was used. At the beginning of each iteration of (5), the field in the aperture of the nth screen is re-sampled at spacings $m\Delta_n$, where Δ_n is calculated according to (7) and with q_n chosen appropriately to ensure that $\Delta_n < \lambda/4$. The re-sampling process was implemented using cubic spline interpolation. At each iteration the value of Δ_{n+1} is then obtained from (8). This procedure reduces the number of simulation points to an acceptable size while avoiding the conditions that lead to the generation of unacceptably large roundoff errors.

III. Simulation Results

Screen heights were assigned random values drawn from a uniform distribution defined over the range $\langle h_b \rangle - \Delta_h/2$ to $\langle h_b \rangle + \Delta_h/2$. A more exact analysis would use the actual distribution of building heights measured from a real city. However, it can be argued that for a given variance of building heights, and for propagation over a large number of buildings, the statistics of the path loss should be nearly independent of the distribution function chosen for the buildings [13]. Consequently, a uniform distribution was used for simplicity.

Table I lists the configuration parameters for each of the systems that was simulated. Transmitter heights ranged from 10 m to 40 m and the screen spacing from 50λ to 1000λ. For each configuration, screen height variations of $\Delta_h = 0, 1, 3, 5, 7$ and 9 m were simulated. The simulations were performed over $N = 100$ screens with the excess loss computed at average rooftop height (*i.e.* x-y co-ordinates of $nd, 0$).

In order to obtain an accurate estimate of the range dependence of the average signal strength for a given value of Δ_h, each configuration was simulated fifty times with a new set of random screen heights used on each simulation run. This resulted in 5000 data points for each configuration. The simulation path loss at average rooftop height from each of the fifty simulation runs were processed to produce a set of average multiple diffraction losses, $\langle L_{md}^{\text{sim}}(n) \rangle$, at distances of $R = nd$, where $n = 1, 2, \ldots 100$.

Figure 4 displays the simulation results for the configuration of $\lambda = 0.125$ m, $H = 10$ m, $d = 400\lambda$ and $\Delta_h = 7$ m. The

λ (m)	H (m)	d (λ)	Δ_h (m)
0.5	20	50	0,1,3,5,7,9
0.5	10	100	0,1,3,5,7,9
0.5	20	100	0,1,3,5,7,9
0.5	40	100	0,1,3,5,7,9
0.2	20	125	0,1,3,5,7,9
0.2	20	250	0,1,3,5,7,9
0.125	10	200	0,1,3,5,7,9
0.125	20	200	0,1,3,5,7,9
0.125	20	400	0,1,3,5,7,9
0.052	10	1000	0,1,3,5,7,9

TABLE I
CONFIGURATION PARAMETERS OF THE SIMULATED SYSTEMS.

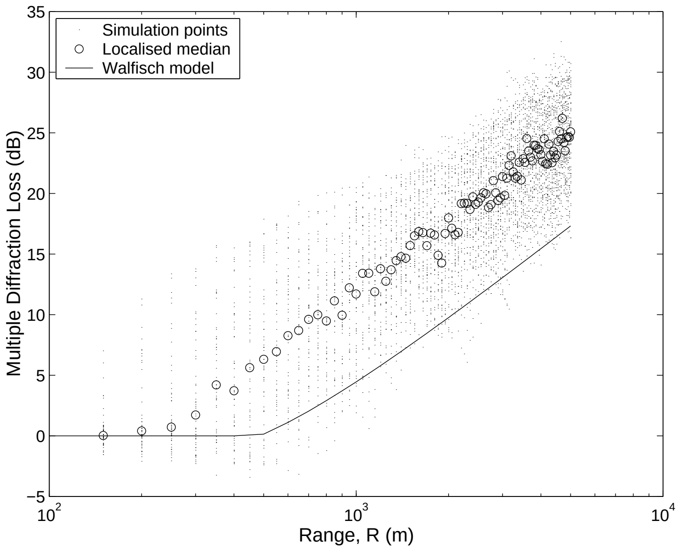

Fig. 4. Simulated excess loss at average rooftop height, $L_{md}^{\text{sim}}(n)$ for $H = 10$ m, $d = 400\lambda$ and $\Delta_h = 7$ m.

ordinate is plotted as excess loss in decibels and the abscissa corresponds to the range from the transmitter. The black dot points are the simulated excess loss data for all fifty runs and the circle points correspond to the average of the excess multiple diffraction loss, $\langle L_{md}^{\text{sim}}(n) \rangle$, at distances of $R = nd$.

