Multiuser cross-correlation channel estimation for SDMA/TDMA systems

E. K. Bartsch and I. J. Wassell
University of Cambridge
Department of Engineering
Laboratory for Communications Engineering
Trumpington Street
Cambridge, CB2 1PZ
United Kingdom
{ekb24, ijw24}@eng.cam.ac.uk

Abstract

A computationally efficient method for constructing zero correlation zone (ZCZ) sequences from optimal channel sounding sequences, which maximizes the length of the zero correlation zone (Z-zone) is proposed. The method is based on cyclic shifting of optimum sounding sequences. The proposed ZCZ sequences are specifically designed for cross-correlation channel estimation for single input multiple output (SIMO) communication systems. Our ZCZ sequence set permits the estimation of M space divison multiple access (SDMA) user channels using only one correlator loaded with the mothersequence. The mother sequence is defined as the first sequence in the set. Further we show how frame synchronization is achieved with the proposed ZCZ sequence set. The proposed method offers computationally efficient real-time cross-correlation channel estimation for SDMA/TDMA systems.

1. Introduction

Digital sequences with good autocorrelation properties (e.g. pseudo-random sequences) can be used for correlation-based channel estimation in time division multiple access (TDMA) systems with single user detection [1], where single user detection means one user is detected in every time slot. Although pseudo-random sequences give acceptable channel estimates, the accuracy of the estimate can be greatly increased by the use of optimum sounding sequences [2].

The performance of TDMA systems can be enhanced by using antenna arrays at the base station to allow multi-user detection at each time slot. This technique is known as space division multiple access (SDMA). Such a combined SDMA/TDMA system is the subject of study in this paper. Further we are interested in broadband fixed wireless access (BFWA). The delay spreads associated with such channels can be detrimental when high data rates are considered and the intersymbol interference (ISI) can span several symbol periods. We therefore study frequency selective fading environments and we assume time-invariant channels during each burst. Further we study SDMA in the up-link (i.e. receive diversity). This means our base station is equipped with multiple antennas and subscriber units with single antennas which transmit at the same time and at the same frequency.

Channel estimation for single-input-multiple-output (SIMO) SDMA systems has concentrated on least squares channel estimation [3]. Recently the authors found out about a method, which uses a similar approach to ours for CDMA/TDMA systems [4]. However, m-sequences were used for sequence construction, whereas in our case we employ optimum complex sounding sequences. Further we derive our technique based on the cross-correlation properties of such sequences. We show specifically how to extend the optimum sequences proposed in [2] to a sequence set of size M appropriate for estimating M SDMA user channels.

Accurate correlation-based channel estimation for SDMA/TDMA systems requires M optimum sounding sequences for M users, where M is the sequence set size. In addition to optimum periodic autocorrelation properties, the sequences in the set need to have a zero cross-correlation zone (Z-zone), which should be as long as possible. These sequences, known as ZCZ sequences, have recently attracted a lot of research interest in CDMA systems [5, 6, 7] and a generalised description of ZCZ sequences can be found in [8]. ZCZ sequence sets are superior to Walsh, Gold and Kasami sequence sets, whose correlation properties are good, but not optimal. By optimal it is meant that the periodic autocorrelation function (ACF) of the individual sequences should be zero everywhere except for zero time shift and the periodic cross-correlation function (CCF) of all the individual sequences should be zero for every time shift. It can be shown, that such a sequence set cannot be designed [9], however ZCZ sequence sets do have optimum properties albeit over a reduced range of time shifts. Consequently ZCZ sequences employed in CDMA can eliminate the effects of co-channel interference and intersymbol interference completely [5] bounded by the zero correlation zone of size Z. In our study the Z-zone is an important parameter, because it specifies the length of the channel dispersion which can be handled with the proposed method. The specific properties required of the ZCZ-sequences so that they are optimum for cross-correlation channel estimation for joint multiuser detection in SDMA/TDMA systems are investigated.

The paper begins by defining optimum correlation properties. In Section III we show how to construct a ZCZ sequence set optimal for a SIMO cross-correlation channel estimation. Our SDMA/TDMA system is shown in section IV. Section V shows the performance of our estimation scheme. Finally we draw conclusions in section VI.

