Exact and Asymptotic Outage Probability Analysis for Decode-and-Forward Networks

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***Abstract*—We consider decode-and-forward cooperative networks and we derive analytical expressions as well as tractable asymptotic approximations for the outage probability of a network node. Our analysis sheds more light on the interplay between the channel conditions, the network size and the adopted transmission scheme, and provides a useful tool for the design of cooperative networks.**

***Index Terms*—Cooperative systems, relays, fading channels, outage probability.**

I. Introduction

Node cooperation [1] is an alternative means of spatial diversity in wireless networks. Both decode-and-forward (DF) and amplify-and-forward (AF) protocols can achieve full diversity [2], however AF transmission requires that the destination has knowledge of the channel conditions between cooperating nodes, which is not possible in many practical scenarios [3]. In this paper, we consider *uncoordinated* DF cooperation, according to which network nodes that successfully decode the data of some or all their partners assist them in their transmission, even though cooperation might not be reciprocal. By contrast, *coordinated* cooperation ensures that a node will be assisted only by those partners to whom it can also offer assistance; this approach requires regular handshakes among the nodes for the formation of partnerships. Moreover, coordinated protocols limit the number of independent paths that data can follow and, hence, achieve a lower diversity than uncoordinated protocols, unless cooperative beamforming is performed [4].

Analysis of the error probability of coordinated and uncoordinated DF networks has been carried out in [5]–[7]. In particular, Souryal and Vojcic use a threshold-based model and propose an approximation for the packet error probability that is accurate for two-node networks employing turbo codes [5]. The performance of uncoordinated multi-node DF networks was studied by Sadek *et al.* [6] and Zhao *et al.* [7] who derived expressions for the symbol error probability and the outage probability, respectively. However, they both treated cooperative networks as equivalent relay networks; that is, information flows in one direction, from the source through the relays to the destination, and relays remain idle for the duration of the cooperation frame when they cannot assist the source since they do not have data of their own to transmit.

The motivation for this paper is to derive accurate closed-form expressions for the outage probability of uncoordinated DF networks as well as tractable approximations that asymptotically approach the exact expressions at high signal-to-noise ratio (SNR) values. In contrast to [6] and [7], we allow nodes that cannot assist a partner to retransmit their own data instead. Furthermore, we consider bi-directional channels between communicating nodes and we study the impact of channel reciprocity and channel independence on the outage probability.

The rest of the paper is organized as follows. Section II introduces the system model. Outage probability expressions are derived in Section III and asymptotic approximations are presented in Section IV. Analytical and simulation results are compared and discussed in Section V. The paper concludes in Section VI with a summary of the main contributions.

II. System Model

We consider a network of M nodes, denoted as $\mathrm{U}_1, \ldots, \mathrm{U}_M$, that transmit to the same destination D. Channels between nodes and the destination are referred to as *uplink channels*, whilst channels that link nodes are known as *internode channels*. All channels are subject to frequency-flat Rayleigh fading and additive white Gaussian noise. Nodes transmit on orthogonal channels, which allows the destination to detect each node separately. Node cooperation occurs in two successive stages. Quasi-static fading is considered, hence each channel realization remains constant for the duration of the two-stage frame but changes independently from frame to frame.

During the first stage of cooperation, each node dedicates a time step to broadcast its own packet of coded bits to the other nodes and the destination. At the end of the first stage, each node has received $M-1$ coded packets, i.e., one from each partner. Let us assume that a node $\mathrm{U} \in \{\mathrm{U}_1, \mathrm{U}_2, \ldots, \mathrm{U}_M\}$ failed to decode m packets but successfully decoded the remaining $M-1-m$ packets. During the second stage, node U will re-encode and relay the successfully retrieved packets of the $M-1-m$ corresponding partners and it will transmit m copies of its own packet to the destination, over a period

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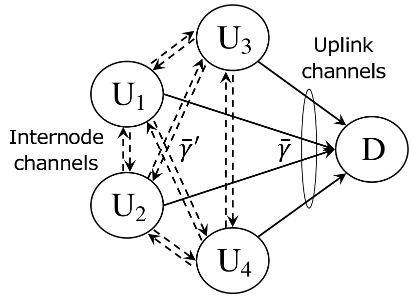

Fig. 1. Block diagram for a cooperative network of $M=4$ nodes. Note that internode channels are characterized by the same average SNR, $\bar{\gamma}'$. Similarly, the average SNR of all uplink channels is $\bar{\gamma}$.

of $M-1$ time steps. Retransmission of m copies of a node's own packet aims to improve reliability while ensuring that the transmit energy per cooperation frame remains constant. It is important to note that the same channel code is employed by all nodes in both stages of a cooperation frame. Furthermore, channel state information is available to the receiving end of all nodes and the destination, therefore coherent detection is possible.

