Transmit Antenna Selection in Linear Receivers: a Geometrical Approach

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Abstract: We consider transmit antenna subset selection in spatial multiplexing systems. In particular, we propose selection algorithms aiming to minimize the error rate when linear detectors are used at the receiver. Previous work on antenna selection has considered capacity and post-processing SNR selection criteria. However, in this work we consider a geometrical interpretation of the decoding process which also permits us to develop a suboptimal algorithm that yields a considerable complexity reduction with only a small loss in performance.

Introduction: Wireless systems employing multiple antennas at the transmitter and at the receiver (MIMO systems) are a solution to increase the capacity of the wireless channel [1]. One major concern in the implementation of these systems is the high cost due to the price of the RF chains (analog-digital converters, low noise amplifiers, downconverters, etc.) attached to each antenna. A technique to reduce the cost of the MIMO system while maintaining part of the high performance is the use of antenna selection. The idea behind antenna selection is to employ a large number of inexpensive antenna elements and to use only the *best* subset. Then, only a limited number of the more expensive RF chains is necessary. The question that arises is how to select the best antenna subset among all the available antennas. An intuitive approach is to select the antenna subset that maximizes the mathematical expression of the mutual information between the transmitter and the receiver (see [2] and references therein). Using this approach, in [3] it is shown that the diversity obtained using antenna selection in spatial multiplexing systems is the same as the diversity achieved when the whole set of antennas is used. This result strongly motivates the use of antenna subset selection.

On the other hand, it is known that capacity results only give a bound on the performance of a system having infinite complexity. In practical systems, designers are interested in the error rate performance which will depend on the specific receiver. The antenna subset maximizing channel capacity does not necessarily minimize the error rate in a practical receiver and different optimization criteria should be tailored to the specific detection algorithm. In this letter, we will consider spatial multiplexing systems with linear receivers. In [4], selection criteria have been proposed which attempt to minimize the error rate when linear receivers are used. In that work, the signal-to-noise ratio prior to the slicing operation is considered as the objective function to be optimized. In this letter, we propose a selection metric based upon the geometrical interpretation of the decoding process in a linear receiver.

System Description: Consider the system shown in Figure 1 with n_T transmit and n_R receive RF chains. We assume that the receiver is equipped with equal number of antennas and RF chains whereas the transmitter is equipped with N_T antenna elements. Thus, the selection algorithm consists of selecting the best n_T transmit antennas out of the $\binom{N_T}{n_T}$ different combinations according to certain optimization criterion. The wireless channel is assumed to be quasi-static and flat fading and can be represented by a $(n_R \times N_T)$ matrix $\boldsymbol{H}$ whose element h_{ij} represents the complex gain of the channel between the j-th transmit antenna and the i-th receive antenna. Denote each of the transmit antenna subsets as $\omega_i = \{Ant_1, ..., Ant_{n_T}\}$ (e.g., $\omega = \{2, 3, 4\}$ indicates the selection of the second, third and fourth transmit antennas). Define the set of all $P = \binom{N_T}{n_T}$ antenna subsets as

$\Omega = \{\omega_1, ..., \omega_P\}$ and denote $\boldsymbol{H}_\omega$ as the $(n_R \times n_T)$ submatrix corresponding to the columns of $\boldsymbol{H}$ selected by ω. We assume that the channel state information is available at the receiver but not at the transmitter. Thus, the selection algorithms are implemented at the receiver and the antennas indices to be used are fedback to the transmitter assuming that there exists a low rate link between the receiver and the transmitter.

In spatial multiplexing systems, different data streams are transmitted from different antennas. Assume that $\boldsymbol{s} = [s_1, ..., s_{n_T}]^T$ is the transmitted symbol vector with $E\{s_i^* s_i\} = 1$. Then, the received signal when the transmit antenna subset selected is ω can be expressed as $\boldsymbol{y} = \sqrt{\frac{\rho}{n_T}}\boldsymbol{H}_\omega \boldsymbol{s} + \boldsymbol{n}$, where $\boldsymbol{y} = [y_1, ..., y_{n_R}]$ is the received signal vector, $\boldsymbol{n}$ is the received noise vector distributed as $\mathcal{N}_c(\underline{0}, \boldsymbol{I}_{n_R})$ and ρ is the total signal-to-noise ratio independent of the number of transmit antennas. In linear receivers, a spatial linear equalizer $\boldsymbol{G}_\omega$ is applied to recover the transmitted symbol vector. The equalizer can be optimized according to the ZF criterion, $\boldsymbol{G}_\omega = \sqrt{\frac{n_T}{\rho}}\boldsymbol{H}_\omega^\dagger$, where $\dagger$ denotes the pseudo-inverse, or the MMSE criterion, $\boldsymbol{G}_\omega = \sqrt{\frac{\rho}{n_T}}\boldsymbol{H}_\omega^H(\frac{\rho}{n_T}\boldsymbol{H}_\omega \boldsymbol{H}_\omega^H + \boldsymbol{I}_{n_R})^{-1}$. Since at high signal-to-noise ratio with antenna selection the MMSE solution tends to the ZF solution, we will focus on the ZF solution. As has been recently shown in [5], the decision regions in linear receivers consist of n_T-dimensional complex parallelepipeds formed by the column vectors of $\boldsymbol{H}_\omega$. Therefore, from a geometrical perspective, we propose a simple transmit antenna selection criterion consisting of selecting the columns of $\boldsymbol{H}$ such that the decision region minimizes the error rate. At a high signal-to-noise ratio, the error rate performance will be limited by the minimum error vector that makes a symbol fall out of the decision region. Denote $\boldsymbol{h}_{w,1}, ..., \boldsymbol{h}_{\omega,n_T}$ as the n_T columns of $\boldsymbol{H}$ selected by ω. Then, considering that the symbol is located in the center of the n_T-dimensional parallelepiped, the minimum length of a vector to make an error is