2. Definitions

We define the sequence set S as:

$$S = \begin{Bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,N} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{M,1} & s_{M,2} & \cdots & s_{M,N} \end{Bmatrix} \qquad (1)$$

The individual sequences in the set are defined as $s_i(n)=\{s_{i,1}, s_{i,2}, \ldots s_{i,N}\}$. Given a sequence set S of size M $\{s_i(n)\}$, each individual sequence having length N, the definition of the periodic auto-correlation function (ACF, r=s) and the periodic cross-correlation function (CCF, r≠s) is given by [6]:

$$R_{r,s}(\tau) = \sum_{n=1}^{N} s_r(n) \cdot s_s^*(n+\tau) \qquad (2)$$

where r,s=1,2,…,M, n=1,2,…,N, superscript * denotes complex conjugation and the addition n+τ is performed modulo N.

A sequence set is said to be orthogonal if the set has the following property:

$$R_{r,s}(\tau) = \begin{cases} N & \text{for} \quad \tau = 0, r = s \\ 0 & \text{for} \quad \tau = 0, r \neq s \end{cases} \qquad (3)$$

Walsh sequences are examples of such sequences. Further a sequence set is said to be Z-orthogonal or is a ZCZ sequence if the set has the following property:

$$R_{r,s}(t) = \begin{cases} N & \text{for} \quad t = 0, r = s \\ 0 & \text{for} \quad t = 0, r \neq s \\ 0 & \text{for} \quad 0 < |t| < Z \end{cases} \qquad (4)$$

Z is defined to be equal to the minimum value of the Z-zone of the ACF or the CCF (whichever is the lower value):

$$Z = \min(Z_{ACF}, Z_{CCF}) \qquad (5)$$

Further s_1, the first sequence in the set is called the mothersequence. Finally an ideal channel sounding sequence, i.e. a sequence with an impulsive periodic autocorrelation function, has the following property:

$$R_{r,s}(t) = \begin{cases} N & \text{for} \quad t = 0, r = s \\ 0 & \text{for} \quad 0 < |t| < N, r = s \end{cases} \qquad (6)$$

Such a sequence is optimal with respect to cross-correlation channel estimation. The properties defined in this section will be illustrated in the next section where we explain the construction method of our sequence set.

3. ZCZ sequence set construction

The authors of [2] proposed constant magnitude optimal complex sequences for channel sounding. Our method is based on these sequences, since the mothersequence s_1 in our set is derived using the construction method of [2]. These sequences have some very interesting properties, which can be exploited for cross-correlation channel estimation of multiple users. First these sequences have the property of equation (6). Consequently through the shift property of the periodic ACF of period N, every cyclic shifted version of such a sequence is also an optimal sounding sequence:

$$R_{r,s}(\tau) = \sum_{n=1}^{N} s_r(n) \cdot s_s^*(n+\tau) = \sum_{n=1\text{-cSft}}^{N\text{-cSft}} s_r(n) \cdot s_s^*(n+\tau), \; r = s \qquad (7)$$

where cSft denotes the cyclic shift value (Note, a cyclical shift of the mothersequence is the same as changing the sum bounds in equation (7)). Further all CCFs of the mothersequence with cyclically shifted versions have only one correlation peak with a height of magnitude N and are zero elsewhere. This is an essential property and having found such a mothersequence the following recursion is applied, which calculates the relative cyclic shift values among the sequences in such a way that the Z-zone is maximised. We are interested in maximising the Z-zone simply because this parameter specifies the maximum channel dispersion which our method can cope with. The following recursion generates three arrays, namely the cyclic shift value array cSft(i), the user array users(i) and the Z-zone array Z(i).

```
i=2, cSft(1)=N, users(1)=1, Z(1)=N-1
while cSft(i) > 1
    cSft(i)  = $\left\lfloor \frac{cSft(i-1)}{2} \right\rfloor$
    users(i) = $\left\lfloor \frac{N}{cSft(i)} \right\rfloor$
    Z(i)     = $cSft(i) - 1$
    i        = i+1
end
```

