Euclidean Distances in Quantized Spaces with Pre-stored Components for MIMO Detection

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***Abstract*— This paper proposes a technique to reduce the complexity involved in the maximum likelihood detection of multiple input multiple output (MIMO) spatial multiplexing systems. Both the received lattice and the components of the received signal are quantized, corresponding to a mapping into a multidimensional space divided into hypercubes. In the new space the maximum likelihood detection criterion can be applied making use of a small look-up table storing all the exact possible distance components in each dimension of the quantized space. The number of pre-stored elements can be as small as the number of quantization levels per dimension. This procedure eliminates the multiplications involved in the calculation of the squared Euclidean distances. The impact of the quantization error is assessed by means of simulations over fast flat fading channels. Near optimum performance is achieved with only 5 bits representing each dimension of the received signal.**

I. Introduction

Multiple input multiple output (MIMO) systems have been at the centre of the research in wireless communications during the last decade [1, 2]. MIMO systems make use of the existence of different channels between the transmitter and the receiver due to the existence of multipath propagation between the two parties. The systems can be designed in order to exploit that diversity either to maximize the diversity of the link (space-time coding systems) or to maximize the overall bit throughput (spatial multiplexing systems). Research into the tradeoffs of the two frameworks is still being researched and recently a scheme which considers switching between the two frameworks was proposed [3]. Spatial multiplexing allows raising considerably the bit rate of wireless links; however this comes at the expense of an enormous increase in the complexity of optimum detection as the number transmitting antennas is incremented and modulations with increasing spectral efficiency are used. For the case of equally probable symbols the maximum a posterior (MAP) detection strategy corresponded to the maximum likelihood (ML) criterion, leading to a number of comparisons that grows exponentially with the number of transmission antennas. For this reason, research on sub-optimal receivers has always been central to spatial multiplexing, leading to both linear and non-linear receivers [2, 4] which encompass a range of different power performances. The receivers using zero-forcing (ZF), and minimum mean square error (MMSE) criteria constitute the linear receivers whilst the most used example of non-linear receivers is the vertical Bell Laboratories layered space-time receiver, also called ordered successive interference cancellation (OSIC) receiver [4, 5]. The sub-optimality of all these receivers does not allow them to fully extract the diversity available, i.e., the curves representing the number of errors against the signal to noise ratio (SNR) are less steep. The call for near ML performance originated research into low complexity exact methods where sphere decoding [4], and lattice reduction [5, 6] are the most prominent ones. The later retains the diversity of ML, exhibiting performance curves parallel to those of ML (corresponding to a power penalty) at only a fraction of the computational cost. Attempts to simplify the brute force approach undertaken by ML receivers are always limited by the number comparisons needed, and assessments of the complexity of the algorithms are chiefly made by the total number of multiplications required [7].

This paper presents a technique to simplify the calculation of the squared Euclidean distances required in ML detection. The proposal is inspired by similar problems in computer graphics and image processing where approximations for the Euclidean distance are used [8, 9] (not for the squared Euclidean distances though). As is the case of the approximations that modify the forward-backward MAP algorithm into a max-log MAP algorithm [10-Sec.4.3]. The application of completely multiplication-free Euclidean distances have been used in MIMO with minimum penalty [11], however in that case the approximation occurs in the bidimensional space of the transmitted constellation. Instead, this paper proposes the simplification of the evaluation of the Euclidean distances in the received multidimensional space where ML search is undertaken. A look-up table technique is used in order to obtain the components of the squared Euclidean distances [12].

