Power Control for Turbo Coded Symmetrical Collaborative Networks

Weisi Guo[1], Ioannis Chatzigeorgiou[1], Ian J. Wassell[1] and Rolando Carrasco[2]
[1]University of Cambridge, Computer Laboratory, Cambridge CB3 0FD, United Kingdom
Email: {wg209, ic231, ijw24}@cl.cam.ac.uk
[2]Newcastle University, School of EE&C Engineering, Newcastle-upon-Tyne NE1 7RU, United Kingdom
Email: r.carrasco@newcastle.ac.uk

***Abstract*—Optimal power allocation for pure relay networks utilizing repetition codes has drawn a great deal of attention. However, power allocation in a cooperative network, where users who act as relays for each other and who utilize non-trivial codes, remains a challenge. With the aid of theoretical frame error rate expressions, we explore the performance of adaptive power control for decode-and-forward (DF) collaborative networks containing M users, employing turbo codes over statistically similar Rayleigh block faded channels. We conclude that power control can greatly benefit a DF collaborative network in a block fading environment.**

I. Introduction

Collaborative networks involve users cooperating with each other by sharing information and transmitting each other's data to the destination [1], which can be seen as a more complex form of a relaying network [2]. Diversity arising from the use of different fading paths can improve system performance. We will utilize this cooperative scheme in the context of a fixed wireless access (FWA) network. The motivation for this paper is to propose a power control scheme, that in its most complex form relies on the use of channel state information (CSI) at both the users and the destination, but can be simplified so that it does not rely on CSI. We show that in certain channel conditions, equal power allocation can be optimal. Simulations are used to verify the accuracy of our expressions.

Previous work regarding power allocation focus primarily on relay networks, where a single transmitting source is aided by one or multiple relays. Schemes exist in the form of water-filling algorithms for amplify-and-forward (AF) relays [3], [4] as well as networks employing coded cooperation [5]. The codes utilized in decode-and-forward (DF) have mostly been trivial repetition codes [2] or cyclic redundancy checks (CRC)[6], where the capacity expressions are known [7]. Solutions for DF relay networks having M users [8] have limited the resource optimization to only the relay-to-destination paths [9] due to complexity. There has been only limited work on power allocation involving cooperating users, who act as relays for each other [10]. Therefore, we believe that power allocation for DF networks with M users remains a challenge, especially when utilizing non-trivial codes whose performance can be characterized by a signal-to-noise ratio (SNR) threshold, such as turbo codes. We propose a method appropriate for turbo coded DF cooperative networks that optimizes power allocation in all the channels involved, under a longer term power constraint than [9]. This idea is based on our previous work on block codes in DF networks [11].

This work is supported by EPSRC under Grant EP/E012108/1.

In Section II, we define the system model and the channel environment. In Section III, we introduce theoretical FER expressions of turbo codes established in [12]. We aim to optimize the system's FER through power allocation factors determined both via a search based approach and deterministic scheme in Section IV. Our schemes are then compared to the equal power allocation and the performance is analyzed.

II. System Model

Transmission is assumed to take place over a Rayleigh block fading channel impaired by additive white Gaussian noise (AWGN). This is appropriate for a slowly changing environment such as that experienced in a FWA system. Within a block period, the channel gain coefficient remains constant, and any power adaption occurs between the blocks. Each user's transmission is assumed to take place over orthogonal channels to avoid interference and using a binary modulation scheme. Whilst this approach is intended for any code that can be described by a threshold, we will use a turbo code with generator polynomials $(1, 5/7, 5/7)$ in octal form, a threshold of -4.4dB, and an input frame size of 256. In our system we define:

- *inter-user channel*: the channel linking a user to another user.
- *uplink channel*: the channel linking a user to the destination.

We define γ as the instantaneous SNR of the uplink channel, $\overline{\gamma}$ as the channel's average SNR, and $\overline{\gamma}_R$ as the average inter-user channel SNR. We assume each user has statistically similar uplink channels (symmetric), and statistically similar reciprocal inter-user channels.

In a collaborative network, M users try to transmit independent information to the destination, using each other as their relays. For each fading block, there are M steps. The first step is always defined as a broadcasting step, where all M users broadcast their own data to each other and the destination with power $\mathrm{P_B}$. After which, the remaining $M-1$ steps are left to be distributed between cooperation and no cooperation operations. We define m as the number of steps in which cooperation occurs, and $M-1-m$ for no cooperation, where $0 \leq m \leq M-1$. Cooperation occurs when a user can decode another user's data, which it subsequently re-encodes and transmits to the destination at power $\mathrm{P_C}$. No cooperation occurs when there are any remaining steps unused, in which case the user will retransmit its own data at power $\mathrm{P_{NC}}$. If there is cooperation, the destination will perform maximum ratio combining (MRC) between the broadcast frame from the first step, the m subsequent cooperation steps, and the $M-1-m$ retransmission steps. In no cooperation, the destination will combine the broadcast and retransmission step $(\mathrm{P_C}+\mathrm{P_{NC}})$ with no diversity gain.

The scheme for $M=2$ is illustrated in Fig. 1 and in the case of equal power allocation, the power allocation factors are set to $\mathrm{P_B}=\mathrm{P_C}=\mathrm{P_{NC}}=\frac{1}{M}$. Users may choose to cooperate despite no cooperation by the other partners, or form a consensus whereby cooperation only occurs mutually. The former scenario will be named "unselfish cooperation", whereby all second step scenarios shown in Fig. 1 are possible.

We now make a sub-optimal simplification to the above scheme, whereby in order for the destination to perform conventional MRC with symmetrical inputs, we have chosen $\mathrm{P_B}$ = $\mathrm{P_C}$ and only utilize the retransmission when there is absolutely no cooperation. Thus, the destination will combine the broadcast step and the m cooperation steps, both of which are transmitted at $\mathrm{P_C}$.

III. Error Rate Expressions

The instantaneous uplink SNR is chi-squared distributed, with two degrees of freedom for the case of a direct channel, its probability density function is given by:

$$p_{\overline{\gamma}}(\gamma) = \left(\frac{1}{\overline{\gamma}}\right)e^{-\frac{\gamma}{\overline{\gamma}}}. \tag{1}$$

Let T be the convergence threshold of a turbo code; using results derived in [13], the direct channel FER of a block faded turbo code is [12]:

$$\mathrm{FER_{Direct}}(\overline{\gamma}, \mathrm{P_C}) = 1 - e^{-\frac{T}{\overline{\gamma}\mathrm{P_C}}}. \tag{2}$$

Similarly, we can derive the FER for the maximum ratio combining (MRC) of $m \geq 1$ channels which yields:

$$\begin{aligned}\mathrm{FER_{MRC}}(m, \overline{\gamma}, \mathrm{P_C}) &= 1 - e^{-\frac{T}{\overline{\gamma}\mathrm{P_C}}} \sum_{k=1}^{m} \frac{(\frac{T}{\overline{\gamma}\mathrm{P_C}})^k}{k!} \\ &\approx \frac{(\frac{T}{\overline{\gamma}\mathrm{P_C}})^{m+1}}{(m+1)!}.\end{aligned} \tag{3}$$

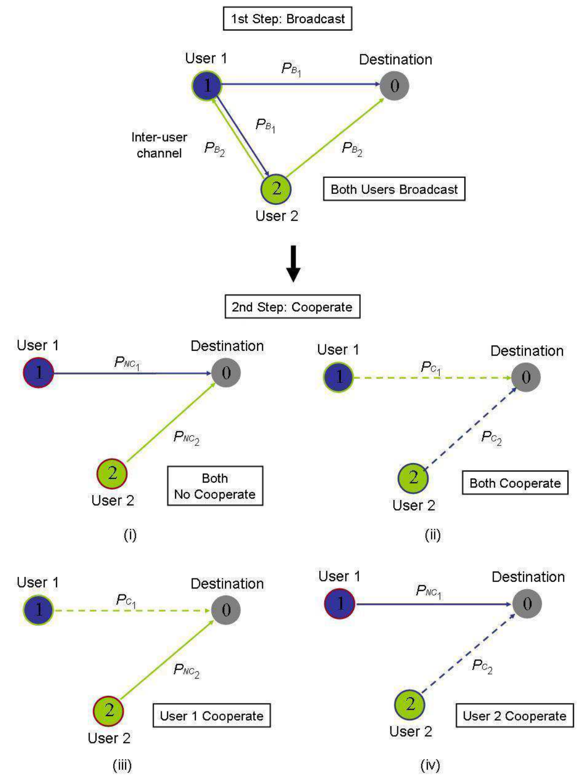

Fig. 1. The Two Steps of a Decode-and-Forward Cooperation Block for $M=2$ Users

Note that all $m+1$ channels are assumed to be symmetrical. In an "unselfish" collaborative network, where M total users always cooperate when possible, each user at any particular time may cooperate with m users, where $0 \leq m \leq M-1$. The system FER for M symmetrical users is the same as the FER for a particular user:

$$\begin{aligned}&\mathrm{FER_{DF}} = (1-\wp)^{M-1}\mathrm{FER_{Direct}}(\overline{\gamma}, \mathrm{P_C}+\mathrm{P_{NC}}) + \\ &\sum_{m=1}^{M-1}\binom{M-1}{m}\wp^m(1-\wp)^{M-1-m}\mathrm{FER_{MRC}}(m, \overline{\gamma}, \mathrm{P_C}),\end{aligned} \tag{4}$$

where the probability of cooperation $\wp$ is the probability of no frame errors in the interuser channel:

$$\begin{aligned}\wp &= 1 - \mathrm{FER_{Direct}}(\overline{\gamma}_R, \mathrm{P_C}) \\ &= e^{-\frac{T}{\overline{\gamma}_R \mathrm{P_C}}}.\end{aligned} \tag{5}$$

By substituting (2), (3) and (5) into (4), we produce:

$$\begin{aligned}\mathrm{FER_{DF}} = (1 - e^{-\frac{T}{\overline{\gamma}_R\mathrm{P_C}}})^{M-1}[(1 - e^{-\frac{T}{\overline{\gamma}(\mathrm{P_C}+\mathrm{P_{NC}})}}) \\ + \frac{T}{\overline{\gamma}_R\mathrm{P_C}}\sum_{m=2}^{M}\binom{M}{m}\frac{m}{M}\frac{z^m}{m!}],\end{aligned} \tag{6}$$

where $z = \frac{T\wp}{\overline{\gamma}\mathrm{P_C}(1-\wp)}$. Under medium-high $\overline{\gamma}$, we take the dominant terms of the series in (6) and simplify to yield

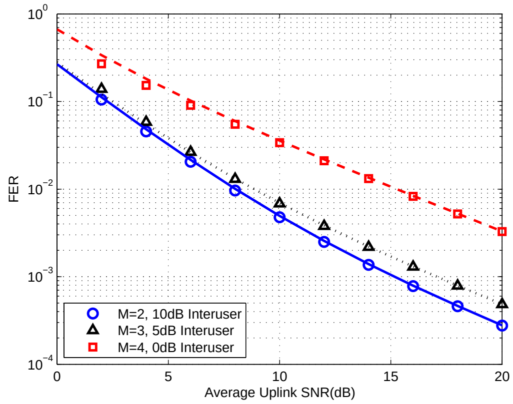

Fig. 2. FER theory and simulation for $M = 2, 3, 4$ users and with interuser channel SNR of $10, 5, 0$dB respectively

the following objective function:

$$f(\mathrm{P_C}, \mathrm{P_{NC}}, \overline{\gamma}, \overline{\gamma}_R) = (1 - e^{-\frac{T}{\overline{\gamma}_R \mathrm{P_C}}})^{M-1} \\ [(1 - e^{-\frac{T}{\overline{\gamma}(\mathrm{P_C}+\mathrm{P_{NC}})}}) + \frac{(M-1)T\overline{\gamma}_R}{2\overline{\gamma}^2 \mathrm{P_C}}]. \tag{7}$$

We shall define (7) as the objective FER function. In Fig. 2, we demonstrate the accuracy of (7) by comparing it to simulation results for a variety of M and SNR values.

IV. Power Allocation

In the previous section, we introduced the system FER function, which we shall now attempt to optimize using the power allocation factors, namely $\mathrm{P_C}$ and $\mathrm{P_{NC}}$. We will first introduce the power constraint, then use a brute force search algorithm to find the optimal power allocation factors. After which, we prove the convexity of our FER and constraint functions and derive a deterministic power allocation scheme accordingly.