Array users(i) gives the number of SDMA channels which can be supported by having a relative cyclic shift amongst all sequences in the set of cSft(i) resulting in a Z-zone of Z(i), i.e. running the recursion for a mothersequence of length N=16, gives:

```
cSft(i)  = [16  8  4  2  1]
Z(i)     = [15  7  3  1  0]
users(i) = [ 1  2  4  8 16]
```

Looking at the array elements with index 3, this means 4 users can be supported by cyclically shifting every sequence with respect to the previous sequence in the set by 4, which will result in a Z-zone of 3 for this set. This example is further illustrated in Fig. 1.

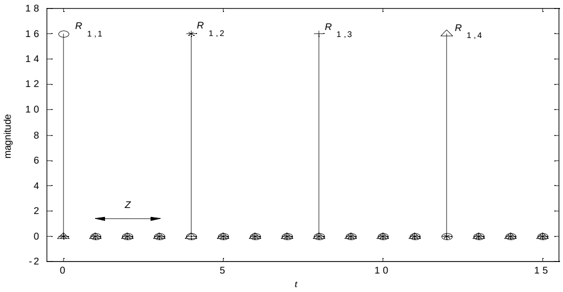


Fig. 1: correlation functions of a sequence set of size 4

It can be seen that all correlation functions have only one peak of height N. It is essential that this condition is fulfilled, eitherwise our channel estimate, will have different scaling factors. It can be shown that the sum of mean square values of the CCF between two complex valued sequences are bounded by an upper and a lower bound [9]:

upper bound:

$$\sum_{\tau=0}^{N-1}|\mathrm{CCF}_{x,y}(\tau)|^2 \le \mathrm{ACF}_x(0)\cdot\mathrm{ACF}_y(0) + \sqrt{\left(\sum_{\tau=1}^{N-1}|\mathrm{ACF}_x(\tau)|^2\right)}\cdot\sqrt{\left(\sum_{\tau=1}^{N-1}|\mathrm{ACF}_y(\tau)|^2\right)} \tag{8}$$

lower bound:

$$\sum_{\tau=0}^{N-1}|\mathrm{CCF}_{x,y}(\tau)|^2 \ge \mathrm{ACF}_x(0)\cdot\mathrm{ACF}_y(0) - \sqrt{\left(\sum_{\tau=1}^{N-1}|\mathrm{ACF}_x(\tau)|^2\right)}\cdot\sqrt{\left(\sum_{\tau=1}^{N-1}|\mathrm{ACF}_y(\tau)|^2\right)} \tag{9}$$

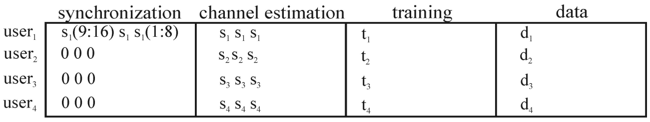
For optimal complex sounding sequences which satisfy equation (6), the terms in the square roots are zero and therefore the upper bound and the lower bound are equal:

$$\sum_{t=0}^{N-1}\left|\mathrm{CCF}_{x,y}(\tau)\right|^2 = \mathrm{ACF}_x(0)\cdot\mathrm{ACF}_y(0)$$

This means that the sum of mean squares of the CCF of the individual sequences in our set is N^2. Since the CCFs of our sequence set have only one non-zero element, the CCF peak = ACF peak = N.

4. SDMA/TDMA System

We assume that our system is synchronized, e.g. the individual user signals arrive at the same time at the base station. Synchronization is achieved through the frame structure shown in Fig. 2 for an example system with four SDMA users per TDMA time slot.



	synchronization	channel estimation	training	data
$user_1$	$s_1(9{:}16)\ s_1\ s_1(1{:}8)$	$s_1\ s_1\ s_1$	t_1	d_1
$user_2$	0 0 0	$s_2\ s_2\ s_2$	t_2	d_2
$user_3$	0 0 0	$s_3\ s_3\ s_3$	t_3	d_3
$user_4$	0 0 0	$s_4\ s_4\ s_4$	t_4	d_4

Fig. 2: SDMA packets

During the synchronization period, only one user, namely $user_1$ is allowed to transmit. Fig. 3 illustrates the concept.

