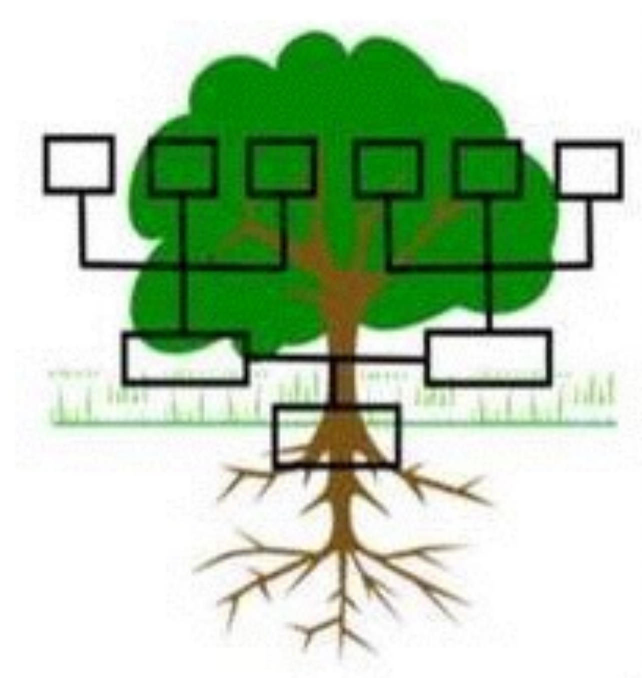
Brick diagrams, string diagrams, proof trees, k-d trees