$$d_\omega = \min_{1 \le i \le n_T} \frac{1}{2} \|\pi^\perp(\boldsymbol{h}_{\omega,i})\|^2, \tag{1}$$

where $\pi^\perp(\boldsymbol{h}_{\omega,i})$ denotes the projection of $\boldsymbol{h}_{\omega,i}$ on $span(\{\boldsymbol{h}_{\omega,1}, ..., \boldsymbol{h}_{\omega,n_T}\} \backslash \boldsymbol{h}_{\omega,i})^\perp$ and $(\cdot)^\perp$ denotes the orthogonal complement. Then, the selection criterion becomes

$$\omega^* = \arg\max_{\omega \in \Omega} \left\{ \min_{1 \le i \le n_T} \frac{1}{2} \|\pi^\perp(\boldsymbol{h}_{\omega,i})\|^2 \right\}. \tag{2}$$

a) Low Complexity Algorithms: The selection process in (2) could be highly complex when the number of antenna combinations is large. One solution to reduce the complexity consists of employing sub-optimal incremental or decremental greedy algorithms similar to that proposed in [3] for the capacity case. In the decremental approach, we start considering the whole N_T columns and at every step, we remove the column that has the minimum projection onto the orthogonal complement of the span of the remaining $N_T - 1$ columns. The process is repeated with the remaining columns until only n_T columns are left. The inconvenience of this approach is that the system requires not only $n_R \ge n_T$ but $n_R \ge N_T$ which is not always true. In the incremental approach, we start by selecting one column that has the maximum 2-norm. Then, at every step of the algorithm, we add the column with the largest projection onto the orthogonal complement of the subspace spanned by the columns already selected. This approach greatly reduces the complexity in the situation where n_T is small in comparison to N_T. A very low complexity implementation of incremental selection is given in Algorithm 1. In the algorithm, $\mu_{p,j}$ denotes the Gram-Schmidt coefficient $\mu_{p,j} = \hat{\boldsymbol{h}}_p^H \boldsymbol{h}_j$ and Θ_i represents the subset of antennas selected up to the i-th step.

Simulations Results: In Figure 2 we show the performance of the antenna selection algorithms in a system with $n_R = 4$ receive antennas and $N_T = 8$ antennas where only

Algorithm 1 Reduced complexity incremental selection

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INPUT: all column vectors h_1,...,h_{N_T} in H
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$k_1 = \arg\max_{i \le N_T} \{\boldsymbol{h}_i^H \boldsymbol{h}_i\}$;

$\hat{\boldsymbol{h}}_1 = \boldsymbol{h}_{k_1} / \|\boldsymbol{h}_{k_i}\|$; $\Theta_1 = \{k_1\}$;

FOR $i = 2 : n_T$

  FOR EVERY $j \in \{\{1, ..., N_T\} \backslash \Theta_{i-1}\}$

    $\boldsymbol{b}_j = \boldsymbol{h}_j - \sum_{p=1}^{i-1} \mu_{p,j} \hat{\boldsymbol{h}}_p$;

  END FOR

  $k_i = \arg\max_j \{\boldsymbol{b}_j^H \boldsymbol{b}_j\}$;

  $\hat{\boldsymbol{h}}_i = \boldsymbol{b}_{k_i} / \|\boldsymbol{b}_{k_i}\|$; $\Theta_i = \{\Theta_{i-1}\} \cup \{k_i\}$;

END FOR

OUTPUT: selected antenna indices: Θ_{n_T}

$n_T = 4$ are actually used. We average the results over several channel realizations. In the same figure we also show the error rate of a system employing a selection criterion that maximizes the minimum eigenmode [4] and also the error rate of a system without antenna selection. It is seen that the geometrical approach obtains the best performance although its complexity is very high (although similar to the complexity of the eigenmode criterion). On the other hand, the much less complex incremental algorithm only shows a small loss of performance.

References

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Figure Captions

Figure 1: MIMO system with antenna selection at the transmitter.

