

Pseudo-random Puncturing: A Technique to Lower the Error Floor of Turbo Codes

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Abstract—It has been observed that particular rate-1/2 partially systematic parallel concatenated convolutional codes (PCCCs) can achieve a lower error floor than that of their rate-1/3 parent codes. Nevertheless, good puncturing patterns can only be identified by means of an exhaustive search, whilst convergence towards low bit error probabilities can be problematic when the systematic output of a rate-1/2 partially systematic PCCC is heavily punctured. In this paper, we present and study a family of rate-1/2 partially systematic PCCCs, which we call pseudo-randomly punctured codes. We evaluate their bit error rate performance and we show that they always yield a lower error floor than that of their rate-1/3 parent codes. Furthermore, we compare analytic results to simulations and we demonstrate that their performance converges towards the error floor region, owing to the moderate puncturing of their systematic output. Consequently, we propose pseudo-random puncturing as a means of improving the bandwidth efficiency of a PCCC and simultaneously lowering its error floor.

I. INTRODUCTION

Although in certain applications, such as satellite communications, link reliability is of essence and low rate codes are used to support it, bandwidth occupancy is more important in wireless communications and hence high rate codes are preferred. A high rate convolutional code can be obtained by periodic elimination, known as puncturing, of particular codeword bits from the output of a parent low rate convolutional encoder. Extensive analyses on punctured convolutional codes have shown that their performance is always inferior to the performance of their low rate parent codes (e.g. see [1], [2]).

The performance of punctured parallel concatenated convolutional codes (PCCCs), also known as punctured turbo codes, has also been investigated. Design considerations have been derived by analytical [3]–[5] as well as simulation-based approaches [6]–[8], while upper bounds on the bit error probability (BEP) were evaluated in [5], [9]. Punctured turbo codes are usually classified as systematic, partially systematic or non-systematic depending on whether all, some or none of their systematic bits are transmitted [7]. Recent papers [7]–[9] have demonstrated that partially systematic PCCCs yield lower error floors than systematic PCCCs of the same rate.

In [10] we showed that rate-1/2 non-systematic PCCCs can achieve error floors, which are lower even than those of their rate-1/3 parent PCCCs. This interesting outcome is valid when maximum-likelihood (ML) decoding is employed.

When suboptimal iterative decoding is used, the absence of received systematic bits causes erroneous decisions, which prohibit the iterative decoder from converging to the error floor. Nevertheless, we demonstrated that rate-1/2 child codes, whose BEP performance converges towards an error floor which is lower than that of their rate-1/3 parent PCCC, can still be found by means of an exhaustive search. During this process, the union bound on the BEP of each rate-1/2 punctured PCCC is computed and compared to the union bound of the rate-1/3 parent PCCC. Note that the union bound coincides with the error floor of the code for high values of E_b/N_0 [11]. Punctured PCCCs that achieve a bound lower than that of their rate-1/3 parent PCCC are selected.

Computation of the exact union bound on the BEP of a punctured PCCC becomes intensive as the interleaver size increases. In [12] we presented a simple technique to approximate the union bound of a turbo code and we demonstrated that this approximation is very accurate when a large interleaver size is used. We used our technique to identify a family of rate-1/2 partially systematic PCCCs, which we called pseudo-randomly punctured PCCCs (PRP-PCCCs). Although we did not explore their BEP performance in detail, we observed that particular PRP-PCCC configurations could achieve a lower error floor than that of their parent codes.

This paper builds upon the work carried out in [10] and [12]. Initially, we provide analytical expressions for the parameters that influence the bit error performance of PCCCs. We then evaluate those parameters and compute the union bound approximations for both rate-1/3 parent PCCCs and rate-1/2 PRP-PCCCs. We demonstrate that the latter always exhibit a lower error floor than the former, when large interleaver sizes are considered. In order to verify our theoretical analysis, we compare analytic results to simulations for specific PCCC configurations. The paper concludes with a summary of the main contributions.

II. PERFORMANCE EVALUATION OF PCCCs

Turbo codes, in the form of symmetric rate-1/3 PCCCs, consist of two identical rate-1/2 recursive systematic convolutional encoders separated by an interleaver of size N [13]. The information bits are input to the first constituent convolutional encoder, while an interleaved version of the information bits are input to the second convolutional encoder.