II. Traditional Receivers

A MIMO system under flat fading can be represented in a complex baseband model as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}\ , \qquad (1)$$

where $\mathbf{x} = \left[\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_T}\right]^T$, that is, each component is transmitted from each one of the n_T transmitter antennas, $\mathbf{y} = \left[\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{n_R}\right]^T$, where each component corresponds to the signal in each one of the n_R receiver antennas and the noise vector $\mathbf{n} = \left[\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_{n_R}\right]^T$ is composed of complex Gaussian

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random variables with zero mean and variance σ_n^2=1 (0.5 in both real and imaginary parts). The components in **x** are taken from a set $\Xi_{\mathbf{C}}$ of symbols belonging to an *M*-ary complex constellation with real and imaginary parts taken from the set $\Xi_{\mathbf{R}}$. The channel matrix **H** for this $n_R \times n_T$ system is

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n_T} \\ h_{21} & h_{22} & \cdots & h_{2n_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R1} & h_{n_R2} & \cdots & h_{n_Rn_T} \end{bmatrix} \quad (2)$$

where all the elements are i.i.d complex circularly symmetric Gaussian random variables with zero mean and unit variance (i.e., with variance 0.5 in both the real and imaginary components). This model can be converted into one with the double number of dimensions where all the elements are real numbers by stacking the real and imaginary parts of **x** and **y** and constructing a new equivalent channel matrix

$$\begin{bmatrix} \Re\mathbf{y} \\ \Im\mathbf{y} \end{bmatrix} = \begin{bmatrix} \Re\mathbf{H} & -\Im\mathbf{H} \\ \Im\mathbf{H} & \Re\mathbf{H} \end{bmatrix} \begin{bmatrix} \Re\mathbf{x} \\ \Im\mathbf{x} \end{bmatrix} + \begin{bmatrix} \Re\mathbf{n} \\ \Im\mathbf{n} \end{bmatrix} \quad (3)$$

where the symbols $\Re$ and $\Im$ indicate the real and imaginary components respectively. Equation (3) can be written as

$$\mathbf{y}_r = \mathbf{H}_r \mathbf{x}_r + \mathbf{n}_r . \quad (4)$$

Thus $\mathbf{x}_r = \left[\mathbf{x}_{r,1}, \mathbf{x}_{r,2}, \ldots, \mathbf{x}_{r,2n_T}\right]^T$, $\mathbf{y}_r = \left[\mathbf{y}_{r,1}, \mathbf{y}_{r,2}, \ldots, \mathbf{y}_{r,2n_R}\right]^T$, and $\mathbf{n}_r = \left[\mathbf{n}_{r,1}, \mathbf{n}_{r,2}, \ldots, \mathbf{n}_{r,2n_R}\right]^T$. Also, $\mathbf{H}_r$ doubles the dimensions with respect to **H**. The problem of detecting the transmitted symbols is optimally solved by the maximum likelihood procedure based in squared Euclidean distances, i.e.,

$$\hat{\mathbf{x}}_{r(ML)} = \min_{\mathbf{x}_r \in \Xi_{\mathbf{R}}^{2n_T}} \left\{ \left\| \mathbf{y}_r - \mathbf{H}_r \mathbf{x}_r \right\|^2 \right\}. \quad (5)$$

This implies the measurement and comparison of M^{n_T} squared Euclidian distances per component of the $\mathbf{R}^{2n_R}$ space or per component of the complex space $\mathbf{C}^{n_R}$ in (1). Hence, the search increases exponentially with the number of transmission antennas for a given modulation.

The simplest linear receiver corresponds to a ZF criterion, i.e., an inversion of the channel matrix. In the general case, as the channel matrix **H** is not necessarily square (corresponding to n_T=n_R), then the Moore-Penrose pseudo-inverse of **H** is used, which is given by $\mathbf{H}^+ = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$ where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transposition. In this case the "filtering" matrix

$$\mathbf{W}_{ZF} = \mathbf{H}^+ . \quad (6)$$

This procedure induces noise enhancement in the constellation space where the decisions of constellation symbols are to be made. The MMSE receiver takes the noise into account, resulting in the following "filtering" matrix [2]

$$\mathbf{W}_{MMSE} = \left(\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}_{n_T} \right)^{-1} \mathbf{H}^H \quad (7)$$

The OSIC-ZF receiver uses (6) in an initial iteration. The component of y_r with the smallest noise amplification is selected and detected (by inspection of the rows of $\mathbf{W}_{\mathrm{ZF}}$ [5]. The next step is to remodulate that symbol and subtract its effect from the original received $\mathbf{y}_r$. This procedure is repeated for the new signal, originating the detection of a second component of **y**, and is repeated until all components have been detected. The OSIC-MMSE receiver operates similarly by applying (7) instead of (6). The complexity of OSIC is only polynomial with n_T [7] but the fact that an erroneous decision in a component cascades the error throughout the components to be subsequently detected explains the fact that OSIC is not able to exploit entirely the diversity available in the system.