The quality of a channel in our system model is characterized by its average receive SNR value. We assume that the uplink channels and the internode channels are statistically similar and we denote the average receive SNR for each set of channels as $\bar{\gamma}$ and $\bar{\gamma}'$, respectively. Even though this assumption significantly simplifies our analysis, it can still be used to determine the contribution of each network parameter to the overall performance of various practical network configurations. Examples include networks in which nodes are clustered far away from the destination or the destination uses feedback channels to dictate a specific receive SNR from all nodes. Fig. 1 shows the block diagram for a cooperative network of four nodes, depicting the various channels and their average SNR levels. Note that each pair of nodes is linked by two internode channels, i.e., one for each direction, which are usually modeled as being either mutually independent or reciprocal. Internode channel independence can be assumed in frequency division/time division multiple access (FD/TDMA) networks, where nodes employ different frequencies. On the other hand, internode channel reciprocity is a common assumption in TDMA networks, where all nodes employ the same frequency but transmit at different intervals. In the following sections, both cases are considered.

III. Outage Probability Analysis

At the end of the two-stage process, node U has transmitted $m+1$ copies of its own packet: one copy during the first stage and $m\geq 0$ copies during the second stage. Let us assume that $\ell\geq 0$ copies of the same packet have also been relayed to the destination by the partners of U. The destination will combine the $m+\ell+1$ copies to obtain a better estimate of the packet of U.

If γ_Σ is the instantaneous output SNR of the maximal ratio combiner at the destination and $f(\gamma_\Sigma)$ is the probability density function (pdf) of γ_Σ, the outage probability can be obtained in terms of the average uplink SNR, $\bar{\gamma}$, as follows

$$P(\bar{\gamma}) = \Pr\{\gamma_\Sigma \leq \gamma_o\} = \int_0^{\gamma_o} f(\gamma_\Sigma)\, d\gamma_\Sigma \tag{1}$$

where γ_o is an SNR threshold. This expression is also a good approximation of the packet error probability when γ_o characterizes the error correction capability of the transmission scheme [8].

A. Conditional Outage Probability

In order to derive the outage probability, conditioned on $m+\ell+1$ copies of a packet being received at the destination, we identify two unique scenarios in the second stage of cooperation. In the first scenario, which we refer to as *full cooperation*, node U successfully decodes the packets of all its partners and relays them to the destination. In the second scenario, which we call *partial cooperation*, node U decodes the packets of some of its partners and thus transmits both copies of partners' packets and copies of its own packet to the destination. Therefore, our objective is to determine the power density functions $f_{\mathcal{F}}(\gamma_\Sigma)$ and $f_{\mathcal{P}}(\gamma_\Sigma)$ as well as expressions for $P_{\mathcal{F}}(\bar{\gamma})$ and $P_{\mathcal{P}}(\bar{\gamma})$, where indices $\mathcal{F}$ and $\mathcal{P}$ refer to full and partial cooperation, respectively.

In full cooperation, node U does not transmit additional copies of its own packet during the second stage of cooperation ($m=0$), hence the destination combines $\ell+1$ copies which have been transmitted over independent and identically distributed channels. In this case, it is well known that γ_Σ has a central chi-square distribution, whose pdf is equal to [9]

$$f_{\mathcal{F}}(\gamma_\Sigma) = \frac{\gamma_\Sigma^\ell}{\ell!\,\bar{\gamma}^{\ell+1}}\, e^{-\frac{\gamma_\Sigma}{\bar{\gamma}}}. \tag{2}$$