Note that the synchronization field of $user_1$ is a modified version of sequence s_1. First the correlator output produces the frame synchronization pulse. After an interval of 2.5 times the length of the mothersequence the channel estimate appears. The correlator outputs between the synchronization pulse and the channel estimate contains inaccurate channel estimates due to aperiodic cross-correlation.

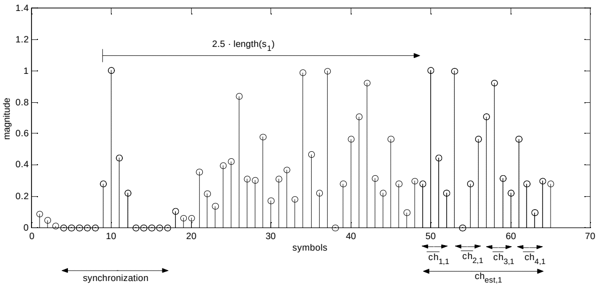


Fig. 3: correlator output on antenna 1

Following synchronization the actual channel estimation pilot sequences p_1 to p_M are transmitted by the users. Each pilot sequence p_i consists of the assigned sequence s_i repeated three times. This is done to assure periodic correlation properties.

$$p_i = \{s_i\ s_i\ s_i\} \tag{10}$$

In other words $user_1$ is assigned the mothersequence s_1 and all the other users are assigned cyclically shifted versions of the mothersequence. Following channel estimation further symbols are reserved for training e.g. of space-time equalizers. Then the actual data symbols for each user are transmitted in parallel. The SDMA system model for the up-link is depicted in Fig. 4.

All the pilot sequences are convolved with individual channel realizations for each antenna and then superimposed over each other. The received pilot sequence on each antenna can be represented as:

$$p_{rx,a} = \sum_{i=1}^{M} p_i * ch_{i,a} + N_a\ ,\ a = 1 \ldots A \tag{11}$$

where $ch_{i,a}$ is the channel impulse response between $user_i$ and antenna a and N_a is additive white Gaussian noise (AWGN) with variance σ_N. The * operator denotes convolution. For each antenna processing channel there is one correlator, namly $correlator_i$ estimating M user channels. Each correlator is loaded with the mothersequence s_1. The final step is to cross-correlate the conjugate of the received pilot signal $p_{rx,a}$ with s_1. This operation gives the channel estimate $ch_{est,a}$ (i.e. $ch_{est,a}(i) = \frac{1}{N} R^*_{s1,p_{rx,a}}(i+N)$, i=1…N) for all user channels. The output of the correlator $R_{s1,p_{rx,a}}$ of antenna a is given by:

$$R_{s_1,p_{rx,a}}(i+1) = \sum_{n=1}^{N} s_1(n) \cdot p^*_{rx,a}(n+i), i = 0...2 \cdot N \qquad (12)$$

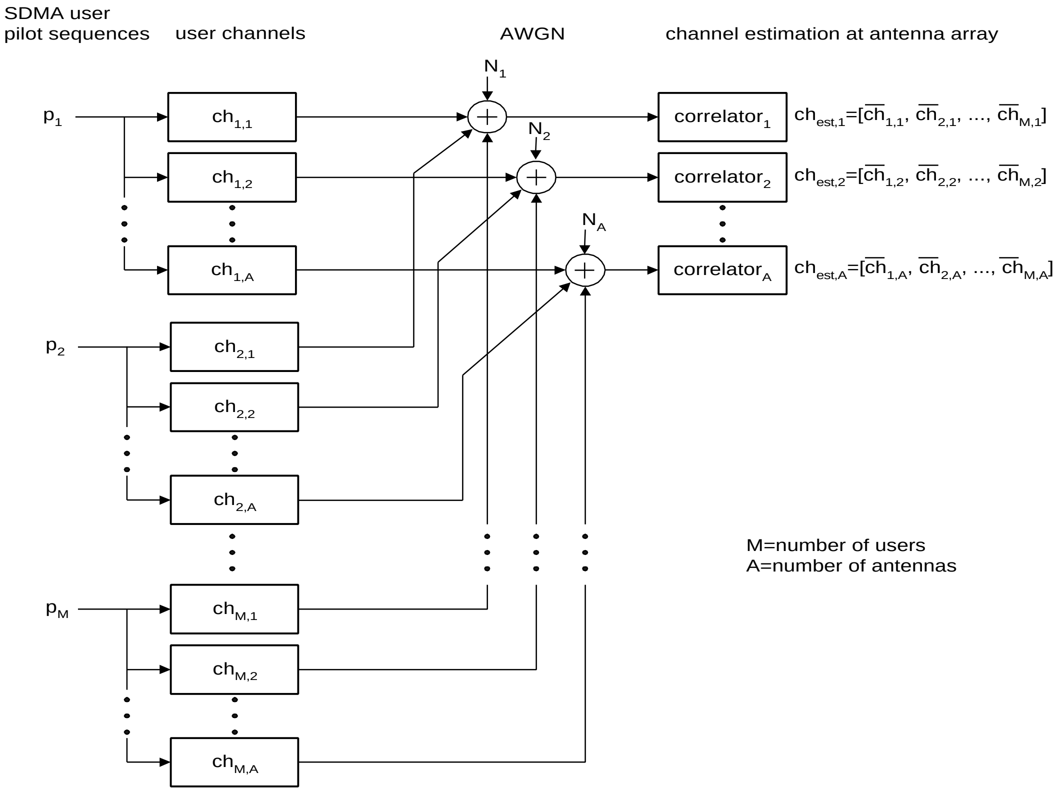


Fig. 4: SDMA system model

The obtained channel estimate $ch_{est,a}$ on each antenna contains the individual channel estimates $ch_{i,a}$ of all the $users_i$, which are superimposed at the input of antenna a. In the next section we illustrate how the channel estimates are composed of all the individual user channels $ch_{i,a}$.

5. Numerical Simulation Results

First we draw our attention back to the example in section III to illustrate the concept. All the channel taps $ch_{i,a}$ are symbol spaced. The sequence set S is of size M=4 and contains sequences s_i of length N=16. The following sequence s_i were used:

s_1=[3302120331001001]
s_2=[1001330212033100]
s_3=[3100100133021203]
s_4=[1203310010013302]

where the symbols are defined as: 0 = $+\sqrt{0.5}+\sqrt{0.5}i$, 1 = $-\sqrt{0.5}+\sqrt{0.5}i$, 2 = $-\sqrt{0.5}-\sqrt{0.5}i$, 3 = $+\sqrt{0.5}-\sqrt{0.5}i$ and the individual user channels for antenna 1 are:

$ch_{1,1}=[0.28e^{0.785i}, 1.00e^{0.100i}, 0.45e^{-0.463i}, 0.22e^{-1.107i}]$
$ch_{2,1}=[1.00e^{0.000i}, 0.00e^{0.000i}, 0.28e^{0.785i}, 0.57e^{-0.785i}]$,
$ch_{3,1}=[0.71e^{-0.142i}, 0.92e^{-0.219i}, 0.32e^{1.249i}, 0.22e^{1.107i}]$
$ch_{4,1}=[0.57e^{-0.785i}, 0.28e^{0.785i}, 0.10e^{-1.571i}, 0.30e^{-1.571i}]$.

We consider only antenna 1 and set the AWGN term in equation (10) to zero. The cross-correlation channel estimation process on the other antennas is simply applied repetitively in the same manner. The recursion procedure indicates for this example that channel estimation for channels of 4 symbol periods in duration (i.e. the channel dispersion must be ≤ Z+1 symbol periods) with 4 SDMA users is possible. If the channel dispersion is longer than Z+1 (i.e. 4) the individual channel estimates in $ch_{est,1}$ will overlap. Fig. 5 shows the results of using our method with this example.

Fig. 5 shows that if no AWGN is present, our method gives optimum channel estimates (i.e. $ch_{i,1} = \overline{ch}_{i,1}, i = 1...4$). Fig. 6 shows the same example with a signal to noise ratio (SNR) of 20dB. Therefore the accuracy of the channel estimate is only affected by

AWGN and the use of cyclically shifted versions of optimum sounding sequences is therefore optimum for joint multiuser channel estimation.

Fig. 7 shows the mean square error (MSE) of the channel estimate at various signal to noise ratios (SNRs). The MSE between the actual channels $ch_{i,j}$ and the estimated channels $\overline{ch}_{i,j}$ is computed with 100 channel realizations at each SNR. It is seen that the MSE decreases as the SNR is increased.

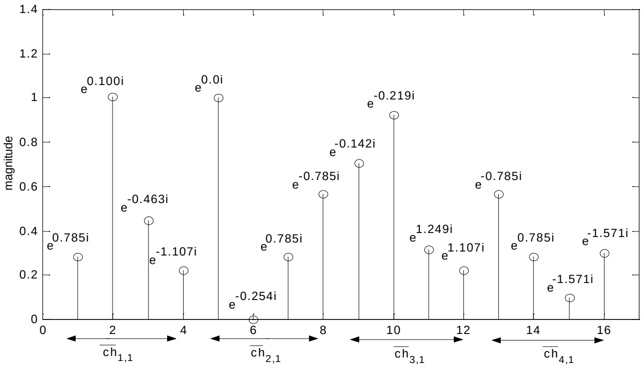


Fig. 5: channel estimate of antenna 1 $ch_{est,1}$ without AWGN

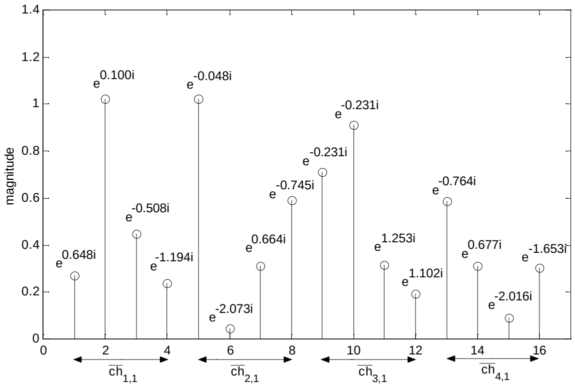


Fig. 6: channel estimate of antenna 1 $ch_{est,1}$ with an SNR of 20dB

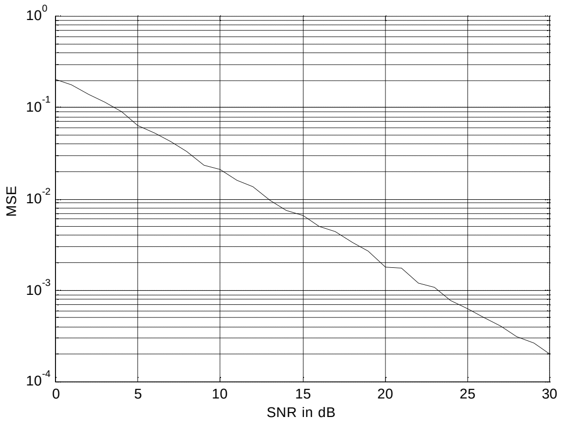


Fig. 7: MSE vs SNR

6. Conclusions

It has been shown how the optimum sounding sequence proposed in [2] can be extended to a ZCZ sequence set, which is optimal for cross-correlation channel estimation for SIMO systems. Once an optimum complex sounding sequence with constant magnitude is found (i.e. the sequences from [2]) a ZCZ sequence set is simply constructed by cyclic shifting of this sequence. The proposed sequence set has the advantage compared to other ZCZ sequence sets that multiple SDMA channels can be estimated using only one correlator loaded with the mothersequence s_1. The application of the proposed ZCZ sequence set for cross-correlation channel estimation in a SDMA/TDMA BFWA system shows that highly accurate channel estimates can be obtained with our method.

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