III. Detection in Quantized Spaces

The proposed receiver starts by stacking the imaginary components of **y**, generating $\mathbf{y}_r$, as described in (3)-(4). Then the received n_T– dimensional space is quantized and all the possible points $\mathbf{y}_r^{(i)}$ on the lattice are mapped into the quantized space. Denoting the quantization process by $Q(\cdot)$, the resulting quantized vector is

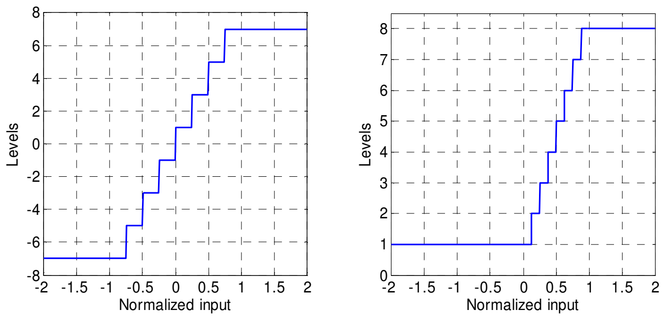
$$\left[\tilde{y}_{r,1}, \tilde{y}_{r,2}, \cdots, \tilde{y}_{r,2n_T}\right]^T = Q\left(\left[y_{r,1}, y_{r,2}, \cdots, y_{r,2n_T}\right]^T\right). \quad (8)$$

Each one of these component $\tilde{y}_{r,i} \in \{c_1, c_2, c_3, \cdots, c_L,\}$, which are the L=2^b possible quantization levels (described by *b* bits) with a uniform step

$$q = c_i - c_{i+1},\ i \in \{1, 2, \cdots, L\}. \quad (9)$$

Defining $\mathbf{y}_r^{(l)} = \mathbf{H}_r\mathbf{x}_r^{(l)}$ as each one of the points in the lattice constituted by all possible received vectors, the Euclidean distances needed to compute in (5) are of the form

$$\left\| \mathbf{y}_r - \mathbf{y}_r^{(l)} \right\|^2 = \sum_{i=1}^{n_R} \left(\tilde{y}_{r,i} - \tilde{y}_{r,i}^{(l)} \right)^2,\ l=1, 2, \cdots, M^{n_T}. \quad (10)$$



(a) Bipolar (b) Unipolar

Fig. 1. Quantizer with *L*=8 levels (3 bits) for each dimension.

In [12] the authors consider a unitary increment *q* and positive c_i whereas we propose that $q \in \mathbf{Z}$. In fact, the restriction to *q*=1 would bring difficulties to the implementation of the technique that will be presented in the next section. Each component of $\mathbf{y}_r^{(i)}$ can be either positive or negative and therefore $Q(\cdot)$ should be able to deal with both cases. The extension to negative components would require a central zero level and hence an even number of levels.

Figure 1 (a) depicts the appropriate bipolar quantizer. It should be noticed that both the received signal and the lattice itself are bounded to $[-m_L, +m_L]$ per real dimension, corresponding to the clipping imposed by $Q(\cdot)$. We define this maximum value for the lattice as $m_L = \max\{y_r^{(l)}\}$, taken over all the possible $y_r^{(l)}$ points of the lattice for each different channel realization. The use of the quantizer in Figure 1 (b) would require shifting each one of the complex lattices (comprising M^{n_T} points in each received dimension) by $\frac{m_L}{2} + i\frac{m_L}{2}$.