Figure 2: Selection criteria comparison ($n_T = 4, n_R = 4$ and $N_T = 8$).

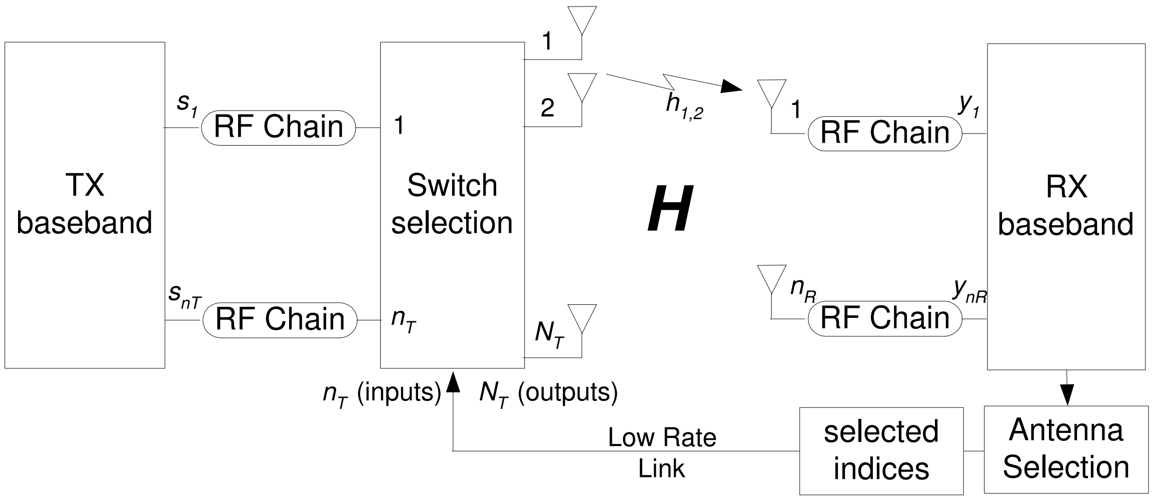
TX
baseband
s1
snT
RF Chain
RF Chain
1
nT
Switch
selection
1
2
NT
h1,2
H
1
nR
RF Chain
RF Chain
y1
ynR
RX
baseband
nT (inputs)
NT (outputs)
Low Rate
Link
selected
indices
Antenna
Selection

Figure 1:

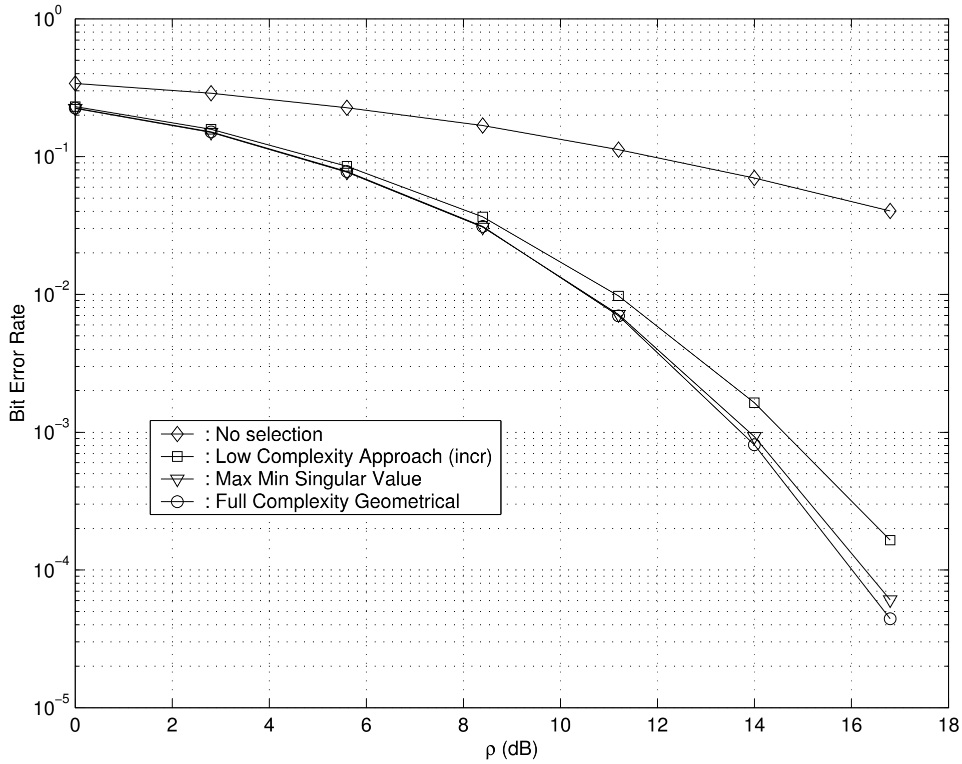
Bit Error Rate
ρ (dB)
: No selection
: Low Complexity Approach (incr)
: Max Min Singular Value
: Full Complexity Geometrical

Figure 2: