Common Phase Error Correction for OFDM in Wireless Communication

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Abstract—**Orthogonal Frequency Division Multiplex (OFDM) systems are very sensitive to phase noise caused by oscillator instabilities. In this paper the phase noise is resolved into two components, namely the Common Phase Error (CPE), which affects all the subchannels equally and the Inter Carrier Interference (ICI), which is caused by the loss of orthogonality of the subcarriers. We present a technique to estimate and correct the CPE component and demonstrate its effectiveness when applied to a Broadband Fixed Wireless Access (BFWA) data transmission system for the Multichannel Multipoint Distribution Service (MMDS) band. We show a performance increase up to 6 dB when applying the CPE correction in terms of the tolerance to the phase noise variance, σ_ψ^2.**

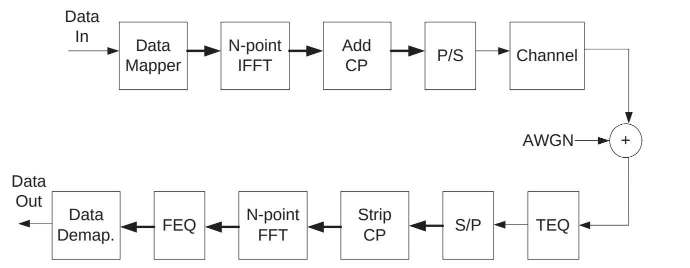

Fig. 1. Block Diagram of the OFDM System

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has become increasingly popular for wireless data transmission in Broadband Fixed Wireless Access (BFWA) systems due to its robustness under multipath effects. It looks even more attractive as modulation and demodulation can be done digitally by computationally efficient Fast Fourier Transforms (FFT) of finite length, N.

The orthogonality of the consecutive OFDM symbols is maintained by appending a length v cyclic prefix (CP) at the start of each symbol. The CP is obtained by taking the last v samples of each symbol and consequently the total length of the transmitted OFDM symbols is $(N + v)$ samples. For each OFDM symbol to be independent and to avoid any Inter Symbol Interference (ISI) or Inter Carrier Interference (ICI), the length of the Channel Impulse response (CIR) should be less than $v + 1$ samples. The receiver discards the CP and takes only the last N samples of each OFDM symbol for demodulation by the receiver FFT. Consequently, the effects of the CIR can then be easily equalized by an array of one-tap Frequency Domain Equalizers (FEQ) following the FFT. This is because the frequency selective fading channel can be approximated as a sum of flat fading channels, provided the number of subchannels is large. Sometimes a Time Domain Equaliser (TEQ) may be used to shorten the effect of the CIR [1]. Figure 1 shows the block diagram of the system.

Unfortunately, OFDM has been proven to be very sensitive to oscillator phase noise. The usual scenario for BFWA transmission is that of a point to multipoint system. Here, a single base station (BS) transmits to many subscriber units (SU) placed at the user locations. The system feasibility depends heavily on the cost of the SUs. The use of low cost components, particularly the oscillators, is a major issue since their accuracy and stability are directly related to cost. Hence it is imperative that if OFDM is to be used for BFWA transmission, it should be able to operate effectively using oscillators with only moderate performance and cost.

An OFDM system effectively consists of N sinusoidal subcarriers with frequency spacing $1/T$, where T is the active symbol period of each subcarrier. The kth subcarrier will thus be at $f_k = f_0 + k/T$, where f_0 is a reference frequency. Without loss of generality, we can assume $f_0 = 0$. The modulated subcarriers overlap spectrally, but since they are orthogonal over a symbol duration, they can be easily recovered as long as the channel does not destroy the orthogonality. An unwindowed OFDM system has rectangular symbol shapes. Hence, in the frequency domain the individual subchannels will have the form of sinc functions where the first sidelobe is only some 13 dB below the main lobe of the subcarrier. A practical oscillator has spectral components around the centre frequency. These components cause the loss of orthogonality of the OFDM carriers. In the frequency domain it can be viewed as interference caused on a particular subcarrier by the high sidelobes of the adjoining carriers. This explains the higher susceptibility of OFDM to phase noise. Reference [2] gives a good overview of the problem.