A. Power Constraint

For power allocation to be fair and comparable to other schemes, we add a power constraint whereby the amount of power available to each user is fixed over a long period of time. This period should be long enough so that the probability of cooperation ($\wp$) can be reasonably accurate. The overall average power is defined to be unity, allowing a fair comparison to be made with an equal power allocation scheme. Referring to Fig. 1: the first step will broadcast at power $\mathrm{P_C}$, and the subsequent m steps will transmit at either power $\mathrm{P_C}$ or power $\mathrm{P_{NC}}$, depending on whether the user cooperates or not respectively. We define the amount of power used over a period of time to be $\mathbf{E}$, and the expectation is: $\mathbb{E}\{\mathbf{E}\} = 1$. Therefore, the power constraint is:

$$\mathbf{E} = \mathrm{P_C}(1 + m) + \mathrm{P_{NC}}\delta. \tag{8}$$

where $\delta = 1$ for $1 - (\wp)^{M-1}$ and $\delta = 0$ otherwise. The power allocation factors are also positive: $\mathrm{P_C} \geq 0$ and $\mathrm{P_{NC}} \geq 0$. Taking the expectation of (8), where $\mathbb{E}\{m\} = (M-1)\wp$ and $\mathbb{E}\{\delta\} = 1 - \wp^{M-1}$:

$$\mathrm{P_C}[1 + (M-1)\wp] + \mathrm{P_{NC}}[1 - \wp^{M-1}] = 1. \tag{9}$$

We shall define (9) as the constraint function $h(\mathrm{P_C}, \mathrm{P_{NC}}, \overline{\gamma}_R)$.

B. Power Allocation: Search Method

We first utilize a brute force search approach, which searches along all valid FER possibilities, subject to the power constraint. This may be seen as a optimal solution for our error rate minimization approach. The search range for power allocation factors is finite and between 0 and 1 at most in order for both power allocation factors to be positive under (9) .

C. Power Allocation: Deterministic Method

In order for a deterministic power optimization to produce optimal solutions which provide the lowest system FER, we must ensure both the objective system FER equation and the power constraint equation are both convex, as defined in [14]. To do so, we examine the second-order partial derivatives of the functions. When the derivatives are formed into a Hessian matrix, it is used to show whether the functions are positive semi-definite; or in other words: convex. To show that the objective (7) and constraint (9) functions belongs to a convex set, we need to show:

$$v^T \begin{bmatrix} \frac{\partial^2 f}{\partial \mathrm{P_{NC}^2}} & \frac{\partial^2 f}{\partial \mathrm{P_{NC}} \partial \mathrm{P_C}} \\ \frac{\partial^2 f}{\partial \mathrm{P_C} \partial \mathrm{P_{NC}}} & \frac{\partial^2 f}{\partial \mathrm{P_C^2}} \end{bmatrix} v \succeq 0 \tag{10}$$

$$v^T \begin{bmatrix} \frac{\partial^2 h}{\partial \mathrm{P_{NC}^2}} & \frac{\partial^2 h}{\partial \mathrm{P_{NC}} \partial \mathrm{P_C}} \\ \frac{\partial^2 h}{\partial \mathrm{P_C} \partial \mathrm{P_{NC}}} & \frac{\partial^2 h}{\partial \mathrm{P_C^2}} \end{bmatrix} v \succeq 0 \tag{11}$$

for all non-zero vectors $v = [v_1, v_2]$, with real entries ($v \in \mathbb{R}^n$).

We use $\overline{\gamma}_R \gg T$ and $\overline{\gamma} \gg T$ to reduce the Hessian of the FER function (7) to:

$$\mathbf{F} = \begin{bmatrix} \frac{A}{\mathrm{P_C}(\mathrm{P_{NC}}+\mathrm{P_C})^3} & \frac{B}{\mathrm{P_C}(\mathrm{P_{NC}}+\mathrm{P_C})^2} \\ \frac{B}{\mathrm{P_C}(\mathrm{P_{NC}}+\mathrm{P_C})^2} & \frac{2T^2}{\overline{\gamma}_R \overline{\gamma} \mathrm{P_C}(\mathrm{P_{NC}}+\mathrm{P_C})^3} \end{bmatrix}, \tag{12}$$

where A and B are positive functions of uplink and interuser SNRs. From (12), we can see that $\mathbf{F}$ is semi-positive for positive power allocation factors. The Hessian of the power constraint (9) function is:

$$\mathbf{H} = \begin{bmatrix} (C+D)e^{-\frac{T}{\overline{\gamma}_R \mathrm{P_C}}} - E & -Ce^{-\frac{(M-1)T}{\overline{\gamma}_R \mathrm{P_C}}} \\ -Ce^{-\frac{(M-1)T}{\overline{\gamma}_R \mathrm{P_C}}} & 0 \end{bmatrix}, \tag{13}$$