Also shown on the same plot is the predicted excess loss from the Walfisch-Bertoni model (*i.e.* uniform screen heights) obtained from (2). This corresponds to the excess loss above each screen, where the edges of all screens are coincident with the x-axis (*i.e.* zero height). A logarithmic regression of the simulation results reveal that the average excess loss has a slope of 18.6 dB per decade, a value almost identical to that of the Walfisch-Bertoni model, which has a slope that approaches 20 dB per decade. This suggests that the effect of random building heights is to increase the average path loss by an amount δ_L, which in this case is approximately 7 dB, and leave the slope of the excess loss characteristic relatively unchanged, a finding in agreement with [2].

Further support for this argument is provided by statistics of the regression slope for all the simulations in Table I. The mean regression slope was 18.1 dB/decade and the standard deviation

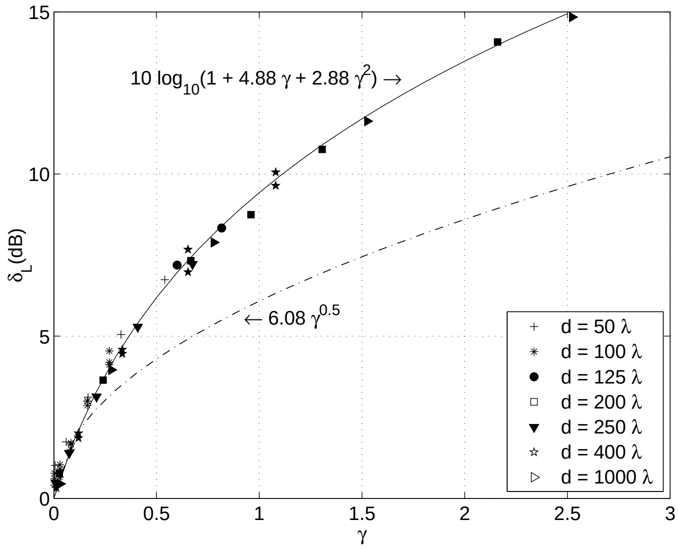

Fig. 5. The increase in the average excess loss at average rooftop height (as compared to the Walfisch-Bertoni model) due to random screen heights. The abscissa is plotted as the dimensionless parameter $\gamma = \frac{\sigma_h^2}{\lambda d}$.

1 dB/decade.

For each system configuration in Table I , δ_L was calculated as an average value according to the following formula:

$$\delta_L = \frac{1}{N - i_o + 1} \sum_{i=i_o}^{N} (\langle L_{md}^{\text{sim}}(i) \rangle - L_{md}^{W}) \qquad (9)$$

where L_{md} is the corresponding Walfisch-Bertoni prediction at $R = id$ obtained from using (2), and i_o is the screen number at which L_{md}^{W} first becomes non-zero.

Figure 5 shows the values of δ_L calculated for all the configurations in Table I. The ordinate is the parameter $\gamma = \frac{\sigma_h^2}{\lambda d}$, where σ_h is the standard deviation of the screen heights. For a uniform distribution σ_h is related to the maximum deviation of the screen heights by $\sigma_h = \frac{\Delta_h}{\sqrt{12}}$. The value of δ_L is observed to be independent of transmitter height and solely determined by the value of γ.

