Frequency Scaled Time Domain Equalization for OFDM in Wireless Communication

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ABSTRACT

In this paper, the use of Orthogonal Frequency Division Multiplexing (OFDM), combined with a Time Domain Equalizer (TEQ), is investigated for broadband fixed wireless access systems. OFDM systems use a Cyclic Prefix (CP) that is inserted at the beginning of each symbol to convert the linear convolution of data and channel into a circular one. If the CP is longer than the channel length, Inter Symbol Interference (ISI) is avoided. However the use of the CP reduces the efficiency of the system. A TEQ is often used to reduce the channel length, enabling a shorter CP to be used. The TEQ schemes that have been proposed to date result in the Effective Channel Impulse Response (EIR), (i.e. including the TEQ) having spectral nulls. This prevents some of subchannels from being used for data transmission. An alternative algorithm is presented to optimise the TEQ coefficients in both the time and frequency domains known as Frequency Scaled Time Domain Equalization (FSTEQ) which results in a flatter spectral response. The simulation results show up to a 100 fold improvement in the BER at an SNR of 20 dB.

I. INTRODUCTION

The concept of Multicarrier Modulation (MCM) was proposed as early as early as the 1960's [1]. The motivation was to transmit the data in a large number of subcarriers, thereby reducing the symbol rate for each subchannel. The theoretically ideal MCM system has an infinite number of subchannels, and consequently an infinite length symbol. If the channel impulse response (CIR) is finite, the MCM system remains immune to distortion. However the idea was not practical until it was found that the modulation can be achieved via a Discrete Fourier Transform (DFT) [2]. Today it looks even more attractive with the use of the computationally efficient Fast Fourier Transform (FFT). The most widely used FFT based MCM methods are Orthogonal Frequency Division Multiplexing (OFDM) and Discrete Multitone (DMT). The former is mainly used for terrestrial broadcasting while the latter is used for Digital Subscriber Lines. We will address only OFDM systems in this paper. Both schemes use symbols generated by a finite length FFT with size N. The orthogonality of the consecutive OFDM symbols is maintained by appending a length v cyclic prefix (CP) at the start of each symbol [3]. The CP is obtained by taking the last v samples of each symbol and

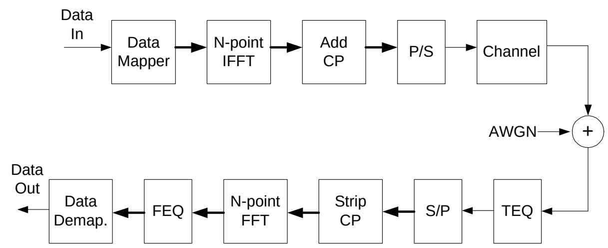

Fig. 1. Block Diagram of the OFDM System

so the total length of the transmitted OFDM symbol is $(N + v)$ samples. By this means the linear convolution of the transmitted signal with the CIR is converted into a circular one. For each OFDM symbol to be independent and to avoid any Inter Symbol Interference (ISI) or Inter Carrier Interference (ICI), the length of the CIR should be less than $v + 1$ samples. Hence the distortion caused by the CIR only affects the samples within the CP. The receiver takes only the last N samples for decoding at the receiver FFT, disregarding the CP. Consequently, the effects of the CIR can then be easily equalized by an array of one-tap Frequency Domain Equalizers (FEQ) following demodulation by the FFT. Figure 1 shows the block diagram of the system, where P/S and S/P mean parallel to serial and serial to parallel conversion respectively.

Due to the robustness of OFDM, it has been adopted as the physical layer standard for example in, Digital Video Broadcasting (DVB) [4] and HIPERLAN-2 [5]. One major disadvantage with the OFDM system is the reduction in the transmission efficiency by a factor $N/(N + v)$ caused by the CP. This is of even more concern when the transmitted symbol rate is higher, because this makes a CIR with the same rms delay spread span a greater number of samples hence requiring a longer CP. Future broadband wireless applications are likely to require data rates in excess of 50 Mb/s. Although the wireless fixed access channel delay spreads are short compared with the symbol rate for transmission rates that are in use today, it will not be the case for higher rates. One way of increasing the efficiency is to increase the FFT size, N. However this increases the complexity of the system and reduces the intercarrier spacing of subcarriers which subsequently makes the system more susceptible to frequency offset and oscillator phase noise. Also a higher number of subcarriers will increase the Peak to Average Power Ratio (PAPR), demanding the use of linear and consequently inefficient power amplifiers. Besides, for data transmission systems, such as HIPERLAN-2, short bursts using low number of subcarriers are used owing to latency considerations. The alternative is to use a Time Domain Equalizer (TEQ) preceding the FFT demodulator at the receiver in order to constrain the length of the Effective Channel Impulse Response (EIR) to be shorter than the selected CP duration. This permits the use of a much shorter CP than could otherwise be employed and so raises the transmission efficiency.

CIR shortening has been proposed as long ago as 1973 to permit the design of practical Maximum Likelihood Sequence Estimation (MLSE) receivers based on Viterbi Algorithm [6]. Later there was a flurry of research to reduce the length of CP in DMT systems [7], [8], [9]. The first two references are based on the Minimum Mean Square Error (MMSE) criterion and the latter is based on minimisation of the Shortened SNR (SSNR), where the ratio between the consecutive v samples of the EIR to the rest of the residuals is considered. To avoid trivial all-zero solutions, additional constraints such as the Unit Energy Constraint (UEC) or Unit Tap Constraint (UTC) are set on the Target Impulse Response (TIR). However the emphasis is to reduce the power of the residuals of the EIR in the time domain. The transfer function of the resulting EIR in the frequency domain often has spectral nulls rendering unusable subchannels with a low SNR. Attempts have also been made to optimise the TEQ in the frequency domain [10], [11]. In particular, [10] rarely results in a global minimum error.

In this paper a new Frequency Scaled Time Domain Equalizer (FSTEQ) is presented that effectively avoids spectral nulls in the frequency domain and yet achieves a better time domain convergence of the EIR. The remaining sections of the paper are organised as follows. In section II we present the background of the TEQ algorithm. In section III we present the FSTEQ algorithm. The channel models used are presented in section IV. We present simulated results in section V and Conclude in section VI.

II. TIME DOMAIN EQUALIZATION BASICS

All simulations and analysis in this paper are done in the complex baseband domain. Channel shortening with the use of a TEQ can be explained with reference to the Block diagram shown in Figure 2. The objective is to shorten the sampled CIR of length N_h, $\underline{h} = [h_0, .., h_{N_h-1}]^T$ to an EIR having significant samples for a length N_b, where $N_b < N_h$, with the use of a TEQ of length N_w, $\underline{w} = [w_0, .., w_{N_w-1}]^T$. The error sequence is generated by comparing the output sequence of the TEQ to that of the transmitted data stream, $\underline{x} = [x(k), .., x(k - N_b + 1)]^T$, sent through a desired TIR, $\underline{b} = [b_0, .., b_{N_b-1}]^T$ of length N_b. Here, k represents the time index and $x(k) = 0$ for $k < 0$. If the TEQ performs perfectly, the overall OFDM system can operate with a shorter CP of length N_b (i.e. $v = N_b - 1$).

If the received data is given by $r(k)$, then

$$r(k) = \underline{h}^T.\underline{x}' + n(k) \tag{1}$$

where $n(k)$ is the zero mean Additive White Gaussian Noise (AWGN) term. Here $\underline{x}' = [x(k), .., x(k - N_h + 1)]^T$. Hence the error signal after the TEQ is given as

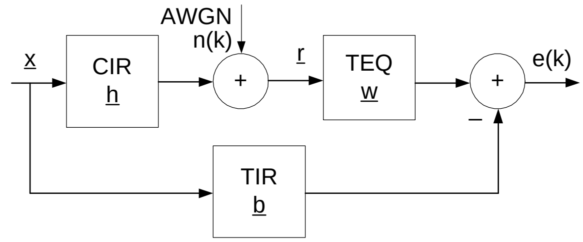

Fig. 2. Block Diagram of the TEQ

$$e(k) = \underline{w}^T.\underline{r} - \underline{b}^T.\underline{x} \tag{2}$$

where $\underline{r} = [r(k), .., r(k - N_w + 1)]^T$. The time index k is defined as before with $r(k) = 0$ for $k < 0$. The squared error is given by

$$\begin{aligned} E\{|e(k)|^2\} &= \underline{w}^T.R_{rr}.\underline{w}^* + \underline{b}^T.R_{xx}.\underline{b}^* - \underline{w}^T.R_{rx}.\underline{b}^* \\ &\quad -\underline{b}^T.R_{rx}.\underline{w}^* \end{aligned} \tag{3}$$

where $(.)^*$ denotes complex conjugation and R_{rr} ,R_{xx} and R_{rx} are the corresponding correlation matrices of $\underline{r}$ and $\underline{x}$. The optimal equalizer tap coefficients can be obtained by solving for the Minimum Mean Squared Error (MMSE) given by

$$d(E\{|e(k)|^2\})/d\underline{w} = 0 \tag{4}$$

which leads to

$$\begin{aligned} \Re[\underline{w}_{opt}] &= R_{rr}^{-1}.R_{rx}.\Re[\underline{b}] \\ \Im[\underline{w}_{opt}] &= R_{rr}^{-1}.R_{rx}.\Im[\underline{b}] \end{aligned} \tag{5}$$

where $\Re$ and $\Im$ are respectively the Real and the Imaginary part of a complex variable. Note that (5) depends on $\underline{b}$. Substituting the above relation in Equation 3 results in

$$E\{|e(k)|^2\} = \underline{b}^T.(R_{xx} - R_{rx}^T R_{rr}^{-1} R_{rx}).\underline{b}^* = \underline{b}^T.O.\underline{b}^* \tag{6}$$

By minimising Equation 6 the optimal coefficients for the TIR b_{opt}, can be found as the eigenvector corresponding to the smallest eigenvalue of the matrix O. Solutions for the case of real data can be found in [12].

An alternative iterative solution is presented in [6] for real data, a modified form of which is presented here for the case of complex data. Assuming the transmitted sequence is known during training, both the TIR coefficients $\underline{b}$, and the TEQ coefficients $\underline{w}$, can be obtained iteratively through steepest gradient methods such as Least Mean Square (LMS). Hence

$$\underline{w}^{k+1} = \underline{w}^k - \Delta_1.e(k).\underline{r}^* \tag{7}$$

and

$$\underline{b}^{k+1} = \underline{b}^k + \Delta_2.e(k).\underline{x}^* \tag{8}$$

where $\underline{w}^k$ and $\underline{b}^k$ are the tap coefficients at k^{th} iteration and Δ_1 and Δ_2 are the LMS convergence control parameters. To avoid the trivial solution, the UEC constraint on the TIR is used. We call the above algorithm the Dual Optimising TEQ (DOTEQ).

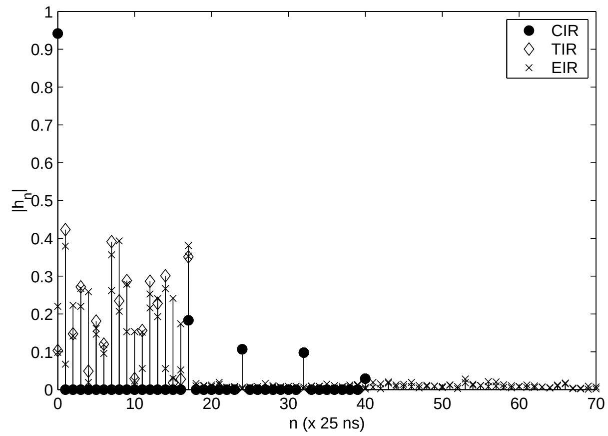

Fig. 3. DOTEQ Performance: Impulse Responses

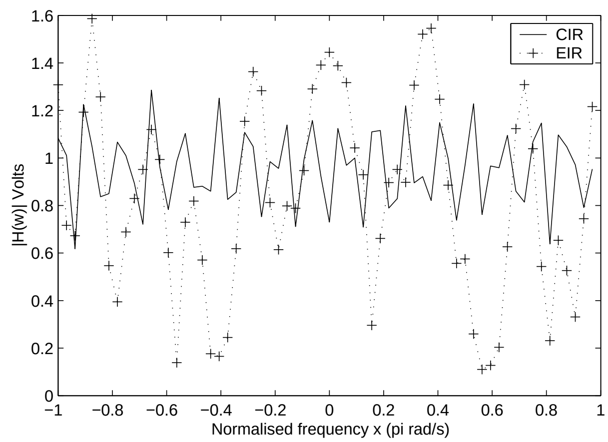

Fig. 4. DOTEQ Performance: Transfer Functions

Figure 3 shows the performance of the DOTEQ algorithm in time domain using the UEC. It can be observed that the EIR is similar to the TIR within the length of the TIR and also that the residuals beyond the TIR length (hence the CP) are low in magnitude compared with the pulses within the TIR length. However, as shown in Figure 4, the EIR in the frequency domain obtained using the DOTEQ algorithm has spectral nulls. Assuming that the transfer function of EIR is constant over each subchannel, the FEQ tap coefficients are calculated as the inverse of the EIR. Consequently the subchannels that fall in the nulls are severely degraded due to the low SNR and subsequently, the AWGN is amplified by the FEQ.

To address this problem, an algorithm will now be presented that achieves both a flatter response of the EIR in the frequency domain and a reduction of residuals of the EIR in the time domain.

III. FREQUENCY SCALED TIME DOMAIN EQUALISER (FSTEQ)

The coefficients required for the FEQs are calculated using a known OFDM training symbol. The inverse of the FEQ coefficients will give an estimate of the CIR.

1. Using the CIR both Upper and Lower frequency domain thresholds are set. The thresholds are selected such that they follow the envelope of the power delay profile of the channel as shown in Figure 5. These thresholds are used to constrain the TIR in the frequency domain, thus

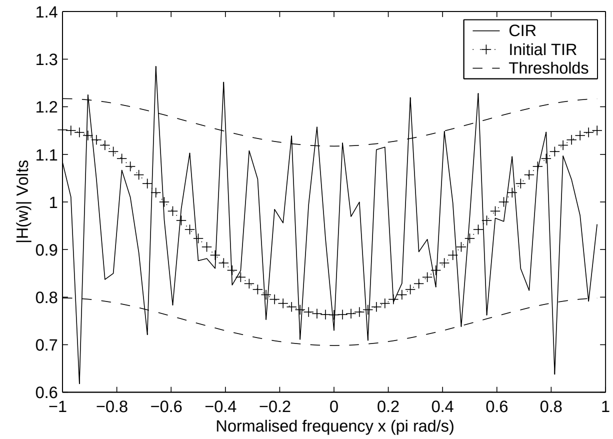

Fig. 5. FSTEQ Algorithm:Initial TIR

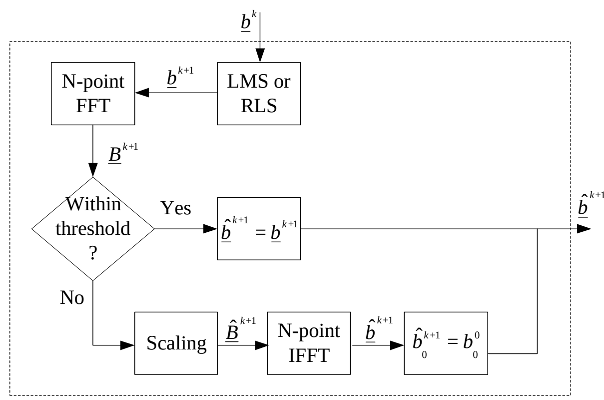

Fig. 6. FSTEQ Algorithm: Block Diagram

reducing the null depths in the EIR.