Using (1), the conditional outage probability assumes the form [7]

$$P_{\mathcal{F}}(\bar{\gamma}) = 1 - e^{-\frac{\gamma_o}{\bar{\gamma}}} \sum_{k=0}^{\ell} \frac{1}{k!} \left(\frac{\gamma_o}{\bar{\gamma}}\right)^k. \tag{3}$$

In partial cooperation, node U transmits $m>0$ copies of its own packet to the destination during the second stage of cooperation. Taking into account the packet that was broadcast during the first stage, the destination receives a total of $m+1$ copies directly from node U at an average SNR of $\bar{\gamma}$. This is equivalent to receiving a single copy of the packet at an SNR value of $(m+1)\bar{\gamma}$. The destination essentially combines this copy with the ℓ copies that were received indirectly through ℓ different partners at an average SNR of $\bar{\gamma}$. The pdf of γ_Σ can now be derived from the moment generating function (MGF), defined as $\mathcal{M}_{\gamma_\Sigma}(s) \triangleq \int_0^\infty e^{s\gamma_\Sigma} f_{\mathcal{P}}(\gamma_\Sigma)\, d\gamma_\Sigma$ [9]. In particular, the MGF of γ_Σ can be expressed as

$$\mathcal{M}_{\gamma_\Sigma}(s) = \frac{1}{1-s\,(m+1)\,\bar{\gamma}} \cdot \frac{1}{(1-s\bar{\gamma})^\ell} \tag{4}$$

where the first term of the product corresponds to the MGF of the direct channel and the second term represents the MGF of the ℓ identically distributed indirect channels. Following a similar approach to that described in [10], one can obtain the desired pdf of γ_Σ, namely $f_{\mathcal{P}}(\gamma_\Sigma)$, by first resolving $\mathcal{M}_{\gamma_\Sigma}(s)$

into partial fractions and then taking its inverse. The operation yields

$$
\begin{aligned}
f_{\mathcal{P}}(\gamma_\Sigma) &= e^{-\frac{\gamma_\Sigma}{(m+1)\bar{\gamma}}}\left(\frac{m+1}{m}\right)^{\ell}\frac{1}{(m+1)\,\bar{\gamma}} \\
&\quad - e^{-\frac{\gamma_\Sigma}{\bar{\gamma}}}\sum_{\lambda=1}^{\ell}\frac{(m+1)^{\ell-\lambda}}{m^{\ell-\lambda+1}}\cdot\frac{\gamma_\Sigma^{\lambda-1}}{(\lambda-1)!\,\bar{\gamma}^{\,\lambda}}\,.
\end{aligned}
\tag{5}
$$

Using (1), we can obtain the conditional outage probability for partial cooperation, that is

$$
\begin{aligned}
P_{\mathcal{P}}(\bar{\gamma}) &= 1 - e^{-\frac{\gamma_{\mathrm{o}}}{(m+1)\bar{\gamma}}}\left(\frac{m+1}{m}\right)^{\ell} \\
&\quad + e^{-\frac{\gamma_{\mathrm{o}}}{\bar{\gamma}}}\sum_{k=0}^{\ell-1}\frac{1}{k!}\left(\frac{\gamma_{\mathrm{o}}}{\bar{\gamma}}\right)^{k}\left[\left(\frac{m+1}{m}\right)^{\ell-k}-1\right].
\end{aligned}
\tag{6}
$$

We established that the value of m determines whether node U fully cooperates ($m=0$) or partially cooperates ($m>0$) with its partners. Thus, the general expression for the outage probability, conditioned on $m+\ell+1$ packets of node U being received at the end of the cooperation frame, assumes the form

$$
P(\bar{\gamma};m,\ell) = \begin{cases} P_{\mathcal{F}}(\bar{\gamma}), & \text{for } m=0 \\ P_{\mathcal{P}}(\bar{\gamma}), & \text{for } m>0 \end{cases}
\tag{7}
$$

for all values of $m\geq 0$ and $\ell\geq 0$, when the average uplink SNR is $\bar{\gamma}$.

