Robin Milner, 1934–2010
His work in theorem proving and verification

John Harrison
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January 28th, 2011 (09:15–09:27)
Invited speaker at TPHOLs 2000?

From: Robin Milner <Robin.Milner@cl.cam.ac.uk>
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Date: Tue, 25 Jan 2000 11:32:39 +0000

Dear John

Thanks very much for inviting me to speak at TPHOLs. I would enjoy it, but the main question is whether I can offer enough of a perspective on automated and interactive theorem proving, as I haven’t done any to speak of for 20 years!
What really sparked me off was getting interested in program verification and what semantics might mean. When I went to Swansea in 1968 I took a research job, I gave up teaching and became a research assistant with David Cooper who was head of the department in Swansea. He had a small group there, working on program verification and automatic theorem-proving and semantics.
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That was at the time when Dana Scott produced his famous domain theory. He gave a series of talks then, in ’69, and I went over to Oxford and heard him. That was very exciting. So, in some sense, it began to move very fast. The idea of a machine proving theorems in logic, and the idea of using logic to understand what a machine was doing ... this double relationship began to inspire me because it was clearly not very simple.
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Dana Scott’s influence on semantics is well-known, but his work was also an important factor in the development of interactive theorem proving.
1970: From automated to interactive proving

I wrote an automatic theorem prover in Swansea for myself and became shattered with the difficulty of doing anything interesting in that direction and I still am. I greatly admired Robinson’s resolution principle, a wonderful breakthrough; but in fact the amount of stuff you can prove with fully automatic theorem proving is still very small. So I was always more interested in amplifying human intelligence than I am in artificial intelligence.
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Interest in ‘interactive’ theorem proving was growing at the time, either because

- Abilities of ATP systems had grown but were plateauing
- More interactive computers made it natural/convenient
1971–2: Move to Stanford and Stanford LCF

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- The set of proof commands could not easily be extended.
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Edinburgh LCF, developed with Malcolm Newey, Lockwood Morris, Mike Gordon and Chris Wadsworth, tackled the two shortcomings of Stanford LCF:

- Did not store complete proofs, just remembering the conclusions of proofs.
- Provided a full programming 'meta-language' (ML) so that the user could extend the set of proof commands.

But how to ensure that theorems were proved correctly, not just arbitrarily asserted or created by buggy user proof commands?

- Make theorems an abstract type in the metalanguage ('thm') with its only constructors being primitive inference rules of the logic.

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How an LCF-style prover works

A logical inference rule such as \( \Rightarrow \)-elimination (modus ponens)

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\frac{\Gamma \vdash p \Rightarrow q \quad \Delta \vdash p}{\Gamma \cup \Delta \vdash q}
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How an LCF-style prover works

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\hline
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\]

becomes a *function*, say \texttt{MP : thm->thm->thm} in the metalanguage.

For example, if \texttt{th1} is the theorem \(\Gamma \vdash p \Rightarrow (q \Rightarrow p)\) and \texttt{th2} is \(p \vdash p\), then \texttt{MP th1 th2} gives \(p \vdash q \Rightarrow p\). Highly automated or convenient derived inference rules can be programmed using these as the basic building-blocks, including support for backward proof via ‘tactics’.
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The LCF diaspora

[LCF] didn’t immediately get applied a great deal, but Mike Gordon brought it to Cambridge [...] He started doing hardware verification. And then one or two other people began to design verifications systems, or rather systems to perform computer-assisted proof, on the model of our system, particularly Constable at Cornell with his NuPrl.
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Despite the name, which has stuck, the LCF approach is not tied to the Logic of Computable Functions, and many other LCF-style provers were written over the years.

- Cambridge LCF (rationalized system for same LCF logic)
- HOL (higher-order logic based on polymorphic type theory)
- Nuprl (Martin-Löf type theory)
- Coq (the Calculus of Inductive Constructions)
- Isabelle (framework supporting multiple object logics)
Design Features of LCF

- Security: no false theorem can be proved, thanks to ML's type discipline
- Automation: LCF accommodates both forward and goal-oriented proof
- Generality: copes with computational problems in context of Scott's theory; user can supplement PPLAMBDA with new types, constants and axioms

Examples of Proofs

- Simple compiling algorithms (Cohn)
- FP-systems and balanced trees (Lesczycyowski)
- Simple parsing algorithms (Milner and Cohn)
- Induction Rules (Jensen and Milner)

Brief History of LCF

1969: Scott invents PPLAMBDA
1971: Milner, Weyhrauch and Newey build Stanford LCF
1973-8: Milner, Morris, Newey, Gordon and Wadsworth build Edinburgh LCF
1975-81: Cohn, Lesczycyowski, Jensen, Milner, et al. do proofs in LCF
1981-present: Joint Edinburgh-Cambridge LCF grant; Paulson and Schmidt join Gordon and Milner

Details

Available on DEC-10 and VAX-UNIX;
ML available on VAX-VMS
The HOL system alone has given rise to numerous different LCF-style implementations of essentially the same logic:
Applications of LCF provers

Several LCF-style systems have been used for major work in formal verification and formalization of mathematics.

- Verification of microprocessors, compilers, floating-point microcode, cryptographic protocols, OS kernels, properties of programming languages and their type systems . . . .

- Formalization of Jordan Curve Theorem, Prime Number Theorem, 4-Colour Theorem . . . . Feit-Thompson theorem and Kepler conjecture (‘Flyspeck project’) in progress.

The ideas that Robin Milner developed almost 40 years ago are central to machine-assisted proof today.
Dear John

I’ve thought a bit more. I believe I can offer, informally, some interesting reminiscences. What I can’t do, given my current preoccupation, is to spend very long on that. (Indeed, a whole talk would be too hard.) At present I am working flat out on the theory I mentioned, hence my interest in how machine assistance could help with it. I can’t afford too much time away from this task over the coming year -- life is short.