IV. Look-up Table Technique

All the possible values of the distance components $\left(\tilde{y}_{r,i} - \tilde{y}_{r,i}^{(l)}\right)$ in (10) are an element of

$$\Omega^{(1)} = \begin{bmatrix} (c_1-c_1)^2 & (c_1-c_2)^2 & (c_1-c_3)^2 & \cdots & (c_1-c_L)^2 \\ (c_2-c_1)^2 & (c_2-c_2)^2 & (c_2-c_3)^2 & \cdots & (c_2-c_L)^2 \\ (c_3-c_1)^2 & (c_3-c_2)^2 & (c_3-c_3)^2 & \cdots & (c_3-c_L)^2 \\ (c_4-c_1)^2 & (c_4-c_2)^2 & (c_4-c_3)^2 & \cdots & (c_4-c_L)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (c_L-c_1)^2 & (c_L-c_2)^2 & (c_L-c_3)^2 & \cdots & (c_L-c_L)^2 \end{bmatrix} \quad (11)$$

An inspection of $\Omega^{(1)}$ allows us to notice its expected symmetry and, furthermore, that it is possible to re-write it as

$$\Omega^{(2)} = \begin{bmatrix} 0 & (c_1)^2 & (c_2)^2 & \cdots & (c_{L-1})^2 \\ (c_1)^2 & 0 & (c_1)^2 & \cdots & (c_{L-2})^2 \\ (c_2)^2 & (c_1)^2 & 0 & \cdots & (c_{L-3})^2 \\ (c_3)^2 & (c_2)^2 & (c_1)^2 & \cdots & (c_{L-4})^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (c_{L-1})^2 & (c_{L-2})^2 & (c_{L-3})^2 & \cdots & 0 \end{bmatrix}. \quad (12)$$

The elements of this matrix can be seen to be associated with the values of the quantization levels by $\Omega_{i,j}^{(2)} = (c_{|i-j|})^2$. Moreover, all the entries in $\Omega^{(2)}$ (i.e., all the squared distance components) belong to the ordered set

$$\Delta = \left[0, (c_1)^2, (c_2)^2, (c_3)^2, \cdots, (c_{L-2})^2, (c_{L-1})^2\right]^T. \quad (13)$$

Using the rule

$$\Omega_{i,j}^{(1)} = \Omega_{i,j}^{(2)} = \Delta_a, \quad \text{with } a = \left(\frac{|c_i - c_j|}{q} + 1\right) \quad (14)$$

it is possible to locate and read the value of the distance component $\left(\tilde{y}_{r,i} - \tilde{y}_{r,i}^{(l)}\right)$ from the values pre-stored in Δ. Note that the division by q in (14) converts the real difference between the two components into the number of integer intervals between them.

V. Simulation Results

The power penalty in comparison to the ML detection introduced by the methods described in Sections III and IV is solely caused by the quantization. The look-up table technique does not introduce any further errors as it is an exact method in the quantized space. Figure 2 shows the quantization error for different precision in the number of bits per dimension.

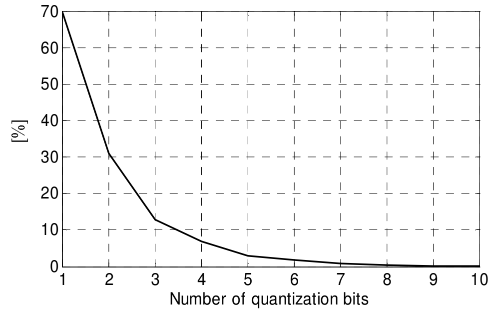


Fig. 2. Relative error for the squared Euclidean distance due to quantization as a function of the number of quantization bits per component.

This quantization error will impact on the performance of the detection in the quantized spaces. The system was simulated for three symmetric configurations (i.e., with n_T=n_R).

The relation between the signal power σ_x^2 and SNR (the noise power was defined in Section II) for the symbols taken from the constellation $\Xi_{\mathbf{C}}$) is obtained according to

$$SNR = \frac{\underset{\mathbf{H,x}}{\mathrm{E}}\left\{\|\mathbf{Hx}\|^2\right\}}{\underset{\mathbf{n}}{\mathrm{E}}\left\{\|\mathbf{n}\|^2\right\}} = \frac{n_T n_R \sigma_x^2}{n_R \sigma_n^2} = \frac{n_T \sigma_x^2}{\sigma_n^2}. \quad (15)$$

Besides the evaluation of the power penalty, Figures 3, 4 and 5 allow comparisons with those of traditional receivers, i.e. ZF, MMSE and OSIC with ZF or MMSE criteria. (Note that results for the traditional receivers and ML can be compared with [2] for 2 × 2 and with [5, 7] for 4 × 4. Notice that the performance measure used in this paper is the symbol error rate (SER) instead of the bit error rate (which for QPSK is about half of the SER if Gray mapping is used as when one symbol is detected in error only one of the 2 bits will be incorrect). The simulations were implemented using the levels $\{c_1, c_2, c_3, \cdots, c_L,\} = \{-(L-1), \cdots, -3, -1, +1, +3, \cdots, (L-1)\}$.

The results for the three configurations show a similar pattern: for b=2 the SER is worse than for any other receiver; for b=3 the SER is close to ZF; for b=4 it is similar to the performance of OSIC-MMSE, for b=5 it is always within 1dB of ML and for b=6 it always coincides with ML. From these results, the total number of bits needed to accurately represent the n_R-dimensional received vector is 5×(2×n_R), i.e., for the most demanding case considered (4 × 4), 40 bits are needed to obtain near the performance of ML.

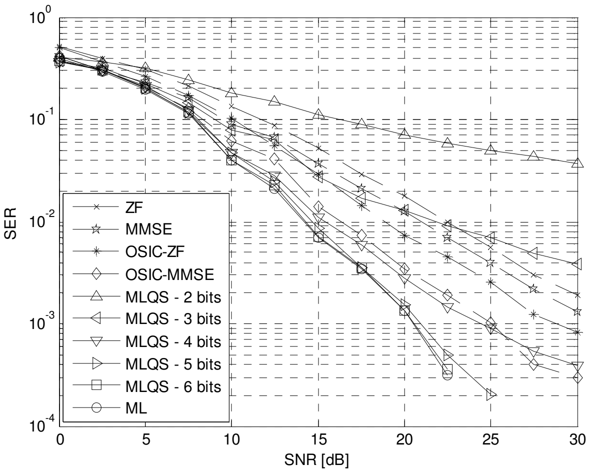


Fig. 3. SER versus average SNR for standard receivers and detection in a quantized space for different levels of quantization per dimension in a 2×2 system using QPSK modulation.

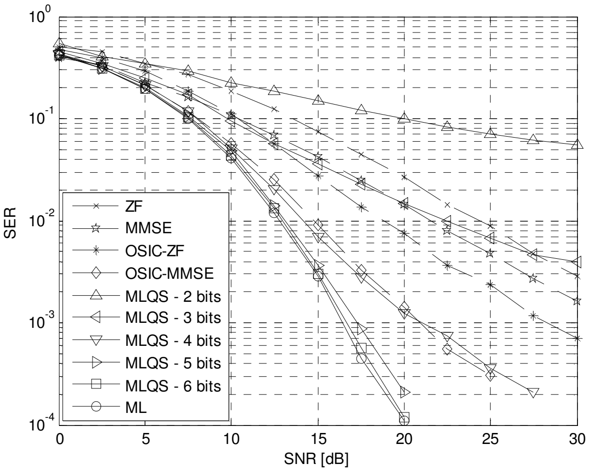


Fig. 4. SER versus average SNR for standard receivers and detection in a quantized space for different levels of quantization per dimension in a 3×3 system using QPSK modulation.

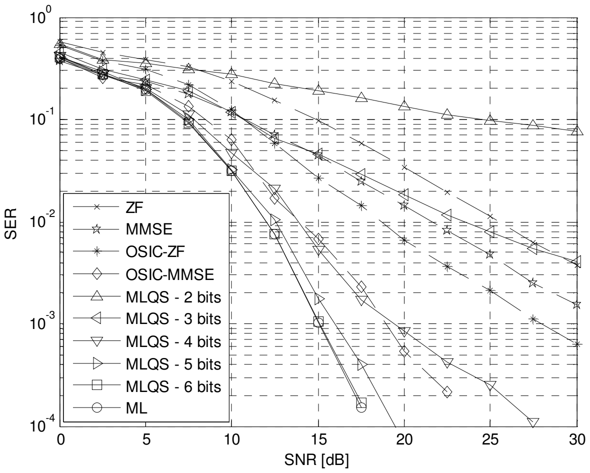


Fig. 5. SER versus average SNR for standard receivers and detection in a quantized space for different levels of quantization per dimension in a 4×4 system using QPSK modulation.