B. End-to-End Outage Probability

Before we determine the end-to-end outage probability for node U, let us first consider the instance when U broadcasts a coded packet and a partner receives it through the corresponding internode channel, whose average SNR is $\bar{\gamma}'$. The probability that the partner will successfully decode the packet of U is given by

$$
p_{\bar{\gamma}'} \;=\; 1-P(\bar{\gamma}';0,0) \;=\; e^{-\gamma_{\mathrm{o}}/\bar{\gamma}'}.
\tag{8}
$$

Here, $P(\bar{\gamma}';0,0)$ represents the outage probability of an internode channel when no packet repetitions take place, and can be obtained using (7) for $m=\ell=0$. Note that $p_{\bar{\gamma}'}$ also represents the probability that a partner cooperates with node U and assists its transmission to the destination.

If node U fails to decode the packets of m partners, it will transmit m copies of its own packet to the destination during the second stage of cooperation. The number of packets m that the destination will receive directly from node U after $M-1$ time steps has a binomial distribution

$$
\Pr\{m\} = \binom{M-1}{m}\left(1-p_{\bar{\gamma}'}\right)^{m} p_{\bar{\gamma}'}^{M-m-1},
\tag{9}
$$

where $\binom{M-1}{m}=(M-1)!/(M-m-1)!m!$ is the binomial coefficient. Similarly, the number of packets ℓ that the destination will receive indirectly through the ℓ partners of node U has also a binomial distribution

$$
\Pr\{\ell\} = \binom{M-1}{\ell} p_{\bar{\gamma}'}^{\ell}\left(1-p_{\bar{\gamma}'}\right)^{M-\ell-1}.
\tag{10}
$$

If the internode channels are mutually independent, m and ℓ vary independently of each other and their joint probability can be written as $\Pr\{m,\ell\}=\Pr\{m\}\cdot\Pr\{\ell\}$. On the other hand, if the internode channels are reciprocal, variable ℓ is dependent on m; the destination will receive m copies of a packet directly from node U and $\ell=M-1-m$ copies through the partners, therefore $\Pr\{m,\ell\}=\Pr\{m\}=\Pr\{\ell=M-1-m\}$.

The end-to-end outage probability for a particular node can be computed by taking the product of the conditional outage probability $P(\bar{\gamma};m,\ell)$ and the probability $\Pr\{m,\ell\}$ over all possible values of m and ℓ. Owing to the symmetry of the network, all nodes have the same outage probability which is given by

$$
P_{\mathrm{ind}} = \sum_{m=0}^{M-1}\sum_{\ell=0}^{M-1}\Pr\{m\}\Pr\{\ell\}\,P(\bar{\gamma};m,\ell)
\tag{11}
$$

$$
P_{\mathrm{rec}} = \sum_{m=0}^{M-1}\Pr\{m\}\,P(\bar{\gamma};m,M-1-m)
\tag{12}
$$

for the case of mutually independent internode channels and reciprocal internode channels, respectively. Both (11) and (12) are closed-form expressions of the outage probability for DF networks and can be further expanded if we substitute $P(\bar{\gamma};m,\ell)$, $\Pr\{m\}$ and $\Pr\{\ell\}$, using (7), (9) and (10).

IV. Asymptotic Performance Analysis

In Section V we will demonstrate that (11) and (12) closely predict the outage probability of a node in a DF network. Even though both expressions are exact, they do not shed much light on the network interdependencies. In this section we investigate the asymptotic behavior of the outage probability, which will provide further insight into the interplay among the network parameters.

In order to simplify the expressions that we derived in the previous sections, we invoke known identities and approximations. In particular, let z be a positive real number such that $z<<1$ and N be a positive integer. The N-th power of $1+z$ can be accurately approximated by the two most significant terms of the binomial expansion, that is $(1+z)^N\approx 1+Nz$. In the same fashion, we can approximate binomial series of z with the most significant term of the series, for example $\sum_{n=0}^{N}\binom{N}{n}\frac{z^{n+1}}{(n+1)!}\approx z$. If the approximation is loose, we can use a tight bound, for example $\sum_{n=0}^{N-1}\binom{N-1}{n}\frac{N!}{(N-n)!}\;z^n<(1+Nz)^{N-1}$. Using the properties of the upper incomplete gamma function $\Gamma(\alpha,z)$ [11], we can obtain the following important approximation

$$
e^{-z}\sum_{n=0}^{N}\frac{z^n}{n!} = \frac{\Gamma(N+1,z)}{N!}\approx 1-\frac{z^{N+1}}{(N+1)!}
\tag{13}
$$

whilst an identity that will also prove useful in our asymptotic analysis is [12]

$$
\sum_{n=0}^{N-1}\binom{N-1}{n}\frac{z^{n+1}}{n+1} = \frac{(z+1)^N-1}{N}.
\tag{14}
$$