Jules Hedges

Max Planck Institute for Mathematics in the Sciences

Jelle Herold

Statebox

We have plenty of stringy proof assistants

Quantomatic

pyZX

homotopy.io

Globular

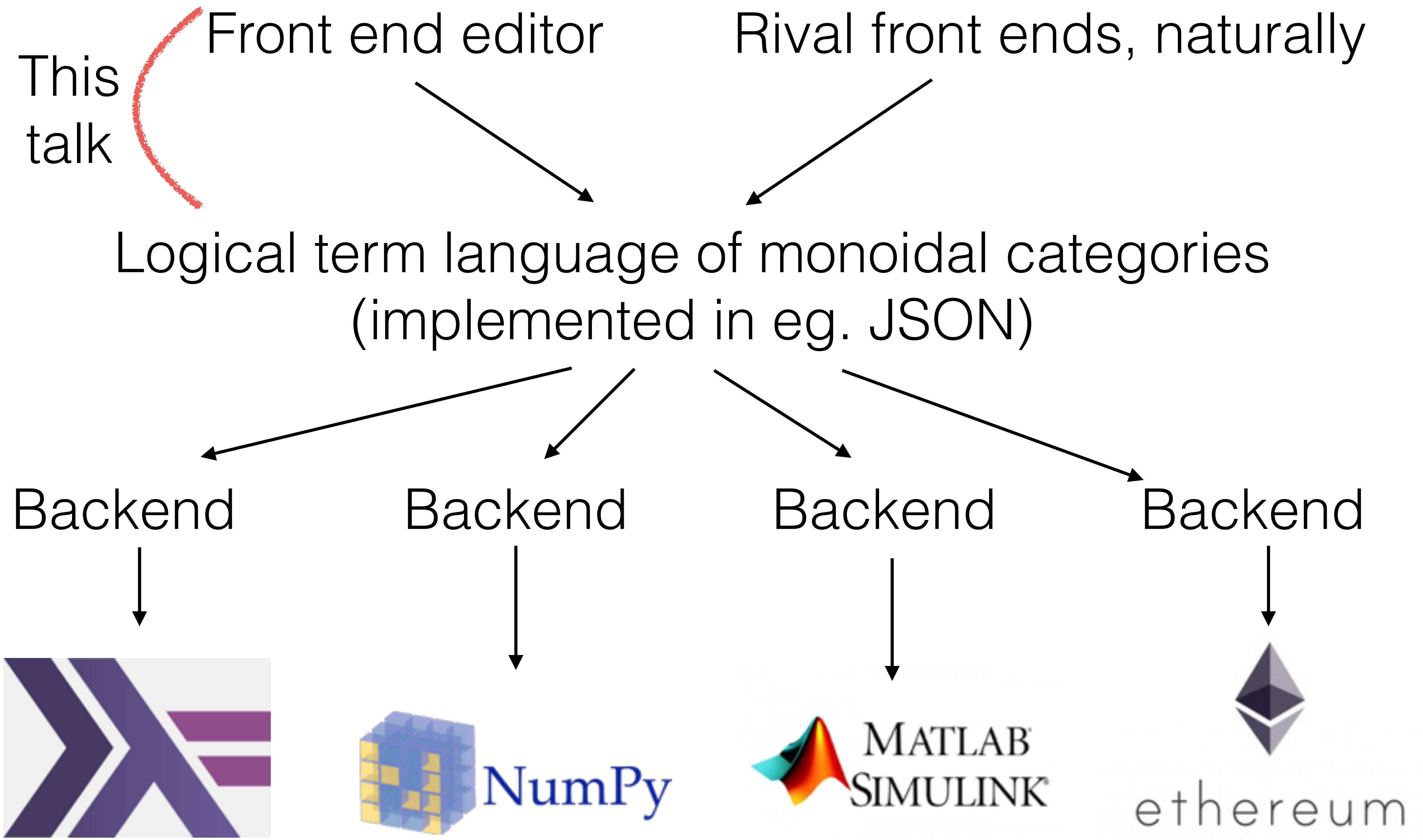
Opetopic

We need a stringy compiler

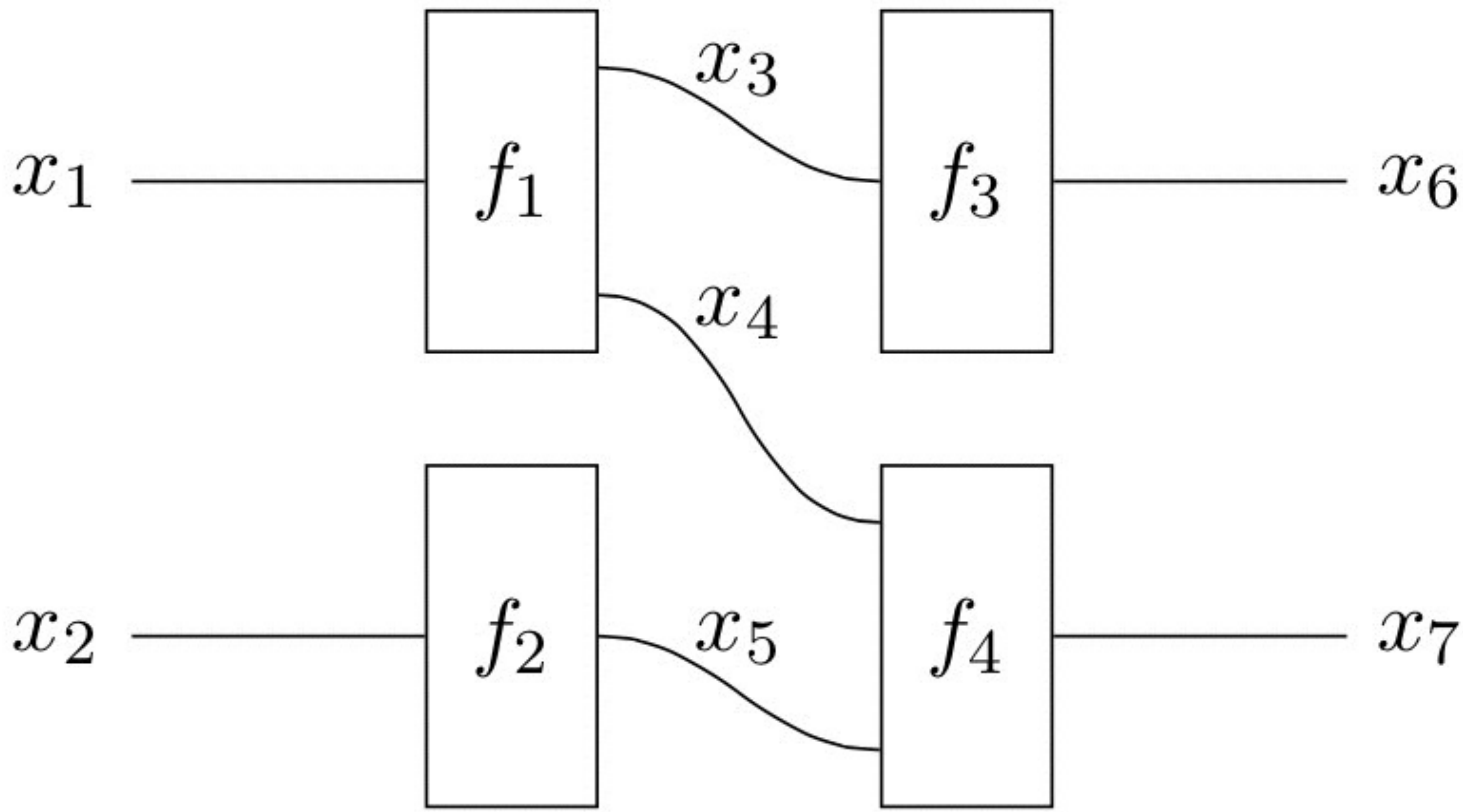
String diagrams are still useful
without a complete proof system…