The output of the turbo encoder consists of the systematic bits of the first encoder, which are identical to the information bits, the parity check bits of the first encoder and the parity check bits of the second encoder.

The bit error probability P_b of a PCCC employing ML soft decoding, on an additive white Gaussian noise (AWGN) channel, is upper bounded as follows

$$P_b \leq P_b^u \quad (1)$$

where the union bound P_b^u is defined as

$$P_b^u = \sum_w P(w). \quad (2)$$

Here, the sum runs over all possible values of input information weight w , with $P(w)$ being the contribution to the union bound P_b^u of only those codeword sequences which were generated by input sequences of a specific information weight w . An individual contribution $P(w)$ is given by [11], [14]

$$P(w) = \sum_d \frac{w}{N} B_{w,d} Q \left(\sqrt{\frac{2R \cdot E_b}{N_0} \cdot d} \right), \quad (3)$$

where N is the interleaver size, R is the code rate of the turbo encoder and $B_{w,d}$ denotes the number of codeword sequences having overall output weight d , which were generated by input information sequences of weight w .

In [11] it was shown that the union bound on the BEP of a PCCC using a uniform interleaver of size N coincides with the average of the union bounds obtainable from the whole class of deterministic interleavers of size N . For small values of N , the union bound can be very loose compared with the actual performance of turbo codes using specific deterministic interleavers. However, for $N \geq 1000$, it has been observed that randomly generated interleavers generally perform better than deterministic interleaver designs [15]. Consequently, the union bound provides a good indication of the actual bit error rate performance of a PCCC operating in the error floor region, when long interleavers are considered.

Derivation of all coefficients $B_{w,d}$ becomes a computationally intensive process as the interleaver size increases, especially when punctured PCCCs are considered [12]. However, the union bound can be approximated as follows

$$P_b^u \approx P(w=2), \quad (4)$$

when long interleavers are used. This approximation is based on a number of observations:

- 1) Codeword sequences, which were generated by input sequences having the minimum possible information weight, become the main contributors to the bit error rate performance, as the size N of the interleaver increases [12], [16].
- 2) Owing to the structure of the constituent encoders, the minimum information weight of an input sequence is always equal to two [16].

Therefore, $P(w=2)$ is the dominant contribution to the union bound over a broad range of bit error probabilities [12], [16] and can be used to predict the error floor of turbo codes.

Throughout this paper, we use the union bound approximation as the basis for the analytic performance comparison of turbo codes. In particular, if \mathcal{P} and \mathcal{P}' are two PCCCs using long interleavers of identical size, we say that \mathcal{P} yields a lower error floor than that of \mathcal{P}' when their bound approximations, $P^{\mathcal{P}}(2)$ and $P^{\mathcal{P}'}(2)$ respectively, satisfy

$$P^{\mathcal{P}}(2) < P^{\mathcal{P}'}(2). \quad (5)$$

The above condition can be expanded using (3) as follows

$$\sum_d B_{2,d}^{\mathcal{P}} Q \left(\sqrt{\frac{2R^{\mathcal{P}} E_b}{N_0} d} \right) < \sum_d B_{2,d}^{\mathcal{P}'} Q \left(\sqrt{\frac{2R^{\mathcal{P}'} E_b}{N_0} d} \right). \quad (6)$$

It was demonstrated in [16] that the free effective distance, d_f , which conveys the minimum weight of a codeword sequence for a weight-2 input information sequence, has a major impact on the performance of a turbo code. Consequently, if $d_f^{\mathcal{P}}$ and $d_f^{\mathcal{P}'}$ denote the free effective distances of \mathcal{P} and \mathcal{P}' respectively, condition (6) collapses to

$$B_{2,d_f}^{\mathcal{P}} Q \left(\sqrt{\frac{2R^{\mathcal{P}} E_b}{N_0} d_f^{\mathcal{P}}} \right) < B_{2,d_f}^{\mathcal{P}'} Q \left(\sqrt{\frac{2R^{\mathcal{P}'} E_b}{N_0} d_f^{\mathcal{P}'}} \right), \quad (7)$$

which only considers the first non-zero, that is the most significant, term of each sum.

