Application of Game Theory for Distributed Dynamic Channel Allocation

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Abstract— **A payoff function used in Game Theory is derived and a mixed strategy is applied to the fully distributed Dynamic Channel Allocation (DCA) problem for a Broadband Fixed Wireless Access (BFWA) network using Packet Reservation Multiple Access (PRMA). DCA using Least Interfered (LI) and Random Channel Allocation (RND) are simulated and their performances are compared with the proposed DCA using Game Theory (GT).**

I. INTRODUCTION

Fig. 1 shows a typical BFWA layout and components. The Subscriber Unit (SU) communicates with an Access Point (AP) using a directional antenna. An AP uses a sectored antenna to communicate with the SUs covered by it. Several APs can be connected to a Control Server where management and authentication are provided.

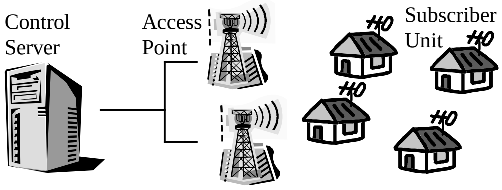

Fig. 1. Broadband fixed wireless access components and layout.

The channels in a BFWA network are usually reused and this causes co-channel and adjacent channel interference resulting in low average Signal to Noise Ratio (SNR) and degrading the performance of the network. Frequency planning is thus required to reduce these interferences but this process is usually time consuming and inflexible – as in Fixed Channel Allocation (FCA). Dynamic Channel Allocation (DCA) can be employed in a BFWA network to reduce the frequency planning process by having the APs adapt to the interference environment.

Existing channel allocation methods found in publications are used mostly for voice calls. These methods can be categorised in a Channel Allocation Matrix shown in Fig. 2.

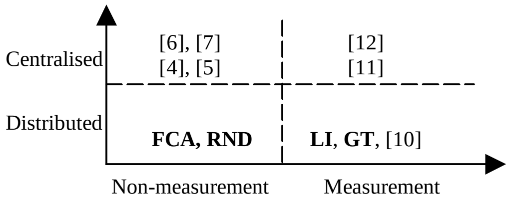

Fig. 2. Channel Allocation Matrix

The vertical axis in Fig. 2 is a measure of the centralisation required in the channel allocation method. The degree of centralisation is defined as being proportional to the number of APs required to communicate with a central controller in order to allocate a channel. A fully centralised system requires every AP in the entire network to communicate with a central controller while in a fully distributed system, the AP can make the decision to allocate a channel on its own. The more centralised the system the greater is the amount of signaling required, which causes high packet or call set-up delay and may result in system instability. The more distributed the system is, the less global knowledge is present at each AP and so the decision is based on partial knowledge and usually the allocation is made to benefit itself. The horizontal axis in Fig. 2 represents the quantity of measurements (e.g. interference power or SNR) made by the APs and/or SUs prior to a channel allocation. Measurement adds to the complexity of the process and needs to be performed quickly to minimise packet delay. In a non-measuring scheme, a-priori knowledge of the network such as the reuse distance and the compatibility matrix [2] are used. The Channel Allocation Matrix can be divided into four quadrants: Distributed Non-measurement, Centralised Non-measurement, Distributed Measurement and Centralised Measurement. Citations to channel allocation methods in each quadrant are shown in Fig. 2.

This paper focuses on the Distributed quadrants of the Channel Allocation Matrix for a data oriented service. Section II defines a payoff function and describes the application of Game Theory to DCA for a data service. Section III describes two existing DCA methods namely the Least Interfered (LI) and Random Channel Allocation (RND) and proposes a new method using Game Theory (GT). Section IV describes the simulation and the results and Section V gives the conclusion.

II. PAYOFF FUNCTION

The simulation assumes an asynchronous BFWA network using asymmetric time division duplex (TDD) with packet reservation multiple access (PRMA) [1]. The BFWA Medium Access Control (MAC) frame structure is shown in Fig. 3.

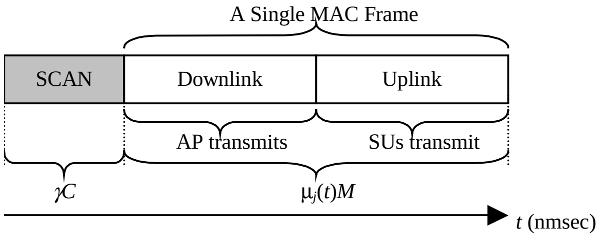

Fig. 3. A single MAC frame structure.

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The unit of time is measured in terms of a normalized second (nmsec), where a normalized second is the time required by an AP or a SU to transmit a data packet (an ATM cell in this case). This unit is defined so that the analysis is independent of transmission rate. The Downlink and Uplink portion of the MAC frame is used for packet transmission and it lasts for $\mu_j(t)M$ nmsec. The length of the transmitting portion varies according to traffic load. M is the maximum MAC frame length and $\mu_j(t)$ is a measure of traffic load (both downlink and uplink) for AP j at time t as a fraction of M. The SCAN portion at the beginning of the MAC frame is used by the AP to measure the interference power of C available frequency channels with a scan time of γ nmsec per channel. Using the measured interference power a channel is selected and is used by the AP for the next F MAC frames before a new SCAN is executed. Let $\mathrm{T}_j(t)=\mu_j(t)MF$ be the period between two scans (both the transmitting and receiving portion) for AP j.

In Game Theory [13], the choices made by every individual within a group affect the outcome of the entire group. This interdependency characteristic is present in the fully distributed DCA scheme where the channels selected by each AP independently of each other change the interference environment. An AP will tend to select the strategy that will give it the highest payoff. For a pair of APs, the payoff function $\pi_{j,k}(t)$ for AP j at time t is defined as the number of packets transmitted (and received) that are interference free from AP k per nmsec and is expressed as:

$$\pi_{j,k}(t)=\mathrm{G}_j(t)\left(\left(1-\mathrm{P_I}(t)\right)\mathrm{O}_{j,k}(t)+\mathrm{S}_{j,k}(t)\right) \tag{1}$$

Where $\mathrm{G}_j(t)$ is the packet throughput for AP j defined as the percentage of time a packet is transmitted or received. $\mathrm{G}_j(t)$ is expressed as:

$$\mathrm{G}_j(t)=\frac{\mathrm{T}_j(t)}{\gamma C+\mathrm{T}_j(t)} \tag{2}$$

$\mathrm{P_I}(t)$ is the probability that AP j and AP k use the same channel and it is dependent upon the DCA method used. $\mathrm{O}_{j,k}(t)$ is the average fraction of $\mathrm{T}_j(t)$ that would coincide with $\mathrm{T}_k(t)$ and $\mathrm{S}_{j,k}(t)$ is the average fraction of $\mathrm{T}_j(t)$ that coincides with the SCAN portion of AP k. This is illustrated in Fig. 4 where $\mathrm{O}_{j,k}(t)$ and $\mathrm{S}_{j,k}(t)$ are the average of $\mathrm{o}_j(t)$ and $\mathrm{s}_j(t)$ respectively.

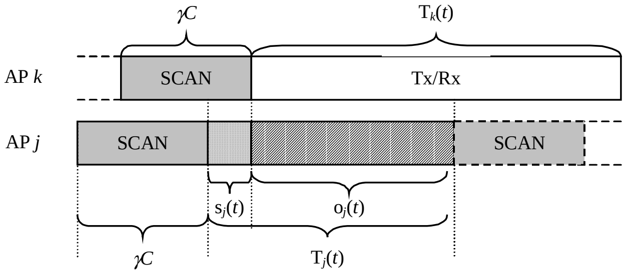

Fig. 4. Fraction of $\mathrm{T}_j(t)$ overlaps $\mathrm{T}_k(t)$ and SCAN portion of AP k.

$\mathrm{O}_{j,k}(t)$ and $\mathrm{S}_{j,k}(t)$ are expressed as follows:

$$\mathrm{O}_{j,k}(t)=\begin{cases}\dfrac{\mathrm{T}_k(t)}{\gamma C+\mathrm{T}_k(t)+\mathrm{T}_j(t)} & ,\mathrm{T}_j(t)\le\gamma C+\mathrm{T}_k(t)\\ \dfrac{\mathrm{T}_k(t)}{2\left(\gamma C+\mathrm{T}_k(t)\right)} & ,\mathrm{T}_j(t)>\gamma C+\mathrm{T}_k(t)\end{cases} \tag{3}$$

$$\mathrm{S}_{j,k}(t)=\begin{cases}\dfrac{\gamma C}{\gamma C+\mathrm{T}_k(t)+\mathrm{T}_j(t)} & ,\mathrm{T}_j(t)\le\gamma C+\mathrm{T}_k(t)\\ \dfrac{\gamma C}{2\left(\gamma C+\mathrm{T}_k(t)\right)} & ,\mathrm{T}_j(t)>\gamma C+\mathrm{T}_k(t)\end{cases} \tag{4}$$

III. DCA METHODS

The DCA methods considered are RND, LI and GT. RND and LI are DCA methods originally used in voice services and in this paper they will be applied to a data service with MAC structure shown in Fig. 3. For each DCA method, the AP selects a channel based on a specific strategy and uses this channel for F MAC frames. The set of strategies $\mathbf{S}$ is represented as:

$$\mathbf{S}=\left\{F\in\mathbf{I}\,|\,0<F<\infty\right\} \tag{5}$$

Where $\mathbf{I}$ is the set of integer numbers.

A. Random Channel Assignment (RND)

In RND [10] each AP randomly selects a channel at the start of every MAC frame (i.e. F=1) without any interference measurements (i.e. $\gamma = 0$). The channels are selected based on a uniform distribution and hence each of the C channels has an equal probability of being selected. The probability $\mathrm{P_I}(t)$ for RND namely, $\mathrm{P_{I\_RND}}(t)$ is:

$$\mathrm{P_{I\_RND}}(t)=\frac{1}{C} \tag{6}$$

B. Least Interfered Method (LI)

In LI [8], the AP scans all available channels and selects the channel with the lowest interference power. If more than one channel shares the same lowest interference power, the channel used previously will be selected and if none were used previously, the channel with the lowest number is selected. The selected channel is used for F frames before the next scan and channel selection.

The probability $\mathrm{D}_{j,k}(t)$ of AP j detecting the channel usage of AP k when AP j is scanning at time t is:

$$\mathrm{D}_{j,k}(t)=\begin{cases}\dfrac{\mathrm{T}_k(t)}{\gamma C+\mathrm{T}_k(t)} & ,\gamma>0\\ 0 & ,\gamma=0\end{cases} \tag{7}$$

If AP j detects the channel usage of AP k, AP j would avoid using this channel. Meanwhile, AP k would benefit from AP j's detection and hence AP k also avoids using the same channel as AP j. The probability $\mathrm{D}(t)$ of at least one AP detecting the channel usage of the other AP after measuring the interference power at time t is:

$$\mathrm{D}(t)=\left(1-\mathrm{D}_{k,j}(t)\right)\mathrm{D}_{j,k}(t)+\left(1-\mathrm{D}_{j,k}(t)\right)\mathrm{D}_{k,j}(t) \tag{8}$$

Hence, the probability $\mathrm{P_I}(t)$ for LI namely $\mathrm{P_{I\_LI}}(t)$) is:

$$\mathrm{P_{I\_LI}}(t)=\frac{\left(1-\mathrm{D}(t)\right)}{C}+\frac{\mathrm{D}(t)}{C^2} \tag{9}$$

For constant $\mathrm{T}_j(t)$ for all $j \in \mathbf{A}$ (**A** is set of all APs in the network), the probability $\mathrm{P}_X(t)$ of X or less APs using the same channel as AP j is:

$$\mathrm{P}_X(t) = \sum_{k=1}^{X} \mathrm{Q}_k(t) \tag{10}$$

Where $\mathrm{Q}_X(t)$ is the probability of exactly X APs out of A APs (total number of APs in **A** or within the interference region) using the same channel and is given as:

$$\mathrm{Q}_X(t) = \frac{A!}{X!(A-X)!}\mathrm{P}_{\mathrm{I\_LI}}(t)^X \left(1-\mathrm{P}_{\mathrm{I\_LI}}(t)\right)^{A-X} \tag{11}$$

In Fig. 5 it can be seen that the probability of interference saturates at $\mathrm{T}_j(t)/M = 2$. Hence the AP using the LI method would choose a strategy F in **S** such that $\mathrm{T}_j(t)/M = 2$. The use of a higher $\mathrm{T}_j(t)/M$ ratio would cause the system to react slowly to interference changes.

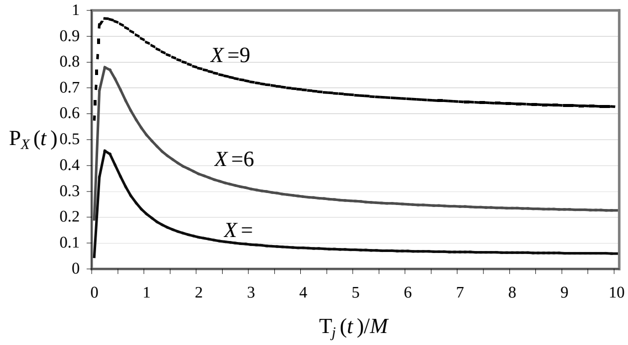

Fig. 5. Plot of $\mathrm{P}_X(t)$ against $\mathrm{T}_j(t)/M$ for A=136 where $\mathrm{T}_j(t) = \mathrm{T}_k(t)$.

C. *DCA using Game Theory (GT)*

GT is similar to LI in that the AP will scan all available channels at the start of a MAC frame and select the channel with the lowest interference power. The selected channel is used for F MAC frames such that $\mathrm{T}_j(t)$ has a value of T_{XL} nmsec with probability p and a value T_{XT} nmsec with probability (1-p) where $T_{XT} > T_{XL}$. Since GT uses the same method as LI for channel selection, the probability $\mathrm{P}_\mathrm{I}(t)$ for GT namely $\mathrm{P}_{\mathrm{I\_GT}}(t)$ is the same as $\mathrm{P}_{\mathrm{I\_LI}}(t)$ as given in equation (9). The derivation for p, T_{XL} and T_{XT} are described in this section.

Fig. 6 is a plot of payoff functions $\pi_{j,k}(t)$ and $\pi_{k,j}(t)$ for AP j and AP k respectively where $\mathrm{T}_k(t)$ is constant. The payoff for both APs is the same (i.e. $\pi_{j,k}(t)$ and $\pi_{k,j}(t) = \pi^*(t)$) when $\mathrm{T}_j(t) = \mathrm{T}_k(t)$. Firstly, as shown in Fig. 6, $\pi_{j,k}(t)$ saturates as $\mathrm{T}_j(t)$ increases and will never be larger than $(C^2\text{-}1)/(2C^2)$ (i.e. when $\mathrm{T}_j(t) = \mathrm{T}_k(t) >> \gamma C$, which is also equivalent to LI with a large $\mathrm{T}_j(t)/M$ ratio). Secondly, there is a peak payoff $\pi_{j,k}(t)=\pi_{\mathrm{P}j}(t)$ for AP j, which has a value larger than $(C^2\text{-}1)/(2C^2)$. However, this peak payoff is reached at the expense of AP k's payoff (i.e. AP k has a lower payoff – smaller than $\pi^*(t)$). The peak payoff $\pi_{\mathrm{P}j}(t)$ for AP j can be found by optimising (1).

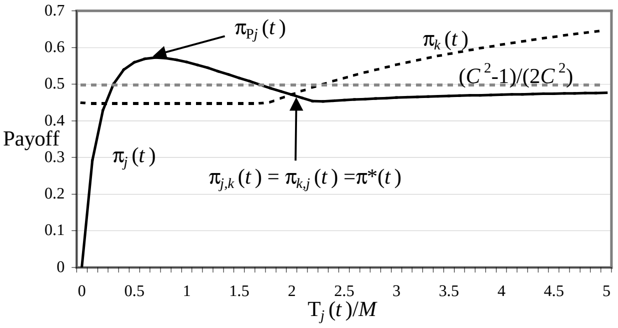

Fig. 6. Payoff function for constant $\mathrm{T}_k(t)/M = 2$.

For every fixed value of $\mathrm{T}_k(t)=T_{XT}$, there exists a value $\mathrm{T}_j(t)=T_{XL}$ such that $\pi_{j,k}(t)=\pi_{\mathrm{P}j}(t)$ while $\pi_{k,j}(t) < \pi^*(t) < \pi_{\mathrm{P}j}(t)$. However, in game theory, both APs would want to achieve a higher payoff and no one would want to stay at the point where its payoff is small. A mixed strategy [13] is introduced so that both APs take turns to reap the peak payoff. Consider only one peak payoff value (i.e. one set of T_{XL} and T_{XT}). Hence each AP can play two strategies s_1 and s_2, where in strategy s_1 an AP will select $F \in \mathbf{S}$ such that its $\mathrm{T}_j(t)$ (or $\mathrm{T}_k(t)$) is T_{XL} and in strategy s_2 an AP will select $F \in \mathbf{S}$ such that $\mathrm{T}_j(t)$ (or $\mathrm{T}_k(t)$) is T_{XT}. When an AP plays strategy s_1, it would spend more time measuring interference power and hence it is *exploring* different channels. While in s_2, an AP would spend more time *exploiting* the channel that it has selected. The extensive form of the game (the possible payoff for each AP) is shown in Fig. 7.

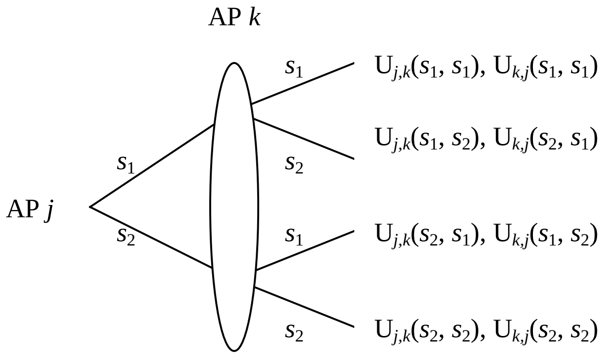

Fig. 7. Extensive form of the game (all possible payoff outcomes).

The function $\mathrm{U}_{j,k}(x, y)$ is the payoff for AP j when AP j plays strategy x and AP k plays strategy y ($x, y \in \{s_1, s_2\}$). Similarly the function $\mathrm{U}_{k,j}(x, y)$ is the payoff for AP k when AP k plays strategy x and AP j plays strategy y. The oval in Fig. 7 means AP j and AP k play their strategies simultaneously and independently.

In a mixed strategy, an AP plays strategy s_1 with probability p and plays strategy s_2 with probability (1-p). If both APs follow this rule, the payoff π_{MIX} (larger than $(C^2\text{-}1)/(2C^2)$) obtained by both APs playing the mixed strategy is thus:

$$\pi_{MIX} = p^2\,\mathrm{U}(s_1,s_1) + p(1-p)\,\mathrm{U}(s_1,s_2) + (1-p)p\,\mathrm{U}(s_2,s_1) + (1-p)^2\,\mathrm{U}(s_2,s_2) \tag{12}$$

Where, $\mathrm{U}(s_1, s_1) = \mathrm{U}_{j,k}(s_1, s_1) = \mathrm{U}_{k,j}(s_1, s_1)$, $\mathrm{U}(s_2, s_2) = \mathrm{U}_{j,k}(s_2, s_2) = \mathrm{U}_{k,j}(s_2, s_2)$, $\mathrm{U}(s_1, s_2) = \mathrm{U}_{j,k}(s_1, s_2) = \mathrm{U}_{k,j}(s_1, s_2)$ and $\mathrm{U}(s_2, s_1) = \mathrm{U}_{j,k}(s_2, s_1) = \mathrm{U}_{k,j}(s_2, s_1)$. Optimising (12) to find the probability p that maximises π_{MIX} leads to:

$$p = \frac{2\,\mathrm{U}(s_2,s_2) - \mathrm{U}(s_1,s_2) - \mathrm{U}(s_2,s_1)}{2\left(\mathrm{U}(s_1,s_1) + \mathrm{U}(s_2,s_2) - \mathrm{U}(s_1,s_2) - \mathrm{U}(s_2,s_1)\right)} \tag{13}$$

IV. SIMULATION AND RESULTS

The three DCA methods (RND, LI and GT) are simulated using OPNET Modeler. A scenario with 37 cells is used with the layout as shown in Fig. 8, where each cell has a radius of 0.5 km. The simulation has 136 APs where each cell has from 2 to 12 APs giving a non-uniform traffic distribution. Boundary effects are reduced using this layout and measurements taken from the indexed cells shown shaded in Fig. 8. A total of 669 SUs are distributed randomly in the layout.

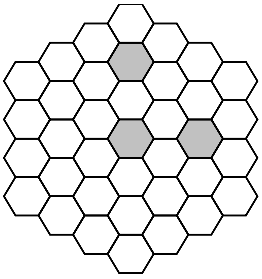

Fig. 8. Simulation layout

An ON-OFF model using a Pareto distribution is used to generate self-similar traffic typical of a packet data networks in both the AP and SU [14]. Pareto's probability distribution function is given by,

$$p(t \mid t > \beta) = \frac{\alpha\beta^{\alpha}}{t^{\alpha+1}} \tag{14}$$

With mean,

$$E[t] = \frac{\alpha\beta}{\alpha - 1} \tag{15}$$

Where β is the minimum OFF (or ON) period and $1 < \alpha < 2$. The values for α are 1.7 for ON-periods and 1.2 for the OFF-periods [14]. The value of β depends upon the data rate and average file size, which is assumed to be 13.9 kbytes for web browsing applications [10].

The radio propagation is assumed to follow the Random Height path loss model [15], which has a path-loss exponent of 2 for distances up to 1km and an exponent of 3.8 thereafter. The lognormal shadow standard deviation is 3.5 dB. Only co-channel interference and thermal noise are assumed in the simulation. The number of available channels is 15 each having a bandwidth of 15 MHz operating in the 5GHz U-NII band.

Fig. 9 shows the cumulative distribution function for received SNR for all the indexed cells for the uplink direction (a similar performance is achieved in the downlink direction). GT has the best SNR performance while RND has the worst SNR performance. Although GT uses the same channel selection as LI, it has a better SNR performance by improving the detection probability.

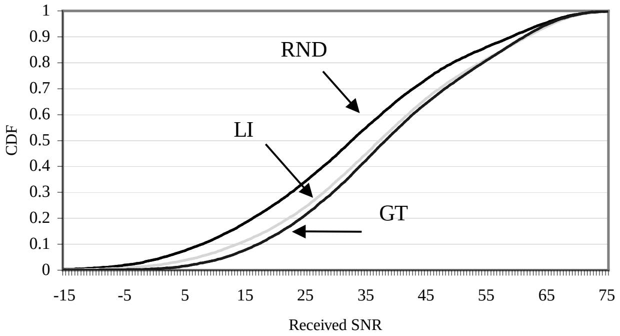

Fig. 9. Received SNR performance in uplink.

The average 1-percentile SNR (i.e. 1% of measured SNR values are below the tabulated value) for all the indexed cells in the uplink and downlink directions is shown in TABLE I, where GT is seen to have a 4.5 dB gain over LI and a 8.2 dB gain over RND.

TABLE I
1-PERCENTILE AVERAGE RECEIVED SNR

DCA	RND	LI	GT
1-percentile SNR	-6.8 dB	-3.1 dB	1.4 dB

We define $P_{>21dB}$ as the probability of a packet being received with a SNR above 21 dB (i.e. the SNR above which a packet is considered to have been received successfully). The overall throughput T_O is the average $T_j(t)$ per nmsec for the APs in the shaded cells in Fig. 8. The average payoff π_{AVG} is thus the number of successful packets transmitted or received by an average AP per nmsec. TABLE II lists $P_{>21dB}$, T_O and π_{AVG} for the 3 DCA schemes considered.

TABLE II
PAYOFF FOR AN AVERAGE AP

DCA	$P_{>21dB}$	T_O	π_{AVG}
RND	0.729	1.0	0.729
LI	0.836	0.931	0.778
GT	0.857	0.982	0.841

RND falls into the Distributed Non-measurement quadrant of the Channel Allocation Matrix and hence has the highest throughput T_O since no scanning is required to select a channel. However, due to the poor SNR performance RND has the lowest overall payoff, while GT has the highest overall payoff owing to reduced interference.

TABLE III shows the total number of data packets (ATM cells sent by the SUs) received per nmsec per AP and also the number of packets received successfully per nmsec (where a data packet is considered to have been received successfully if its SNR is above 21 dB). Once again, GT has the highest data packet throughput compared with the other two methods.

TABLE III
NUMBER OF SUCCESSFUL DATA PACKETS RECEIVED PER NMSEC PER AP

DCA	$P_{>21dB}$ (Uplink)	Packets Received	Successful Packets
RND	0.729	0.658	0.480
LI	0.815	0.595	0.485
GT	0.852	0.610	0.520

V. CONCLUSIONS

Various channel allocation methods are classified in the Channel Allocation Matrix. A payoff function used in Game Theory is applied to a BFWA network. DCA using RND and LI – originally used in voice network – are implemented and analysed for a data network. A DCA method derived from the LI method and applying a mixed strategy borrowed from Game Theory (GT) is proposed and is compared with the LI and RND methods in a simulation of a BFWA network loaded with typical Internet traffic. It is shown that the GT method achieves a SNR gain (first-percentile) of 4.5 dB and 8.2 dB respectively compared with the LI and RND. GT also gives the highest data packet throughput and payoff compared to the other two. LI has a better SNR performance than RND but due to the scanning overhead has an overall packet throughput similar to that of RND.

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