- programming
- complex systems
- DisCoCat
- game theory
- …

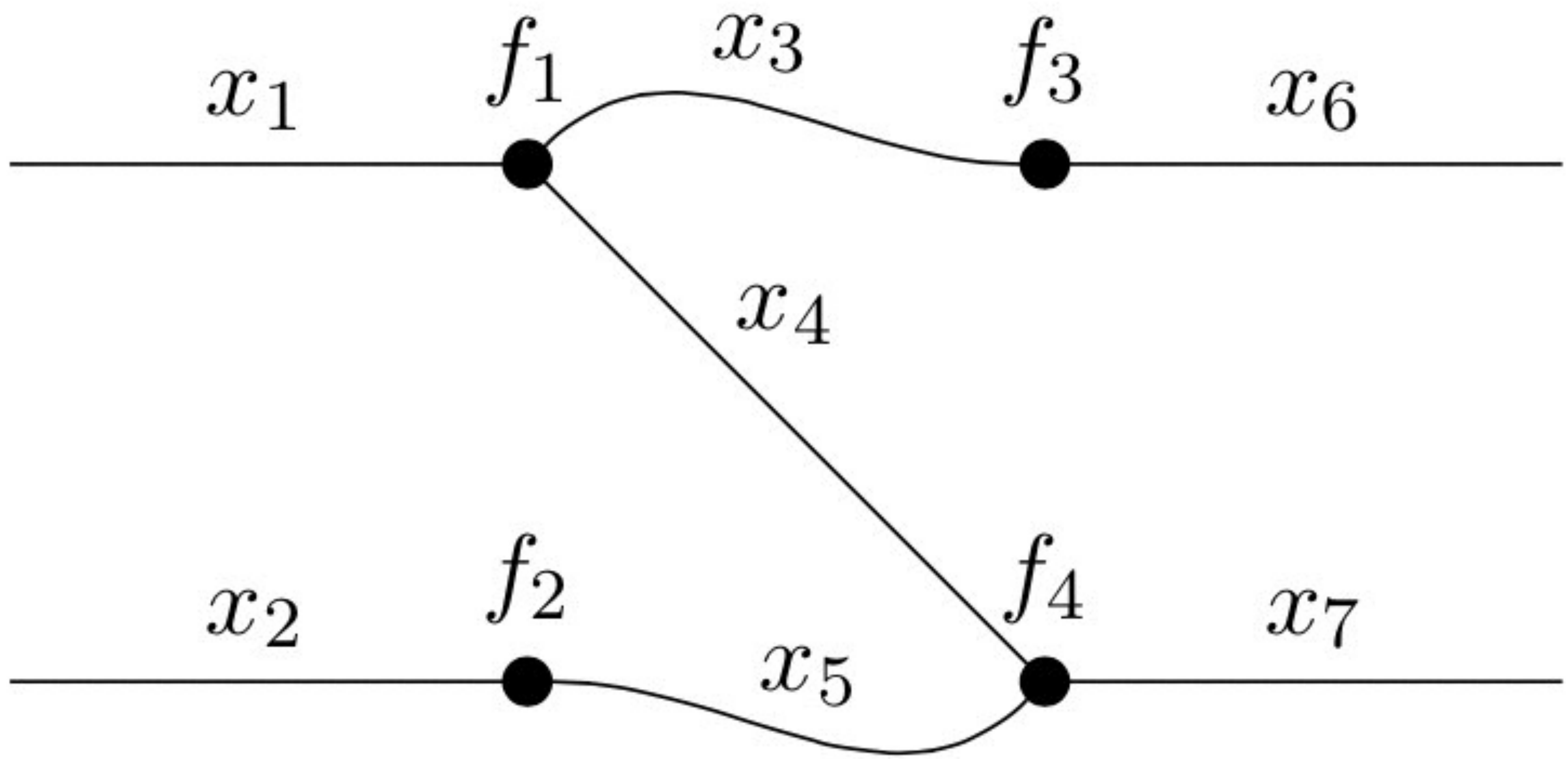
The obvious architecture



What is a string diagram, actually?



Follow the literature…



… Joyal & Street (1991): It's a "topological graph"