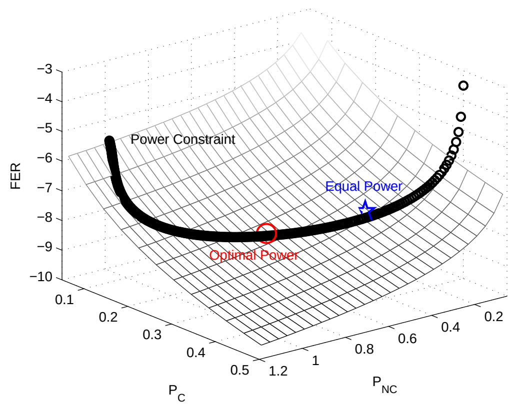

Fig. 3. Plot of Theoretical FER surface with the Power Constraint superimposed: M=3, under interuser SNR 10dB, uplink SNR 20dB

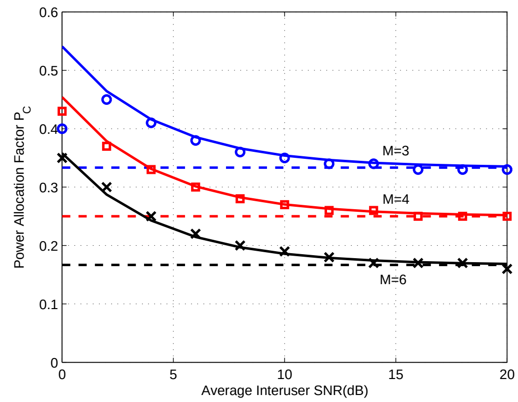

Fig. 4. Power Allocation Factors for 5dB Uplink Channel SNR with $M = 2, 4, 6$ users: symbols indicate search results, and lines indicate deterministic theoretical expressions.

where $C >> D$ and E is small. $\mathbf{H}$ is semi-positive for the following condition:

$$(C + D)e^{-\frac{T}{\overline{\gamma}_R \mathrm{P_C}}} v_1 \geq Ce^{-\frac{(M-1)T}{\overline{\gamma}_R \mathrm{P_C}}}(v_1 + v_2), \tag{14}$$

which reduces to:

$$(1 + \frac{T}{2\overline{\gamma}_R \mathrm{P_C^2}})e^{\frac{(M-2)T}{\overline{\gamma}_R \mathrm{P_C}}} \geq 1 + \frac{v_2}{v_1}. \tag{15}$$

The constraint is not strictly convex but is likely to be so when $M > 2$. Therefore, from (12) and (13), we can conclude that the solution is convex under condition (15). As an example, we refer to Fig. 3, which shows the FER surface as convex and overlapping the surface is the power constraint. The star indicates equal power allocation, and the circle indicates the optimal search power allocation solution. Both of which satisfy the constraint. We shall now derive the deterministic power allocation solutions.

Instead of optimizing the factors through the aforementioned search algorithm, a delay-less deterministic method is now introduced. We use Lagrangian multipliers to find the minimum FER under the power constraint. We define the Lagrangian Λ:

$$\begin{aligned}\Lambda =& f(\mathrm{P_B}, \mathrm{P_{NC}}, \overline{\gamma}, \overline{\gamma}_R) + \lambda h(\mathrm{P_C}, \mathrm{P_{NC}}, \overline{\gamma}_R) \\ =& (1 - e^{-\frac{T}{\overline{\gamma}_R \mathrm{P_C}}})^{M-1}[1 - e^{-\frac{T}{\overline{\gamma}(\mathrm{P_C}+\mathrm{P_{NC}})}} + \frac{(M-1)T\overline{\gamma}_R}{2\overline{\gamma}^2 \mathrm{P_C}}] \\ &+ \lambda \cdot [\mathrm{P_C}(1 + (M-1)\wp) + \mathrm{P_{NC}}(1 - \wp^{M-1}) - 1].\end{aligned} \tag{16}$$