All analysis and simulation in this paper is done in the digital complex baseband domain. The nth sample of the mth OFDM symbol generated by the Inverse FFT (IFFT) at the transmitter is

$$s_{m,n} = \sqrt{\frac{1}{N}} \sum_{k=0}^{N-1} A_{m,k} e^{j2\pi \frac{kn}{N}},\ 0 \le n \le N-1 \qquad (1)$$

$A_{m,k}$ is the data symbol modulated on to the kth subcarrier of the mth OFDM symbol. The data is converted into a serial sequence, then the CP is added. Thus the mth transmitted OFDM symbol is $\underline{s}(m) = [s_{m,N-v}, .., s_{m,N-1}, s_{m,0}, .., s_{m,N-1}]^T$. We assume a finite length CIR of length N_h samples, $\underline{h} = [h_0, .., h_{N_h-1}]^T$, where $v \ge N_h - 1$. The received signal is

$$r(k) = [p(k) + w(k)]e^{j\psi(t)},\ -\infty < k < \infty \qquad (2)$$

where $p(k)$ represents the convolution of the channel, $\underline{h}$, with the serially concatenated transmitted OFDM symbols $\underline{s}(m)$, $-\infty < m < \infty$. Also $w(k)$ is zero mean Additive White Gaussian Noise (AWGN) and $e^{j\psi(t)}$ is the multiplicative error term due to oscillator phase noise. It is assumed that a training sequence occupying OFDM symbol number N_t, $\underline{\tilde{A}} = [A_{N_t,0}, .., A_{N_t,N-1}]^T$ is sent to permit the detection of the start of the OFDM frame by performing a correlation at the receiver with a locally generated

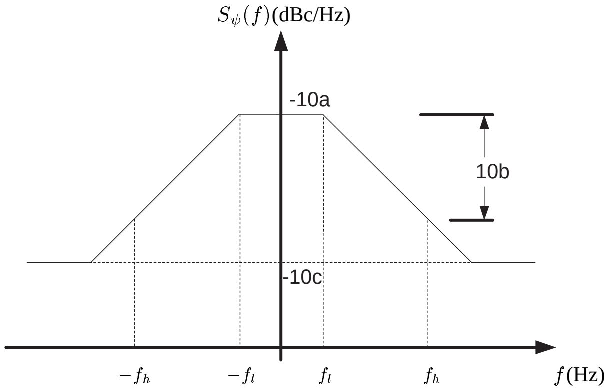

Fig. 2. Phase Noise PSD of a Typical Oscillator

copy of the training sequence. In general, $N_t = 0$. The position of the start of the received copy of $\underline{\tilde{A}}$ is used to determine the start of subsequent OFDM symbols. Once the start of all the received OFDM symbols are known, then for each symbol the CP is discarded and the remainder of the symbol, $r_{m,n}$, $0 \le n \le N-1$ is used for demodulation. The demodulated data symbol of the lth subcarrier of the mth OFDM symbol $Y_{m,l}$ is given by

$$Y_{m,l} = \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} r_{m,n} e^{-j2\pi\frac{ln}{N}},\ 0 \le l \le N-1 \qquad (3)$$

The effect of the channel can now be eliminated by equalisation provided we know the channel frequency response. From the knowledge of the transmitted training symbol, $\underline{\tilde{A}}$ and the corresponding received symbol, $Y_{N_t,l}$, $0 \le l \le N-1$ the tap coefficients for the FEQ are calculated as

$$C_{N_t,l} = \frac{Y_{N_t,l}}{A_{N_t,l}},\ 0 \le l \le N-1 \qquad (4)$$

where $C_{N_t,l}$ approximates the channel transfer function H_l, $0 \le l \le N-1$. The equalised data after the FEQ can thus be realised as $\hat{Y}_{m,l} = Y_{m,l}/C_{N_t,l}$