For the remainder of the paper, we adopt the notation $\tilde{P}$ to denote the asymptotic approximation of a function P. Using (13), we can approximate the conditional outage probabilities

$P_{\mathcal{F}}(\bar{\gamma})$ and $P_{\mathcal{P}}(\bar{\gamma})$, presented in (3) and (6) respectively, as follows

$$\tilde{P}_{\mathcal{F}}(\bar{\gamma}) = \frac{1}{(\ell+1)!}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)^{\ell+1} \quad (15)$$

$$\tilde{P}_{\mathcal{P}}(\bar{\gamma}) = \frac{1}{\ell!\,(m+1)}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)^{\ell+1} \quad (16)$$

for the case when $\bar{\gamma} >> \gamma_{\rm o}$. The asymptotic behavior of the end-to-end outage probability for either mutually independent or reciprocal internode channels can then be investigated if we consider the following two cases:

1) $\bar{\gamma}' >> \bar{\gamma}$ and $\bar{\gamma} >> \gamma_{\rm o}$: This case is representative of nodes that are either in the form of a cluster that is located far from the destination or in the form of a dense mesh around the destination, such that the average quality of the internode channels is better than that of the uplink channels. For high internode SNR values, the probability of node cooperation reduces to $p_{\bar{\gamma}'} \approx 1-(\gamma_{\rm o}/\bar{\gamma}')$, whilst the outage probability expressions (11) and (12) can be simplified into

$$\tilde{P}_{\rm ind} = \frac{1}{M!}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)^{M}\left(1-\frac{\gamma_{\rm o}}{\bar{\gamma}'}\right)^{2(M-1)}\left(1+M\,\frac{\bar{\gamma}}{\bar{\gamma}'}\right)^{M-1} \quad (17)$$

and

$$\tilde{P}_{\rm rec} = \frac{1}{M!}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)^{M}\left(1-\frac{\gamma_{\rm o}}{\bar{\gamma}'}\right)^{M-1}\left(1+\frac{M(M-1)}{2}\frac{\bar{\gamma}}{\bar{\gamma}'}\right) \quad (18)$$

for independent and reciprocal internode channels, respectively. It is worth pointing out that, for $\bar{\gamma}' >> \bar{\gamma}$, the overall performance is greatly affected by outage events that occur when $m=0$. In that case, the node of interest U has successfully recovered the packets of all its partners and does not retransmit its own packets; thus, performance mainly depends on the number of nodes that will cooperate with node U. If we take the ratio between (17) and (18),

$$\begin{aligned}\frac{\tilde{P}_{\rm ind}}{\tilde{P}_{\rm rec}} &= \frac{\left[\left(1-\frac{\gamma_{\rm o}}{\bar{\gamma}'}\right)\left(1+M\frac{\bar{\gamma}}{\bar{\gamma}'}\right)\right]^{M-1}}{\left(1+\frac{M(M-1)}{2}\frac{\bar{\gamma}}{\bar{\gamma}'}\right)} \\ &< \frac{\left(1+M\frac{\bar{\gamma}}{\bar{\gamma}'}\right)^{M-1}}{\left(1+M\frac{\bar{\gamma}}{\bar{\gamma}'}\right)^{\frac{M-1}{2}}} \\ &\approx 1+\frac{M(M-1)}{2}\frac{\bar{\gamma}}{\bar{\gamma}'}, \text{ for } \bar{\gamma}' >> M\bar{\gamma} \end{aligned} \quad (19)$$

we notice that the impact of the internode channel model on the outage probability becomes more pronounced as the network size M increases. As expected, the outage probability is not markedly affected by the statistical model when $\bar{\gamma}' >> M\bar{\gamma}$ and collapses to $\tilde{P}_{\rm ind} = \tilde{P}_{\rm rec} = (1/M!)(\gamma_{\rm o}/\bar{\gamma})^M$ when perfect internode channels are considered, i.e., $\bar{\gamma}' \to \infty$.

2) $\bar{\gamma} >> \bar{\gamma}'$ and $\bar{\gamma} >> \gamma_{\rm o}$: An example for this scenario would be that of a sparse mesh network surrounding the destination, such that the internode distance is greater than the distance between a node and the destination. Following a similar analysis to the above, we can express the asymptotic approximations for $P_{\rm ind}$ and $P_{\rm rec}$ as

$$\tilde{P}_{\rm ind} = \frac{1}{M}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)\left(1-e^{-\frac{\gamma_{\rm o}}{\bar{\gamma}'}}\right)^{M-2}\left(1-e^{-M\frac{\gamma_{\rm o}}{\bar{\gamma}'}}\right) \quad (20)$$

and

$$\tilde{P}_{\rm rec} = \frac{1}{M}\left(\frac{\gamma_{\rm o}}{\bar{\gamma}}\right)\left(1-e^{-\frac{\gamma_{\rm o}}{\bar{\gamma}'}}\right)^{M-1} \quad (21)$$

which provide a clearer picture of the relationship between the outage probability and the network parameters, in the high uplink SNR region. The ratio between (20) and (21), that is

$$\begin{aligned}\frac{\tilde{P}_{\rm ind}}{\tilde{P}_{\rm rec}} &= \frac{1-e^{-M\gamma_{\rm o}/\bar{\gamma}'}}{1-e^{-\gamma_{\rm o}/\bar{\gamma}'}} \\ &\approx \frac{1-(1-M\gamma_{\rm o}/\bar{\gamma}')}{1-(1-\gamma_{\rm o}/\bar{\gamma}')} \\ &\approx M, \quad \text{for } \bar{\gamma}' >> M\gamma_{\rm o}, \end{aligned} \quad (22)$$

demonstrates that a change in the internode channel conditions, from being reciprocal to being independent, incurs a performance penalty that is proportional to the network size M. This outcome is attributed to the unlikely but possible event of an outage caused when node U has not been assisted by any of its partners ($\ell=0$) and thus spatial diversity was not exploited. In that case, if the internode channels are mutually independent, it is very likely that U has decoded and relayed the packets of all its partners since $\bar{\gamma}' >> \gamma_{\rm o}$. Consequently, the destination has received only a single copy of node's U packet that was transmitted directly from U during the broadcast stage. On the other hand, if channel reciprocity is assumed when $\ell=0$, node U has failed to retrieve the packets of its partners and opted to retransmit copies of its own packet. Therefore, the destination has combined M packets from node U and achieved an outage probability that is M times smaller than before. Note that for $\bar{\gamma} \to \infty$, nodes need not cooperate since the direct uplink channels are perfect, hence $\tilde{P}_{\rm ind} = \tilde{P}_{\rm rec} = 0$. If $\bar{\gamma}' \to 0$, nodes fail to cooperate and transmit only on the uplink channel, thus $\tilde{P}_{\rm ind} = \tilde{P}_{\rm rec} = \gamma_{\rm o}/(M\bar{\gamma})$.

V. Results and Discussion

In this section, we validate the derived analytical expressions and their asymptotic approximations by comparing theoretical to simulation results. For this purpose, we consider a network of $M=4$ nodes and we set the SNR threshold to a value that relates to the error correction capability of the adopted transmission scheme. Based on the methodology in [13], an SNR threshold of $\gamma_{\rm o} = -0.441$ dB characterizes a network in which each node encodes packets of 512 information bits, using a rate 1/2 non-recursive non-systematic convolutional (NRNSC) code with octal generator polynomials (15, 17), and then modulates them using binary phase shift keying (BPSK).

In Fig. 2, curves that were obtained using the analytical expression (11) for independent internode channels and the asymptotic approximation (17) for $\bar{\gamma}' >> \bar{\gamma}$ are compared to simulations. A similar comparison for reciprocal internode channels when $\bar{\gamma} >> \bar{\gamma}'$ is presented in Fig. 3. We notice that simulations closely match the exact and the asymptotic theoretical predictions. Furthermore, Fig. 4 and Fig. 5 demonstrate that outage probability ratios are accurately described by (19) and (22), respectively.