The dependence of δ_L on the parameter γ can be explained theoretically by considering the Vogler expression for multiple diffraction over screens of random height. Specifically, in [16] it is shown that for a symmetrical probability distribution of building heights then δ_L can be expressed as a power series in γ:

$$\delta_L = 10 \ \log_{10}(1 + a_1\gamma + a_2\gamma^2 + \dots) \qquad (10)$$

where the coefficients $a_1, a_2, \dots$ may have some dependence on g_p and the number of screens passed. However, the results in Figure 5 indicate that any such dependence is secondary. For $\gamma < 2.5$ a good fit to the numerical simulations can be obtained by considering only the first three terms of the power series in (10). The values of the coefficients a_1 and a_2, were found to be 4.88 and 2.88 respectively, resulting in the following expression for δ_L:

$$\delta_L = 10 \ \log_{10}(1 + 4.88\gamma + 2.88\gamma^2) \qquad (11)$$

(11) is plotted in Figure 5 as a solid black line and agrees to within 1 dB of the simulation results.

An approximating expression for the average multiple diffraction loss component (for uniformly distributed building heights), $\langle L_{md} \rangle$, can therefore be obtained from (11) and (2) as

$$\langle L_{md} \rangle = -20 \; log_{10} \; Q_{\infty}(g_p') \tag{12}$$

where the dimensionless parameter g_p' is specified as

$$g_p' = \frac{\alpha}{(1 + 4.88\gamma + 2.88\gamma^2)^{0.556}} \sqrt{\frac{d}{\lambda}} \tag{13}$$

This formulation for g_p' is derived from consideration of the expression for $Q_{\infty}(g_p)$ in (3). Note that if the screen heights are uniform, then $\gamma = 0$ and the above equation reduces to the standard definition for g_p.

The accuracy of (12) was quantified from the mean and root-mean-square (rms) of the prediction error. The prediction error, e_n, defined as $e_n = \langle L_{md} \rangle - \langle L_{md}^{\text{sim}}(n) \rangle$. For each configuration in Table I , the prediction errors at average rooftop level were calculated at ranges of $R = nd$. From the resulting 100 error values the mean and rms values could be determined. The maximum (in magnitude) of the mean and rms prediction errors for all systems was 0.88 dB and 1.2 dB respectively. These relatively low error statistics indicate that predictions of (12) are in good agreement with the simulation results.

The average value of δ_L calculated from the absorbing screen simulations of Chung *et al.* [2] at a frequency of 900 MHz with $\Delta_h = 8$ m and $d = 50$ m is 3.7 dB. This agrees closely with the predicted value of 4.6 dB from (11).

It is also interesting to compare the prediction of (11) with the results of Chrysanthou *et al.* [13] and Saunders *et al.* [14] calculated for a receiver positioned at street level. Chrysanthou *et al.* calculated that the net effect of random building height variations at 900 MHz and for $d = 50$ m was to increase the average excess loss by an amount equivalent to $6.08\sqrt{\gamma}$. In Figure 5 this expression (shown as a dotted line) is compared to our simulation results. The agreement is good given that the simulations of Chrysanthou *et al.* were performed at only one value of frequency and screen spacing, and only for $\gamma < 0.5$. For an equivalent value of $\gamma = 0.08$, Saunders *et al.* calculated $\delta_L = 0.8$ dB, whereas (11) predicts a reasonably close value of $\delta_L = 1.4$ dB. The differences may be attributed to the fact that only five buildings of random height are considered in [14]. In addition, applying (11) to receivers positioned below rooftop level effectively constrains the last building to the average building height which will introduce some error, whereas the analyses of [13] and [14] allow the last building to have random height.

IV. Conclusions

This paper has examined the effects of random building height variations on the multiple diffraction component of the Walfisch-Bertoni model. A large number of system configurations of mean building separation, building height variance and transmitter height were simulated. It was found that any building height variations act to increase the average of the multiple diffraction loss component, a finding which is in agreement with previous studies.Furthermore, the decibel increase in the average multiple diffraction loss was found to be dependent on the ratio of the building height variance to the product of the wavelength and building separation. For a building height distribution which is symmetrical, a simple extension to the settled field solution of Walfisch *et al.* , which incorporates this dependence is proposed.

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