II. Oscillator Phase Noise

The phase noise is modeled as a phasor $e^{j\psi(n)}$, where the phase noise process $\psi(n)$ is zero-mean and wide-sense stationary with a narrow band Power Spectral Density (PSD) mask, $S_\psi(f)$ given by (5) having a finite power σ_ψ^2 [3].

$$S_\psi(f) = 10^{-c} + \begin{cases} 10^{-a} & f < f_l, \\ 10^{-(f-f_l)(b/f_h - f_l)-a} & f > f_l, \\ 10^{(f+f_l)(b/f_h - f_l)-a} & f < -f_l \end{cases} \qquad (5)$$

Parameter c determines the noise floor of the oscillator and a determines the noise PSD from the center frequency to $\pm f_l$. Parameter b gives the noise fall off rate and at f_h the noise PSD is 10b dB lower than the value at f_l (See Figure 2). Typical parameter values for a 5.2 GHz synthesized source are $a = 8$, $b = 2$, $c = 12$, $f_l = 10$ kHz and $f_h = 100$ kHz [4].

If $\psi(n)$ takes small values, then for analytical purposes a simplifying assumption is $e^{j\psi(n)} \approx 1 + j\psi(n), \psi \ll 1$. Using this assumption and using (1), (2) and (3) and incorporating H_l gives,

$$Y_{m,l} \approx \frac{1}{N}\sum_{n=0}^{N-1}\left\{\left\{\sum_{k=0}^{N-1} A_{m,k} H_k e^{j2\pi\frac{kn}{N}}\right\}(1+j\psi(n))\right\} e^{-j2\pi\frac{ln}{N}} + W_l \qquad (6)$$

where W_l is the contribution due to AWGN. We can further simplify (6) as,

$$Y_{m,l} \approx H_l A_{m,l} \frac{1}{N}\sum_{n=0}^{N-1}(1+j\psi(n)) + \frac{1}{N}\sum_{\substack{k=0\\k\ne l}}^{N-1} H_k A_{m,k} \sum_{n=o}^{N-1}(1+j\psi(n)) e^{-j2\pi\frac{(l-k)n}{N}} + W_l \qquad (7)$$

The first term on the r.h.s. of (7) rotates the useful component $H_l A_{m,l}$ of each subcarriers by an equal amount and is *independent* of the particular subchannel concerned, l. This is commonly known as the Common Phase Error (CPE). The second term is the Inter-Carrier Interference (ICI) caused by contributions from all subcarriers $k \ne l$ on l due to the loss of orthogonality. Let us denote these components as ψ_{CPE} and ψ_{ICI} respectively. Unlike the CPE, ICI is not easy to estimate.

References [3] and [5] use pilot tones with a known phase to estimate the the phase noise. The channel number of the pilot tones change (incremented by 1) after every OFDM symbol. The authors in [6] also use a pilot based scheme. All these proposals are aimed at mitigating the effects of phase noise in broadcast digital TV. However, for BFWA applications, the use of subchannels continuously as pilot tone transmission is not practical.

In this paper, we present a simple yet effective CPE correction (CPEC) algorithm that does *not* use any pilot tones, consequently utilisation of the subchannels is not degraded. The algorithm is based on decision directed feedback compensation and it continuously tracks the CPE. Section III presents the algorithm in detail and section IV show the simulation results. Finally we draw conclusions and present future work in section V.

III. CPE Correction (CPEC) Algorithm

From the Central Limit theorem the effect of ICI can be assumed to be zero mean Gaussian, provided the number of subchannels is large and the data symbols are statistically independent. Consequently, ICI has a similar effect to AWGN. Hence $E(\psi_{ICI}) = 0$, where $E(.)$ is the statistical expectation operator. Also $E(W_l) = 0$. Therefore with reference to (7) an estimate of the CPE for the mth OFDM symbol, $\hat{\psi}_{CPE,m}$, can be made by finding the mean of the phase rotations caused to the subchannels in a particular symbol. However, the CPE estimate could be seriously affected by errors caused by subchannels with a low SNR resulting from spectral nulls in the channel response H_l. Hence we only select those subchannels with $|H_l|$ above a certain threshold. We call this subset of subchannels $\underline{d} \subset [0,..N-1]$. The criteria applied is to select subchannels with $|H_l|$ in excess of a standard deviation above the mean. For post-FEQ symbol m, the outputs of these subchannels $\hat{Y}_{m,d}$ are sent through a slicer to obtain $\tilde{Y}_{m,d}$. If the number of subchannels selected for the CPE estimation is N_d, then the CPE estimate for symbol m is,

$$\hat{\psi}_{CPE,m} = \frac{1}{N_d} \sum_{\substack{l=0 \\ l \in \underline{d}}}^{N-1} (\angle \hat{Y}_{m,l} - \angle \tilde{Y}_{m,l}) \tag{8}$$

The effect of CPE is cancelled by multiplying the post-FEQ symbol $\hat{Y}_{m,l}, 0 \leq l \leq N-1$ by $e^{-j\hat{\psi}_{CPE,m}}$. The CPE has to be estimated for each symbol of data at the output of the FEQ. Although the CPE changes slowly, it can have a considerable variance. The channel estimation procedure required for the FEQ is conventionally performed at the beginning of the data frame and remains the same unless a new training symbol is sent. If the block containing the training symbol has a significant value for $\hat{\psi}_{CPE,N_t}$, then the channel phase estimation, $\angle C_{N_t,l}$ will be offset by a value approximately equal to $\hat{\psi}_{CPE,N_t}$. All estimates of the CPE for the subsequent symbols will be affected by the offset caused by the FEQ. (i.e. the total CPE estimate for subsequent symbols is $\hat{\psi}_{CPE,m} + \hat{\psi}_{CPE,N_t}$). This will cause the magnitude of CPE estimates of subsequent OFDM symbols to become too large which can lead to phase wrapping.

To remove the effect of $\hat{\psi}_{CPE,N_t}$ from subsequent symbols, it is proposed to have a simple moving average filter of length N_w containing CPE estimates from previous symbols (i.e.$\underline{\psi}_{CPE,m} = [\hat{\psi}_{CPE,m-1}, .., \hat{\psi}_{CPE,m-N_w}]$). The output of the filter, $\underline{\bar{\psi}}_{CPE,m}$, is used to update the phase angles of the FEQ for all symbols $m > N_t$. We call it the Feedback (FB) correction factor.

$$\angle C_{m,l} = \angle C_{m-1,l} + \underline{\bar{\psi}}_{CPE,m} \; 0 \leq l \leq N-1 \tag{9}$$

The CPEC algorithm can be summarised as follows.
1. Select the subset of subchannels with peaks in the channel transfer function, $\underline{d}$, at the start of the burst.
2. Calculate $\underline{\bar{\psi}}_{CPE,m}$ by finding the mean of the previous N_w CPE estimates.
3. Use $\underline{\bar{\psi}}_{CPE,m}$ for each symbol m to update the phase of the FEQ coefficients, $\angle C_{m,l}$. This effectively makes the FEQ track the CPE as closely as possible.
4. Obtain $\hat{Y}_{m,\underline{d}}$ from the FEQ output and then $\tilde{Y}_{m,\underline{d}}$ by use of a slicer. Use the difference in angles between $\hat{Y}_{m,\underline{d}}$ and $\tilde{Y}_{m,\underline{d}}$ to get an estimate of the CPE, $\hat{\psi}_{CPE,m}$. Use it to correct the CPE.
5. Move $\hat{\psi}_{CPE,m}$ in to the moving average filter of length N_w to calculate $\underline{\bar{\psi}}_{CPE,m+1}$.

Figure 3 illustrates the CPEC algorithm. The main advantage of CPEC algorithms is its simplicity and low computational demand. Assuming that the determination of the phase angle of a complex number is performed through a look up table, processing of each OFDM symbol requires an additional N_d subtractions, $N_d - 1$ additions and one division for the calculation of the CPE estimate as required in (8) and $N_w - 1$ additions and one division in the calculation of $\underline{\bar{\psi}}_{CPE,m}$ and finally N additions in the updating of the FEQ angles, as shown in (9). Unlike the scheme proposed in [7], which only compensates the CPE estimate $\hat{\psi}_{CPE,m}$, the FB correction factor in our scheme is able to continuously track and correct for the error caused by phase noise in the FEQ.

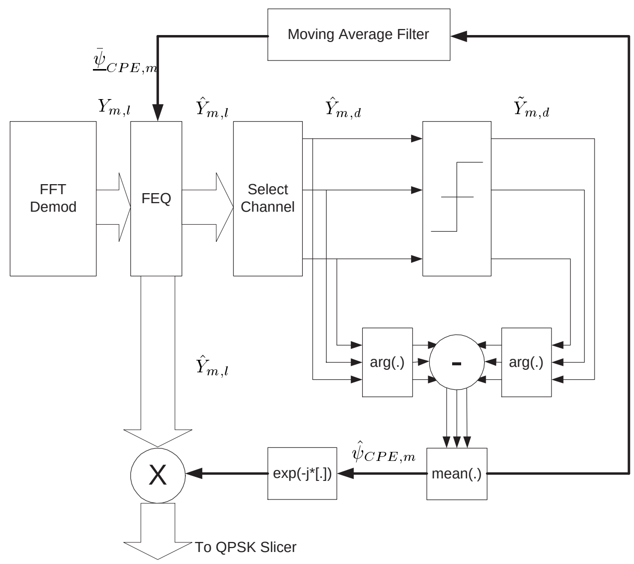

Fig. 3. CPE Correction (CPEC) Algorithm

IV. Simulation Parameters and Results

Appropriate models for broadband fixed wireless access channels are in the process of being defined. The Stanford University Interim (SUI) channels comprise 6 models for the Multichannel Multipoint Distribution Service (MMDS) band for 3 different terrain conditions [8]. All of them are simulated using 3 taps, each having either Rayleigh or Ricean amplitude distributions. The channel is assumed to be wide-sense stationary uncorrelated scattering (WSSUS) and each tap of the CIR is modeled as $h_i = \beta_i e^{j\phi_i}$, where the amplitude β_i and the phase ϕ_i are selected independently [9]. We have selected the SUI-2 channel model, pertaining to terrains with low tree densities and with antennas having directivity of 30 degrees at the SU and 120 degrees at the BS. The channel is characterised by a RMS delay spread of 0.2 μs.

OFDM systems with $N = 64, 128, 256$ have been assumed at a sampling rate of 20 MHz with a guard interval equal to 20 samples, thus the subcarrier spacings are approximately 312 kHz, 156 kHz and 78 kHz respectively. QPSK mapping for all subchannels has been employed and all the subchannels are used. A burst equivalent to 320000 bits is transmitted, which takes less than 10 ms, consequently the channel is assumed constant for the duration of each burst. Each data point in the simulation results is obtained by averaging over 750 such bursts, each experiencing random channel realisations in accordance with the SUI-2 profile. The received Signal to Noise Ratio (SNR) due to AWGN is set at 20 dB for all of the simulations. The value selected for the FB buffer length, N_w is 2.

Figure 4 shows the simulation results for the CPEC algorithm with $N = 64, 128, 256$ for phase noise mask parameters $f_h = 100$ kHz and $b = 4$. The curves are plotted against the inverse of the phase noise variance, σ^2_{ψ}. The received signal power and the LO power are normalised to unity. The phase noise variance is set using the parameters a and c. It can be seen that for $N = 64$ CPEC algorithm achieves a performance gain of 6 dB at BER of 10^{-5} over a system without CPEC. The performance gain can be seen to

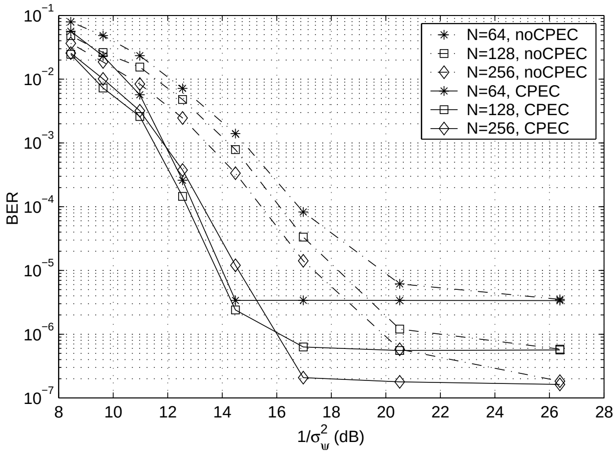

Fig. 4. Performance of the CPE correction algorithm for $b = 4$ and $f_h = 100$ kHz

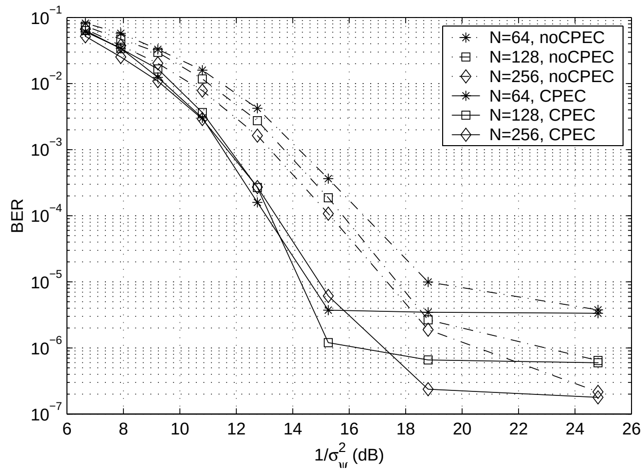

Fig. 5. Performance of the CPE correction algorithm for $b = 4$ and $f_h = 200$ kHz

decrease with increasing N, for example with $N = 256$ the gain is reduced to 3 dB. This can be attributed to the fact that the CPE variance decreases with increasing N, which is confirmed by the theoretical analysis presented in [7]. The improvements are very small in excess of 20 dB owing to AWGN becoming dominant over phase noise. It should be noted that for the SUI-2 channel model at the selected SNR of 20 dB (AWGN only), a BER floor of the order of 10^{-7} is achieved. Obviously, if the SNR was higher, the improvements due to CPEC would be correspondingly greater.

Figure 5 shows the results of the simulations carried out using the phase noise mask with $f_h = 200$ kHz. All other parameters are similar to the previous case. Hence the phase noise will induce more spectral spillage and will have a higher variance for the same values of the parameters a and c. The performance gain is 4 dB for $N = 64$ but is reduced to 2.3 dB for $N = 256$ at a BER of 10^{-5}. The number of selected subchannels in the CPE estimation, N_d averaged 12 for $N = 64$ over all bursts. Hence the total number of additional computations on average was 12 additions, 12 subtractions and 2 divisions per OFDM symbol. The gain due to CPEC algorithm is quite significant considering the low additional computation effort.

V. Conclusion

We have analysed the effect of phase noise in OFDM and have shown that it has a dual effect, one term that affects all subchannels equally, which is termed CPE and another which affects the orthogonality of the subcarriers, termed ICI. We have also presented a simple, yet effective algorithm to correct the CPE. We have shown via simulation that when the algorithm is applied to BFWA systems over MMDS channels at an SNR of 20 dB, performance gains up to 6 dB are achieved. We postulate that the use of the algorithm will allow MMDS equipment manufacturers to use lower quality RF oscillators without performance degradation, thus allowing a significant cost saving.

We also intend to analyse the performance of the algorithm for channel models pertaining to other types of terrains and we will also investigate the performance of the algorithm with other types of multichannel modulation schemes.

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