When the network does not operate in the asymptotic SNR region, we can use the exact expressions (11) and (12) to analyze the outage probability. For instance, Fig. 6 shows

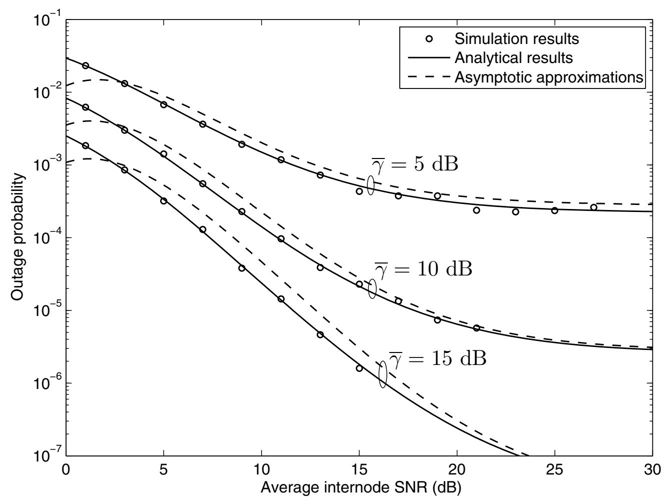

Fig. 2. Comparison between simulation results and theoretical values obtained from the exact outage probability expression for *independent* internode channels (11) and its asymptotic approximation (17) for $\overline{\gamma}' >> \overline{\gamma}$. A network of $M = 4$ nodes is considered and results for uplink SNR values of $\overline{\gamma} = 5$, 10 and 15 dB are presented.

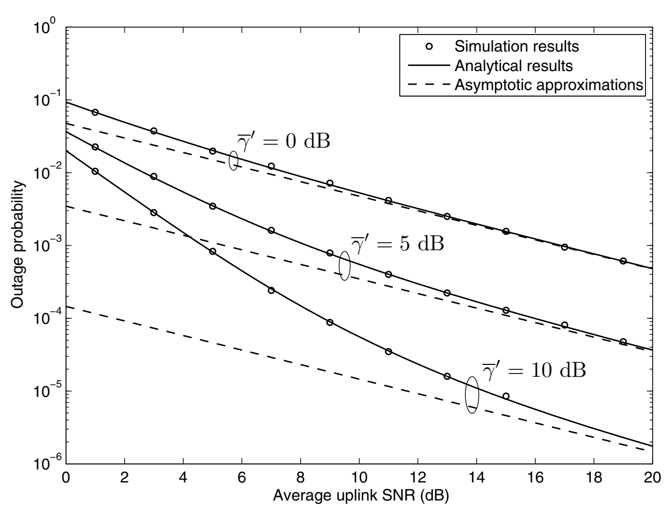

Fig. 3. Comparison between simulation results and theoretical values obtained from the exact outage probability expression for *reciprocal* internode channels (12) and its asymptotic approximation (21) for $\overline{\gamma} >> \overline{\gamma}'$. A network of $M = 4$ nodes has been considered and curves for three internode SNR values, namely $\overline{\gamma}' = 0$, 5, and 10 dB, have been plotted.

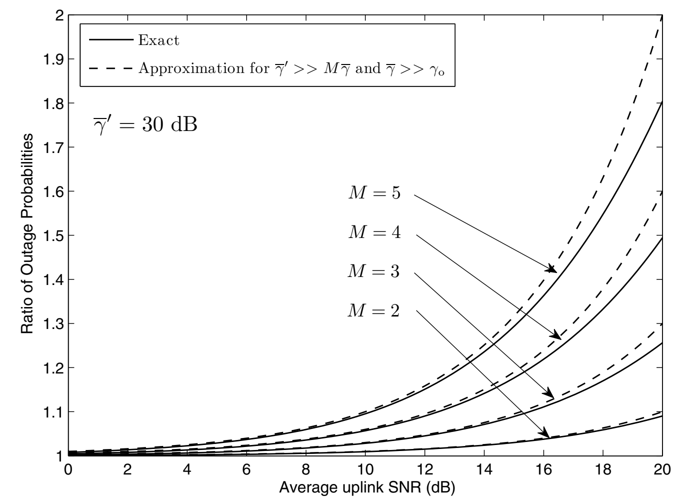

Fig. 4. Comparison between the ratio of the exact outage probabilities (11) and (12), and the approximated ratio (19) for an internode SNR value of $\overline{\gamma}' = 30$ dB.