Function $Q(\xi)$ is a monotonically decreasing function of ξ , where ξ is a real number. Therefore, if ξ_1 and ξ_2 are real numbers, with $\xi_1 > \xi_2$, we deduce that $Q(\xi_1) < Q(\xi_2)$, and vice versa, i.e.,

$$Q(\xi_1) < Q(\xi_2) \Leftrightarrow \xi_1 > \xi_2. \quad (8)$$

Consequently, inequality (7) reduces to

$$R^{\mathcal{P}} d_f^{\mathcal{P}} > R^{\mathcal{P}'} d_f^{\mathcal{P}'}, \quad (9)$$

if

$$B_{2,d_f}^{\mathcal{P}} \leq B_{2,d_f}^{\mathcal{P}'}. \quad (10)$$

When the code rates are equal, the free effective distance of turbo codes plays a role similar to that of the free distance of convolutional codes, since the performance criterion (9) is simplified to

$$d_f^{\mathcal{P}} > d_f^{\mathcal{P}'}. \quad (11)$$

Expressions (9) and (10) will be the basis for the comparison of the BEP performance in the error floor region of two PCCCs.

III. DETERMINATION OF PARAMETERS THAT INFLUENCE THE PERFORMANCE OF TURBO CODES

We will now determine the various parameters that affect performance for two classes of turbo codes: conventional rate-1/3 PCCCs and pseudo-randomly punctured rate-1/2 PCCCs. The turbo codes considered throughout this paper are symmetric, i.e., the two constituent encoders are identical.

A. Rate-1/3 PCCCs

Criteria (9) and (10) require knowledge of the free effective distance d_f and the coefficient B_{2,d_f} of each PCCC. In the remainder of the paper, we use the abbreviation ‘‘Par’’ to denote a rate-1/3 parent PCCC. Its free effective distance d_f^{Par} can be expressed as the sum of the minimum weight d_{\min} of the codeword sequence generated by the first constituent encoder, and the minimum weight z_{\min} of the parity check sequence generated by the second constituent encoder, when a sequence of information weight $w=2$ in input to the PCCC

$$d_f^{\text{Par}} = d_{\min} + z_{\min}. \quad (12)$$

Taking into account that the turbo codes are symmetric and the weight u_{\min} of the systematic output sequence is always 2 since $w=2$, we can write

$$d_f^{\text{Par}} = (u_{\min} + z_{\min}) + z_{\min} = 2 + 2z_{\min}. \quad (13)$$

The number B_{2,d_f}^{Par} of codeword sequences, generated by a turbo encoder using a uniform interleaver of size N , can be associated with the number $B_{2,d_{\min}}$ of codeword sequences having weight d_{\min} , generated by the first constituent encoder, and the number $B_{2,z_{\min}}$ of parity check sequences having weight z_{\min} , generated by the second constituent encoder, if we elaborate on the expressions described in [11]. In particular, we obtain

$$B_{2,d_f}^{\text{Par}} = \frac{B_{2,d_{\min}} \cdot B_{2,z_{\min}}}{\binom{N}{2}}, \quad (14)$$

where $B_{2,d_{\min}}$ and $B_{2,z_{\min}}$ return the same value, since they both consider the same trellis paths. Note that the first index in the above notations refers to the input information weight, which is two.

It was shown in [16] that good rate-1/3 PCCCs are obtained when their feedback generator polynomial G_R is chosen to be primitive, whilst their feedforward generator polynomial G_F is different than G_R . The period L of a primitive polynomial is given by [17]

$$L = 2^\nu - 1, \quad (15)$$

where ν is the order of the polynomial, or equivalently, the memory size of each constituent code.

We demonstrated in [12] that when a primitive feedback generator polynomial is used, the minimum weight z_{\min} and the coefficient $B_{2,2,z_{\min}}$ can be expressed as

$$\begin{aligned} z_{\min} &= 2^{\nu-1} + 2, \\ B_{2,d_{\min}} &= B_{2,z_{\min}} = N - L, \end{aligned} \quad (16)$$

respectively. Consequently, expression (13) assumes the form

$$d_f^{\text{Par}} = 6 + 2^\nu, \quad (17)$$

whilst, if we combine (14) and (16), the coefficient B_{2,d_f}^{Par} can be expressed as a function of the interleaver size N and the period L , as follows

$$B_{2,d_f}^{\text{Par}} = \frac{2(N-L)^2}{N(N-1)}. \quad (18)$$

In the special case when the size N of the interleaver is an integer multiple of the period L of the feedback generator polynomial, i.e., $N = \mu L$, we can rewrite (18) as

$$B_{2,d_f}^{\text{Par}} = \frac{2L(\mu-1)^2}{\mu(\mu L-1)}. \quad (19)$$

B. Rate-1/2 Pseudo-randomly Punctured PCCCs

A high rate PCCC can be obtained by periodic elimination of specific codeword bits from the output of a rate-1/3 parent PCCC. A puncturing pattern \mathbf{P} can be represented by a $3 \times M$ matrix as follows:

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & p_{1,2} & \dots & p_{1,M} \\ p_{2,1} & p_{2,2} & \dots & p_{2,M} \\ p_{3,1} & p_{3,2} & \dots & p_{3,M} \end{bmatrix}, \quad (20)$$

where M is the puncturing period and $p_{i,m} \in \{0,1\}$, with $i = 1, 2, 3$ and $m = 1, \dots, M$. For $p_{i,m} = 0$ the corresponding output bit is punctured, otherwise it is transmitted. The first and second rows of the pattern are used to puncture the systematic and parity check outputs, respectively, of the first constituent encoder. The third row determines which parity check bits from the output of the second constituent encoder will be punctured.

Pseudo-random puncturing has been described in [12], in detail. It is applied to rate-1/3 PCCCs, which use primitive feedback generator polynomials, hence the polynomial period L is also given by (15). The puncturing pattern can be constructed once the parity check sequence $\mathbf{y} = (y_0, y_1, \dots, y_L)$ for an input sequence $\mathbf{x} = (1, 0, \dots, 0)$ of length $L+1$, has been obtained at the output of the first constituent encoder. As long as a trail of zeros follows the first non-zero input bit, the component encoder behaves like a pseudo-random generator, hence the parity check bits from y_1 to y_L form a pseudo-random sequence. We set the elements of the second row of the puncturing pattern to be equal to the bits of this pseudo-random sequence, but circularly shifted rightwards by one, i.e., $p_{2,m+1} = y_m$ for $m = 1, \dots, L$. Note that in pseudo-random puncturing, the puncturing period M is equal to the period L of the feedback polynomial, i.e., $M = L$. The first row of the pattern is set to be the complement of the second row, thus $p_{1,m} = 1 - p_{2,m}$. In order to achieve a code rate of 1/2, we do not puncture the parity check output of the second constituent encoder, hence all the elements of the third row are set to one, i.e., $p_{3,m} = 1$.

As an example, let us consider a rate-1/3 PCCC with generator polynomials $(G_F, G_R) = (5, 7)_8$ in octal form. The memory size of each constituent encoder is $\nu = 2$, thus the period of G_R is found to be $L = 2^2 - 1 = 3$. Consequently, we set the input sequence to $(1, 0, 0, 0)$ and we obtain the parity check sequence $(1, 1, 1, 0)$ at the output of the first constituent encoder. The block of the last $L=3$ parity check bits, i.e., $(1, 1, 0)$, forms a pseudo-random sequence. If we circularly shift the bits of this pseudo-random sequence to the right by one and map them to the elements of the second row of the puncturing pattern, we obtain $[0 \ 1 \ 1]$. Eventually the

puncturing pattern, based on which the rate-1/2 PRP-PCCC is generated from the rate-1/3 parent PCCC, assumes the form

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (21)$$

We emphasize that the puncturing pattern depends on the generator polynomials of the rate-1/3 parent PCCC, hence different polynomials yield different puncturing patterns. Furthermore, a rate-1/2 PRP-PCCC can be obtained only if the parent PCCC uses primitive feedback generator polynomials.