We form the partial derivatives $(\frac{\partial \Lambda}{\partial \mathrm{P_{NC}}}, \frac{\partial \Lambda}{\partial \mathrm{P_C}}, \frac{\partial \Lambda}{\partial \lambda})$ and combining the independent equations. We then use $\overline{\gamma}_R \gg T$ and $\overline{\gamma} \gg T$ to obtain:

$$\mathrm{P_C} \simeq \frac{1}{M} + \frac{T(2 + 2M^2 - 4M)^{\frac{1}{3}}}{M\overline{\gamma}_R \overline{\gamma}^{\frac{1}{3}}}, \tag{17}$$

which is the solution to (16) and produces the minima. The part $\frac{T}{\overline{\gamma}_R \overline{\gamma}^{\frac{1}{3}}}$ is negligible when $\overline{\gamma}_R >> T$. Therefore, whilst (17) relies on CSI, a simplified alternative does not:

$$\mathrm{P_C^*} \simeq \frac{1}{M}. \tag{18}$$

We observe that (18) is in fact equal power allocation. In the following section, we shall analyze the performance of (17) compared to the search method and equal power allocation (18).

V. Results and Discussion

Fig. 4 shows the comparison between deterministic (17) (solid lines), equal (18) (dashed lines) and the search power allocation factors (symbols) found using the search method for a fixed number of users ($M = 4$) and a variety of uplink and interuser SNR values. Fig. 5 shows the comparison between (17) and the search method for a fixed user number and a variety of uplink and interuser SNR values. The dashed line is equal power allocation (18), and we observe that for $\overline{\gamma}_R >> T$, equal power provides the same solution as optimal power allocation without CSI. Fig. 6 shows the performance gain achieved by power allocation for $M = 6$ users at a relay SNR of 5dB. Fig. 7 shows the performance gain achieved by power allocation for $M = 2$ users at an uplink SNR of 15 and 25dB. We show that as the interuser channel achieves sufficiently high SNR to guarantee constant cooperation, optimal power allocation is identical to the equal power allocation solution. From Fig. 6 and Fig. 7, we also show that the approximate deterministic expression (17) matches the exact solutions obtained using the search method. The gain achieved by the deterministic solution over equal allocation can be significant in channel conditions which favor adaptive power allocation ($\overline{\gamma}_R >> T$). Whilst we have demonstrated that the deterministic and search method matches well, it is not always the case at low uplink SNRs as seen in Fig. 6. Nonetheless the deterministic method offers a practical allocation strategy.

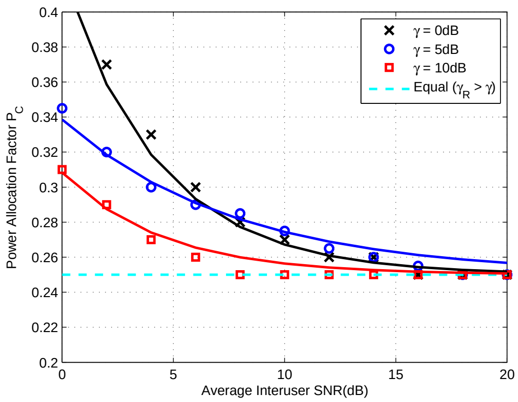

Fig. 5. Power Allocation Factors for 0,5,10dB Uplink Channel SNR with $M = 4$ users: symbols indicate search results, and lines indicate deterministic theoretical expressions.

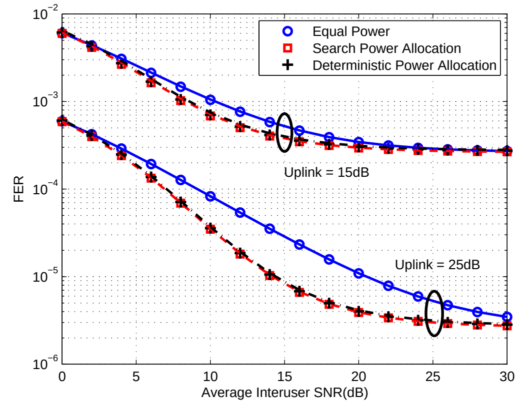

Fig. 7. Effect of Average Interuser and Uplink Channel SNR of 15 and 25dB on the Power Allocation Factors of $M = 2$ users: symbols indicate simulation results, and lines indicate approximate theoretical expressions.