2. The TIR is initialized to be an exponentially decaying function so that the initial TIR is within the two threshold values.

3. At each step of the iteration, the TIR and TEQ are optimised concurrently using the LMS algorithm (Equation 7 and Equation 8) or by the use of the Recursive Least Squares (RLS) algorithm. In other words, the prescaled TIR $\underline{b}^{k+1}$ is calculated from $\underline{b}^k$.

4. Next the frequency domain transfer function $\underline{B}^{k+1}$ of $\underline{b}^{k+1}$ is calculated by an FFT also of size N. (see Figure 6). If $\underline{B}^{k+1}$ exceeds the two threshold values, it is scaled so that the resultant $\hat{\underline{B}}^{k+1}$ is within the thresholds. The updated TIR coefficients, $\hat{\underline{b}}^{k+1}$, are calculated by performing an IFFT on $\hat{\underline{B}}^{k+1}$.

5. The TIR coefficients are updated by the post-scaled values $\hat{\underline{b}}^{k+1}$, except for $\hat{b}_0^{k+1}$ which is forced to be equal to b_0^0 of the initial TIR. Thus the TIR in the frequency domain is constrained to lie within the upper and lower thresholds.

The coefficients required for the FEQs are calculated using a known OFDM training symbol. The training symbol, emanating from the TEQ can be used to estimate the EIR. It was observed that training the TEQ needs at least 1500 iterations. The overhead of transmitting such a long training sequence cannot be justified. This is avoided by sending a shorter training sequence and using accelerated training at the receiver. That is the same received training sequence is used several times until an acceptable level of convergence is achieved.

IV. CHANNEL MODEL USED

Channel models for broadband fixed wireless access channels are in the process of being defined [13]. The Stanford University Interim (SUI) channels that are suggested comprise 6 models for 3 different terrain conditions. All of them are simulated using 3 taps, having either Rayleigh or Ricean amplitude distributions. The model employed here is of this general type. The channel is assumed to be wide-sense stationary uncorrelated scattering (WSSUS) and each tap of the CIR is modeled as $h_i = \beta_i e^{j\phi_i}$, where the amplitude β_i and the phase ϕ_i are selected independently [14]. The model selected is based on 5 taps with the average power defined to be exponentially decaying. The maximum delay is specified to be 1 μs with an rms delay spread of 0.3 μs. The Ricean K factors for each tap are set to be [18 10 0 0 0] dB and the channel delays are specified to be 0, 0.42, 0.6, 0.8 and 1 μs. Figures 3 and 7 show a typical CIR for the chosen model.

V. SIMULATION RESULTS

The required data rate to be transmitted is assumed to be greater than 60 Mb/s. An OFDM system with $N = 64$ has been assumed, which is similar to the HIPERLAN-2 standard, but in this case a higher sampling rate of 40 MHz has been assumed. Without loss of generality, QPSK mapping for all subchannels has been employed and all subchannels are used. This dictates a data rate of $40 * 2 * N/(N + v) * 10^6$ bits/s. For instance, for $v = 15$ the data rate is 64.8 Mb/s.

A burst of 2500 OFDM symbols (i.e. equivalent to $2500 * 64 * 2 = 320000$ data bits) is assumed to be transmitted, which requires a time of less than 5 ms. Hence the channel is assumed constant for each burst. Each data point in the simulation results is obtained by averaging over 200 such bursts, changing the tap coefficients of the channel randomly in accordance with the power delay profile. The received Signal to Noise Ratio (SNR) is set to 20 dB for all the simulations and the TEQ length is set at 60.

Figure 7 shows the performance of FSTEQ algorithm in the time domain for a typical channel. Note that the peak magnitude of the EIR is at the beginning, well within the length of the CP. Figure 8 shows the corresponding transfer functions in the frequency domain. The final EIR shows very close convergence with the Initial TIR (see Figure 5) and more significantly, an absence of deep nulls in the EIR compared with that achieved by the DOTEQ presented in Figure 4. The values used for Δ_1 and Δ_2 in these simulations are 0.005 and 0.08 respectively.

Figure 9 shows the MSE of as a function of the number of iterations for different optimisation algorithms. The

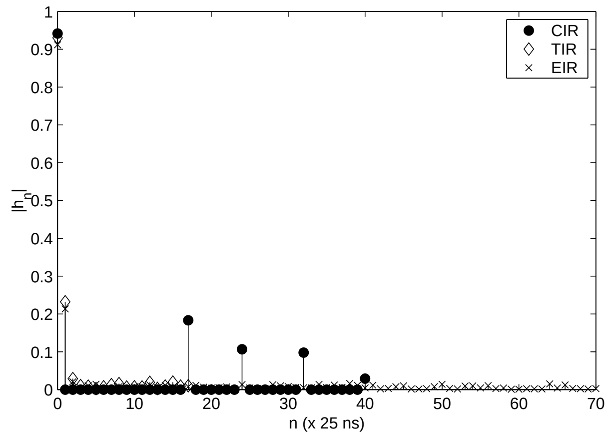

Fig. 7. FSTEQ Performance: Impulse Responses

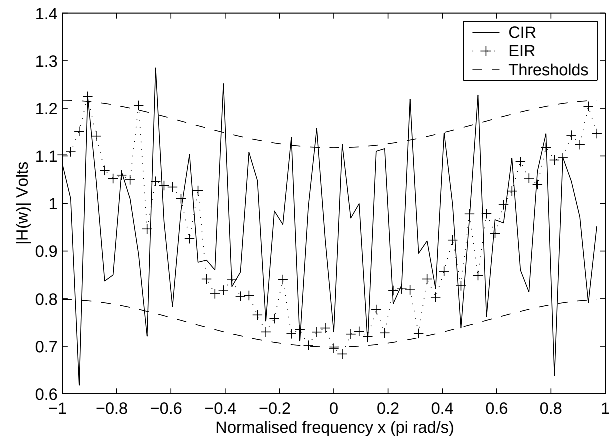

Fig. 8. FSTEQ Performance: Transfer Functions

TEQ training header has a length of 5 symbols and is run through the TEQ 4 times during training. Hence the total number of iterations is $64 * 4 * 5 = 1280$. Note that the MSE of DOTEQ is high even after 1200 iterations. The FSTEQ achieves much lower MSE but the LMS based training seems to be slightly more erratic at the beginning of the training period.

Figure 10 shows a comparison of different TEQ schemes. The FSTEQ utilising RLS achieves a Bit Error Rate (BER) some 10 to 100 times lower than that achieved using DOTEQ. For comparison, the OFDM performance in the absence of TEQ is also included. FSTEQ utilising LMS has a higher BER than that with RLS but achieves a generally better BER than OFDM without TEQ. Interestingly DOTEQ actually performance worse than the OFDM without TEQ. It was also found that DOTEQ performance is very sensitive to the values of the convergence parameters Δ_1 and Δ_2.

VI. FUTURE WORK AND CONCLUSION

We have presented an algorithm that optimises the TEQ design both in the time and frequency domains. We have shown through simulations that the improvement in BER performance can be up to 100 fold compared to that achieved with OFDM without FSTEQ. Admittedly the algorithm needs a reasonable amount of processing time and memory to store the incoming data during the training of the TEQ. However the objective here is not to

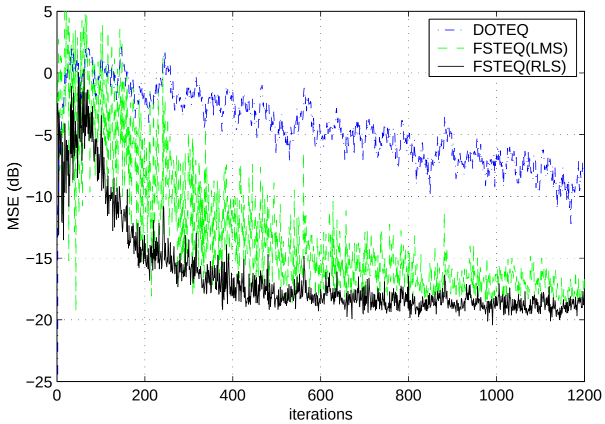

Fig. 9. Comparison of MSE

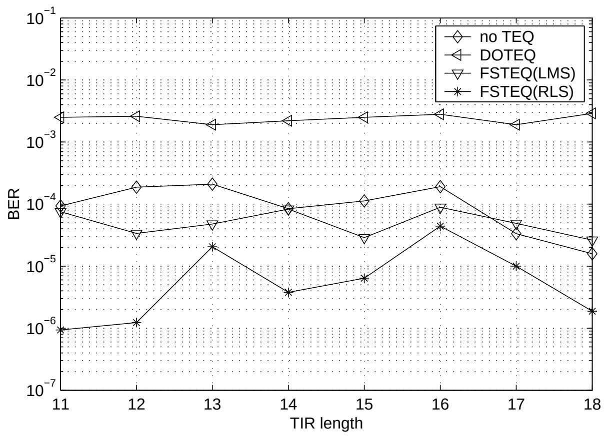

Fig. 10. Comparison of the TEQ schemes

optimise the resources used, but rather to see what performance gains are possible by the use of TEQ for OFDM in broadband fixed wireless scenario. It should be noted that since the FFT is already implemented in the OFDM demodulator, the additional hardware required for the implementation of the FSTEQ is somewhat relieved. In future, the algorithm will be applied to other SUI channels and also techniques to enhance performance of FSTEQ algorithm in terms of its error rate and processing requirements will also be investigated.

REFERENCES

[1] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell System Technical Journal*, pp. 1775–1796, December 1966.

[2] S. Weinstien and P. Ebert, "Data transmission by frequency division multiplexingusing the discrete fourier transform," *IEEE Transaction in Communication Technology*, vol. COM-19, pp. 628–634, October 1971.

[3] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computationally complexity algorithms," in *Proceedings of IEEE International Conference of Acoustics, Speech and Signal Processing*, (Denver), pp. 964–967, April 1980.

[4] P. Shelswell, "The COFDM modulation system: The heart of digital audio broadcasting," *IEE Electronics & Communication Engineering Journal*, vol. 7, pp. 127–136, June 1995.

[5] ETSI, *Broadband Radio Access Networks (BRAN); HIPERLAN-2; Physical Layer*, April 2000.

[6] D. Falconer and F. Magee, "Adaptive channel memory truncation for maximum likelihood sequence estimation," *Bell System Technical Journal*, vol. 52, pp. 1541–1562, November 1973.

[7] J. S. Chow and J. M. Cioffi, "A cost-effective maximum likelihood receiver for multicarrier systems," in *Proceedings of IEEE International Conference on Communication*, (Chicago, USA), pp. 948–952, June 1992.

[8] N. Al-Dhahir and J. M. Cioffi, "Optimum finite-length equalization for multicarrier transceivers," *IEEE Transactions on Communications*, vol. 44, pp. 56–64, January 1996.

[9] P. J. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse response shortening for discrete multitone transceivers," *IEEE Transaction on Communications*, vol. 44, pp. 1662–1672, December 1996.

[10] J. S. Chow, J. M. Cioffi, and J. A. Bingham, "Equalizer training algorithms for multicarrier modulaiton systems," in *Proceedings of IEEE International Conference on Communication*, (Geneva), pp. 761–765, May 1993.

[11] T. Pollet, M. Peeters, M. Moonen, and L. Vandendorpe, "Equalization for DMT-based broadband modems," in *IEEE Communication Magazine*, pp. 106–113, May 2000.

[12] M. Van Bladel and M. Moeneclaey, "Time-domain equalization for multicarrier communication," in *Proceedings of IEEE Globecomm*, (Singapore), pp. 167–171, November 1995.

[13] V. Erceg, K. Hari, *et al.*, "Channel models for fixed wireless applications," tech. rep., IEEE 802.16 Broadband Wireless Access Working Group, January 2001.

[14] K. Pahlavan and A. Levesque, *Wireless Information Networks.* New York: J. Wiley & Sons, 1995.