We have previously determined [12] the minimum weight d'_{\min} of the codeword sequence generated by the first constituent encoder, when a sequence of information weight $w = 2$ in input to the rate-1/2 PRP-PCCC. In particular, we found that

$$d'_{\min} = 2^{\nu-2} + 2. \quad (22)$$

The parity check sequence generated by the second constituent encoder is not punctured, thus its minimum weight is also given by (16). Therefore, we can compute the free effective distance d_f^{PRP} of a rate-1/2 PRP-PCCC as follows

$$\begin{aligned} d_f^{\text{PRP}} &= d'_{\min} + z_{\min} \\ &= (2^{\nu-2} + 2) + 2^{\nu-1} + 2 \\ &= 4 + 3(2^{\nu-2}). \end{aligned} \quad (23)$$

Every time a particular column m of the puncturing pattern is active during the N time steps of the coding process, codeword sequences having minimum weight d'_{\min} are generated. Their exact number, A_m , can be computed using the expressions in [12]. In particular, we find that for $M = L$ the number of minimum-weight codeword sequences A_m , generated when column m is active, is given by

$$A_m = \begin{cases} \lfloor N/M \rfloor - 1, & \text{if } (N \bmod M) < m \\ \lfloor N/M \rfloor, & \text{otherwise,} \end{cases} \quad (24)$$

where $(\xi_1 \bmod \xi_2)$ denotes the remainder of division of ξ_1 by ξ_2 , and $\lfloor \xi \rfloor$ denotes the integer part of ξ . In order to facilitate our analysis, we assume that the interleaver size N is an integer multiple of the puncturing period M , i.e., $N = \mu M$, where μ is a positive integer. Hence, (24) collapses to

$$A_m = \mu - 1, \quad (25)$$

since $(N \bmod M)$ is always zero and $m > 0$.

It has been demonstrated in [12] that minimum-weight codeword sequences can be obtained only when the active column m is in the range $2 \leq m \leq M$; every time one of these $M - 1$ columns of the puncturing pattern is active, A_m minimum-weight codeword sequences are generated. Consequently, the total number of codeword sequences having weight d'_{\min} assumes the value

$$B_{2,d'_{\min}} = (M - 1)A_m, \quad (26)$$

or, equivalently

$$B_{2,d'_{\min}} = (L - 1)(\mu - 1), \quad (27)$$

where M has been replaced by L , since they are equal quantities and they can be used interchangeably.

Similarly to the second constituent encoder of the rate-1/3 parent PCCC, the second constituent encoder of the rate-1/2 PRP-PCCC also generates a total of $B_{2,z_{\min}}$ sequences having weight z_{\min} , since its parity check output is not punctured. Consequently, the coefficient B_{2,d_f}^{PRP} of a rate-1/2 PRP-PCCC can be expressed as

$$\begin{aligned} B_{2,d_f}^{\text{PRP}} &= \frac{B_{2,d'_{\min}} \cdot B_{2,z_{\min}}}{\binom{N}{2}} \\ &= \frac{[(L - 1)(\mu - 1)] \cdot (N - L)}{\binom{N}{2}} \\ &= \frac{2(L - 1)(\mu - 1)^2}{\mu(\mu L - 1)}, \end{aligned} \quad (28)$$

invoking (14), which can be used when PCCCs employing uniform interleavers of size N are considered.

IV. PERFORMANCE COMPARISON OF ANALYTIC TO SIMULATION RESULTS

Having evaluated the parameters that influence the performance of the PCCCs under investigation, we are now in the position to explore whether a rate-1/2 PRP-PCCC exhibits a lower bound approximation than that of its rate-1/3 parent PCCC. We observe that d_f^{PRP} can be expressed in terms of d_f^{Par} , if we subtract (17) from (23)

$$d_f^{\text{PRP}} = d_f^{\text{Par}} - (2 + 2^{\nu-2}). \quad (29)$$

Coefficient B_{2,d_f}^{PRP} can also be represented in terms of B_{2,d_f}^{Par} , if we divide (28) by (19)

$$B_{2,d_f}^{\text{PRP}} = \left(\frac{L - 1}{L} \right) B_{2,d_f}^{\text{Par}}. \quad (30)$$

According to (9) and (10), if both conditions

$$\frac{1}{2} d_f^{\text{PRP}} > \frac{1}{3} d_f^{\text{Par}} \quad (31)$$

and

$$B_{2,d_f}^{\text{PRP}} < B_{2,d_f}^{\text{Par}} \quad (32)$$

are satisfied, a rate-1/2 PRP-PCCC yields a lower bound approximation than that of its rate-1/3 parent code. We deduce from (30) that B_{2,d_f}^{PRP} is always less than B_{2,d_f}^{Par} , thus the second condition holds true. The first condition assumes the following form, if we substitute d_f^{PRP} with its equivalent, based on (29),

$$d_f^{\text{Par}} > 6 + 3(2^{\nu-2}). \quad (33)$$

Nevertheless, we have shown in (17) that the free effective distance of the parent PCCC is given by $d_f^{\text{Par}} = 6 + 2^{\nu}$, which can be rewritten as $d_f^{\text{Par}} = 6 + 4(2^{\nu-2})$. Therefore, d_f^{Par} is always greater than $6 + 3(2^{\nu-2})$, and hence, both conditions are satisfied.

The outcome of this investigation reveals that rate-1/2 PRP-PCCCs using long interleavers are always expected to

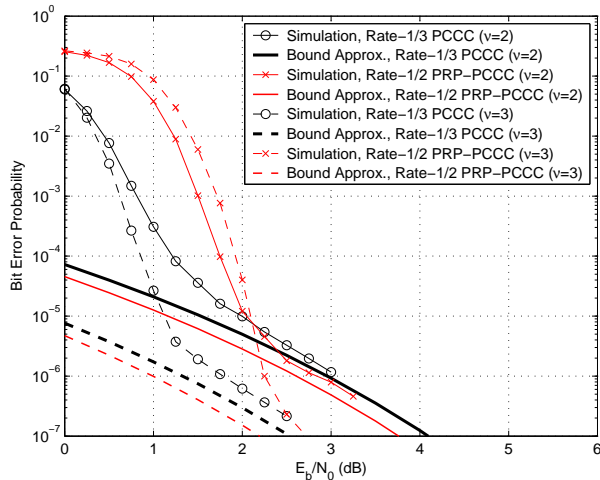


Fig. 1. Comparison of bound approximations to simulation results. The exact log-MAP algorithm is applied over 8 iterations and an interleaver size of 1,000 bits is used.

yield a lower bound approximation, or equivalently a lower error floor, than that of their rate-1/3 parent codes.

Fig.1 compares bound approximations to simulation results for rate-1/3 parent PCCCs and rate-1/2 PRP-PCCCs of memory size $\nu = 2$ and $\nu = 3$, over the AWGN channel. For $\nu = 2$, the generator polynomials of the PCCCs are taken to be $(G_F, G_R) = (5, 7)_8$, whilst for $\nu = 3$, the PCCCs are described by $(G_F, G_R) = (17, 15)_8$. The component decoders employ the conventional exact log-MAP algorithm [18]. A moderate interleaver size of 1,000 bits has been chosen, so as to allow the bit error rate performance of the PCCCs to approach the corresponding bound approximations at BEPs in the region of 10^{-6} to 10^{-7} .

As expected, Fig.1 confirms that for high values of E_b/N_0 , the BEP of each rate-1/2 PRP-PCCC is indeed lower than that of the corresponding rate-1/3 parent code, whilst after 8 iterations the performance curves of all turbo codes approach the respective bound approximation curves.

V. CONCLUSION

In previous work [9], [10], [12] we introduced techniques to evaluate the performance of punctured PCCCs and we observed that, in some cases, the error floor could be lowered by reducing the rate of a PCCC from 1/3 to 1/2. Nevertheless, good puncturing patterns were identified by means of an exhaustive search, whilst convergence towards low bit error probabilities of those rate-1/2 PCCCs whose systematic output was heavily punctured, had to be investigated.

In this paper, we established that rate-1/2 pseudo-randomly punctured PCCCs, which form a subset of rate-1/2 partially systematic PCCCs, not only approach the error floor region for an increasing number of iterations but always yield a lower error floor than that of their rate-1/3 parent codes. Consequently, pseudo-random puncturing can be used to reduce the rate of a PCCC from 1/3 to 1/2 and at the same time achieve a coding gain at low bit error probabilities.

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