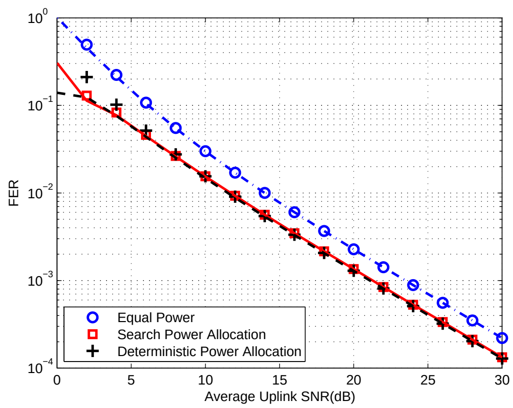

Fig. 6. Effect of Average Uplink and 5dB Interuser Channel SNR on the Power Allocation Factors of $M = 6$ users: symbols indicate simulation results, and lines indicate approximate theoretical expressions.

VI. Conclusion

We set out to optimize power allocation for an unselfish DF cooperative system by finding FER expressions for block faded channels. This analysis can be extended to selfish cooperation, as well as asymmetric channels. This method produced a deterministic power allocation solution which performs close to those of the optimal search method. We showed that in particular channel circumstances, the adaptive allocation is reduced to equal power allocation, which requires no CSI. In general, however, under a long term power budget constraint, we have shown that our proposed adaptive power allocation scheme can improve the system performance over that of an equal power allocation scheme. The proposed approach can be extended to other codes whose error rate performance can be characterized by an SNR threshold.

References

[1] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," in *IEEE Trans. on Wireless Communications*, vol. 42, no. 10, Oct. 2004, pp. 74–80.

[2] J. N. Laneman and G. W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," in *Proc. Wireless Communications and Networking Conference*, Sep. 2000, pp. 7–12.

[3] Y. Zhao, R. Adve, and T. Lim, "Improving amplify-and-forward relay networks: Optimal power allocation versus selection," in *IEEE Trans. on Wireless Communications*, vol. 6, no. 8, Aug. 2007, pp. 3114–3123.

[4] X. Deng and A. Haimovich, "Power allocation for cooperative relaying in wireless networks," in *IEEE Trans. on Communications*, vol. 50, no. 12, Jul. 2005, pp. 3062–3080.

[5] T. E. Hunter and A. Nosratinia, "Coded cooperation under slow fading, fast fading, and power control," in *36th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, USA, Nov. 2002, pp. 118–122.

[6] W. Su, A. K. Sadek, and K. J. R. Liu, "Ser performance analysis and optimum power allocation for decode and forward cooperation protocol in wireless networks," in *Proc. Wireless Communications Networking*, Mar. 2005, pp. 984–989.

[7] T. Cover and A. Gamal, "Capacity theorems for the relay channel," in *IEEE Trans. on Information Theory*, Sep. 1979, pp. 572–584.

[8] I. Maric and R. Yates, "Forwarding strategies for gaussian parallel relay networks," in *Proc. Int. Symp. Information Theory*, Chicago, USA, Jun. 2004, p. 269.

[9] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A.M.Haimovich, "Decode and forward cooperative diversity with power allocation in wireless networks," in *IEEE Trans. on Inform. Theory*, vol. 50, no. 12, Dec. 2004, pp. 3062–3080.

[10] J. Adeane, M. Rodrigues, and I. Wassell, "Optimum power allocation in cooperative networks," in *Proc. Int. Conf. on Telecommunications*, May 2005.

[11] W. Guo, I. Chatzigeorgiou, I. J. Wassell, and R.Carrasco, "Analysis and adaptive power control for block coded collaborative networks," in *International Wireless Communications and Mobile Computing 2009*, Leipzig, Germany, Jun. 2009.

[12] I. Chatzigeorgiou, I. J. Wassell, and R. Carrasco, "On the frame error rate of transmission schemes on quasi-static fading channels," in *Proc. Conf. Inf. Sciences and Systems*, Princeton, USA, Mar. 2008, p. 269.

[13] H. Gamal and J. Hammons, "Analyzing the turbo decoder using the gaussian approximation," in *IEEE Trans. on Inf. Theory*, vol. 47, Feb. 2001, pp. 671–686.

[14] S. Boyd, *Convex Optimization*. Cambridge, UK: Cambridge Uni. Press, 2004.