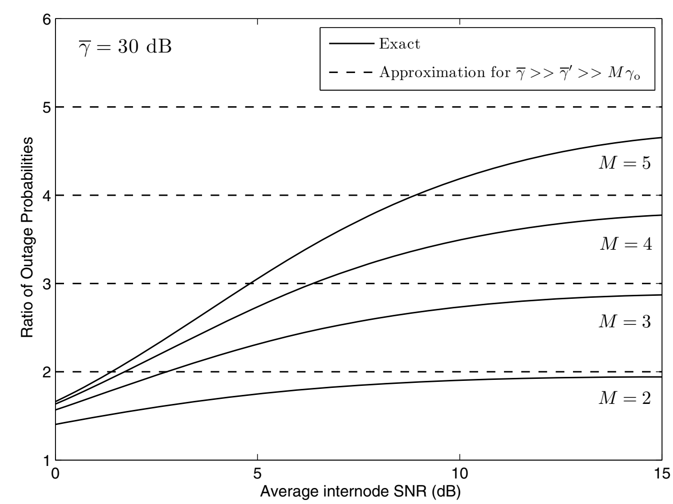

Fig. 5. Comparison between the ratio of the exact outage probabilities, namely (11) and (12), and the approximated ratio (22) for an uplink SNR value of $\overline{\gamma} = 30$ dB.

the interplay among the network parameters when the outage probability is set to the target value of 10^{-2}. Here, the SNR threshold γ_o is shown on the horizontal axis; note that the better the error correction capability of the transmission scheme is, the lower the SNR threshold will be. On the vertical axis, we measure the performance gain which is defined as the uplink SNR reduction that is required to maintain the outage probability at the target value when the internode SNR improves from $-\infty$ (no cooperation) to a finite value $\overline{\gamma}'$, expressed in dB. According to Fig. 6, powerful error correction schemes markedly increase the performance gain when the internode channel quality is poor (e.g., $\overline{\gamma}' = 2$ dB) but the gain is mainly determined by the network size at high internode SNR values (e.g., $\overline{\gamma}' = 18$ dB).

VI. Conclusions

In this paper, we considered decode-and-forward cooperative networks in a quasi-static fading environment and we derived exact closed-form expressions that accurately predict the outage probability of a node communicating over either reciprocal or independent internode channels. We also obtained tractable asymptotic approximations that clearly illustrate the dependence of the outage probability on the various network parameters, such as the network size, the adopted transmission scheme and the channel conditions. We validated our theoretical results by comparing them to simulations and we established that in particular scenarios, a node can experience a performance degradation, which is proportional to the network size, when the internode channel conditions change from being reciprocal (e.g., in TDMA networks) to being independent (e.g., in FD/TDMA networks). We are hopeful that the derived expressions and their approximations

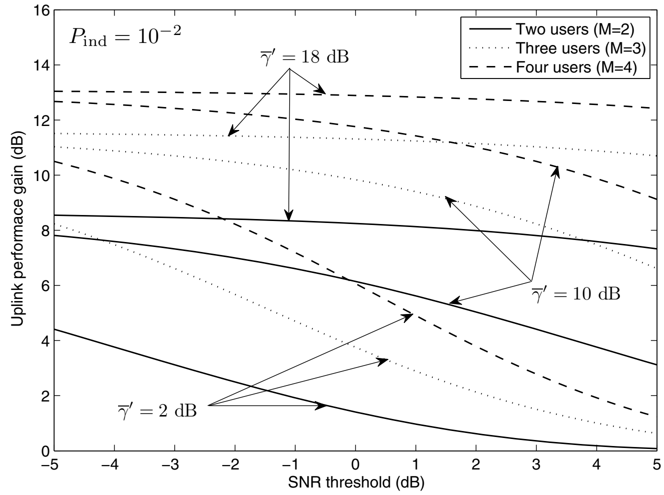

Fig. 6. Effect of the network parameters on the performance gain for a target outage probability of 10^{-2}. Mutually independent internode channels are considered (worst case scenario).

will provide a useful reference tool for the analysis and design of decode-and-forward cooperative networks.

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