VI. CONCLUSIONS

The number of multiplications (squares) involved in the ML detection of one received vector is $(2n_R)M^{n_T}$. This paper presents a technique which enables a multiplication-free computation of the components of the squared Euclidean distances by means of a look-up table, which is specially adequate for VLSI (very large scale integration) architectures. The number of bits needed to represent both the received vectors and the lattice associated to each channel realization is small. It is expected that this number can be further reduced if the adaptive clipping of the signal space is carried out independently in each real received dimension. Additionally, the number of required pre-stored components constitutes a very small table with merely L positions.

The reduction of the complexity becomes more significant as the number of antennas increases. It should be noticed that the number of comparisons needed remains exponential with the number of transmit antennas, however the overall complexity is reduced.

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REFERENCES

[1] David Gesbert, “Breaking the barriers of Shannon’s capacity: An overview of MIMO wireless systems”, Telenor’s Journal: Telektronikk, Vol. 98, pp. 53–64, Jan. 2002.

[2] Arogyaswami Paulraj, Rohit Nabar, Dhananjay Gore, *Introduction to Space-Time Wireless Communications*, Cambridge Univ. Press, 2003.

[3] Robert W. Heath, Jr., and Arogyaswami J. Paulraj, “Switching between diversity and multiplexing in MIMO systems”, *Transactions on Communications*, Vol. 53, No. 6, pp. 962–968, June 2005.

[4] T. Kailath, H. Vikalo, B. Hassibi, H. Bölcskei (Editor), D. Gesbert (Editor), C. B. Papadias (Editor), A.-J. van der Veen (Editor), “MIMO receive algorithms”, in *Space-Time Wireless Systems: From Array Processing to MIMO Communications*, Cambridge Univ. Press, 2006.

[5] Christoph Windpassinger, “Detection and precoding for multiple input multiple output channels”, Ph.D. thesis, University of Erlangen-Nurenberg, Nuremberg, Germany, 2004.

[6] Huan Yao and Gregory W. Wornell, “Lattice-reduction-aided detectors for MIMO communication systems, in *Proc. of. IEEE GLOBECOM’02 – 2002 IEEE Global Telecommunications Conf.*, Vol. 1, Taipei, Taiwan, pp. 424–428, Nov. 2002.

[7] Hozun Sung, Jee Woong Kang, Kwang Bok Lee, “A simple maximum likelihood detection for MIMO systems”, *IEICE Transactions on Communications*, Vol. E89-B, No. 8, pp. 2241–2244, August 2006.

[8] Yoshikazu Ohashi, “Fast linear approximations of Euclidean distances in higher order dimensions”, *Graphics Gems IV*, Morgan Kaufmann, pp. 120–124, 1994.

[9] Mauro Barni, Fabio Buti, Franco Bartolini and Vito Cappelini “A quasi-Euclidean norm to speed up vector median filtering”, *IEEE Trans. on Image Processing*, Vol. 9, No. 10, pp. 1704–1709, Oct. 2000.

[10] Gianluigi Ferrari, Guilio Colavolpe, Riccardo Raheli, *Detection Algorithms for Wireless Communications: With Applications to Wired and Storage Systems*, John Wiley and Sons, 2004,

[11] Markus Rupp, Gerhard Gritsch, Hans Weinrichter, “Approximate ML detection for MIMO systems with very low complexity”, in *Proc. of ICASP’ 04 – The 2004 IEEE Intern. Conf. on Acoustics, Speech, and Signal Processing*, Vol. IV, pp.809–812, Montreal, Canada, 2004.

[12] Syed A. Rizvi, Nasser M. Nasrabadi, “An efficient Euclidean distance computation for vector quantization using a truncated look-up table”, *IEEE Transactions on circuits and systems for video technology*, Vol. 5, No. 4, pp. 370–371, August 1995.