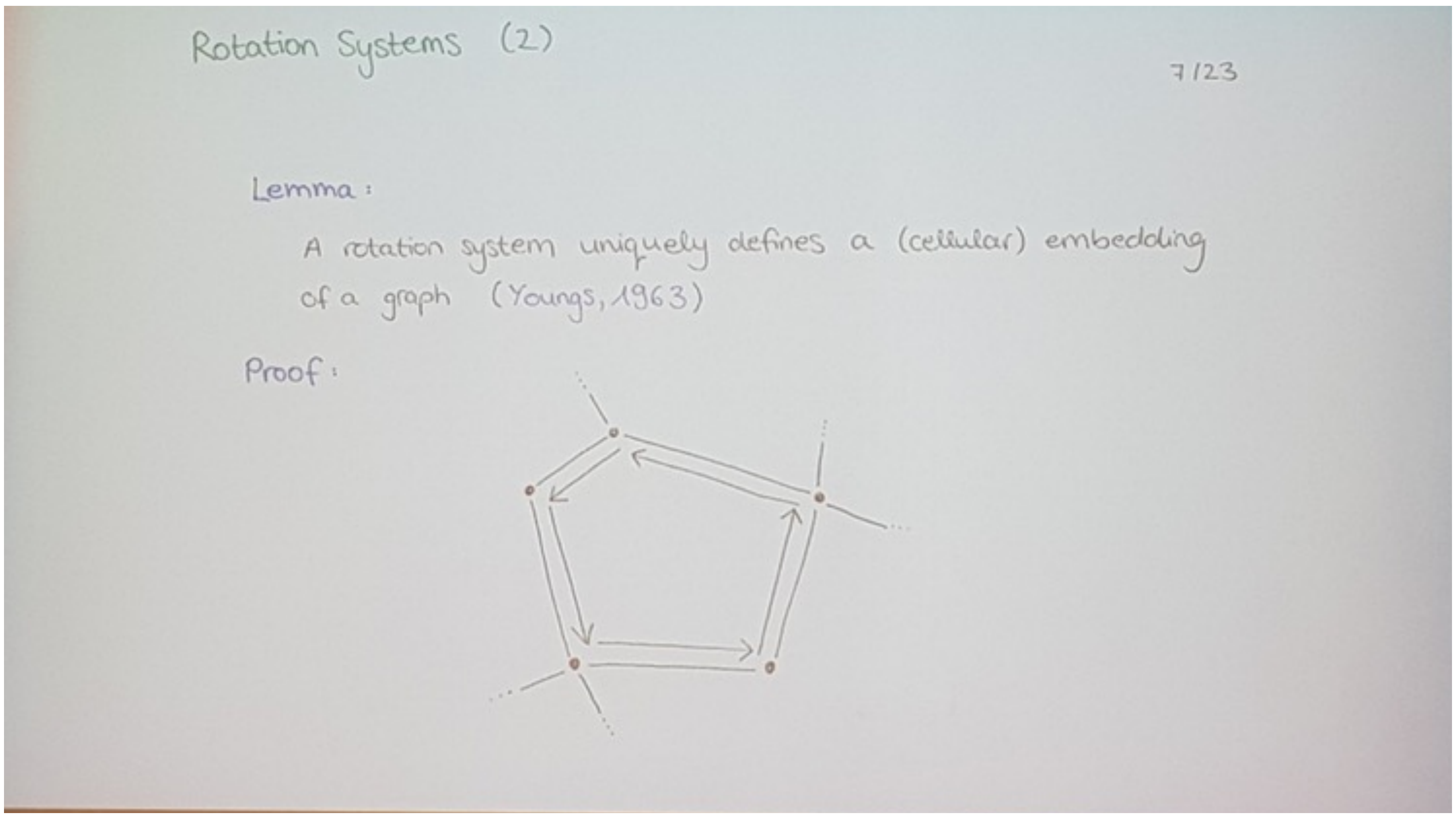
String diagrams as graphs

- duh
- Used by Quantomatic & pyZX
- Graphs = CCCs, DAGs = SMCs

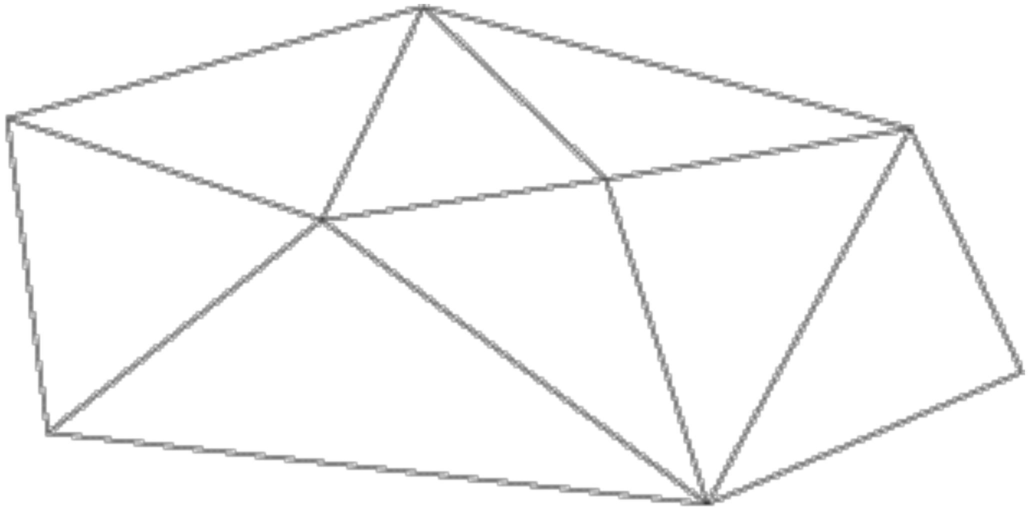
Planar graphs

- We might care about non-symmetric category, e.g. linguistics
- We might want to control where symmetries go, e.g. compiling for quantum computers
- Planar graphs are annoyingly complicated

Rotation systems



Polygonal complexes



The Joyal-Street Theorem

- String diagrams modulo isotopy are the morphisms in the free monoidal category on a signature
- Equivalently: every interpretation induces an isotopy-invariant interpretation
- Everybody knows this instinctively

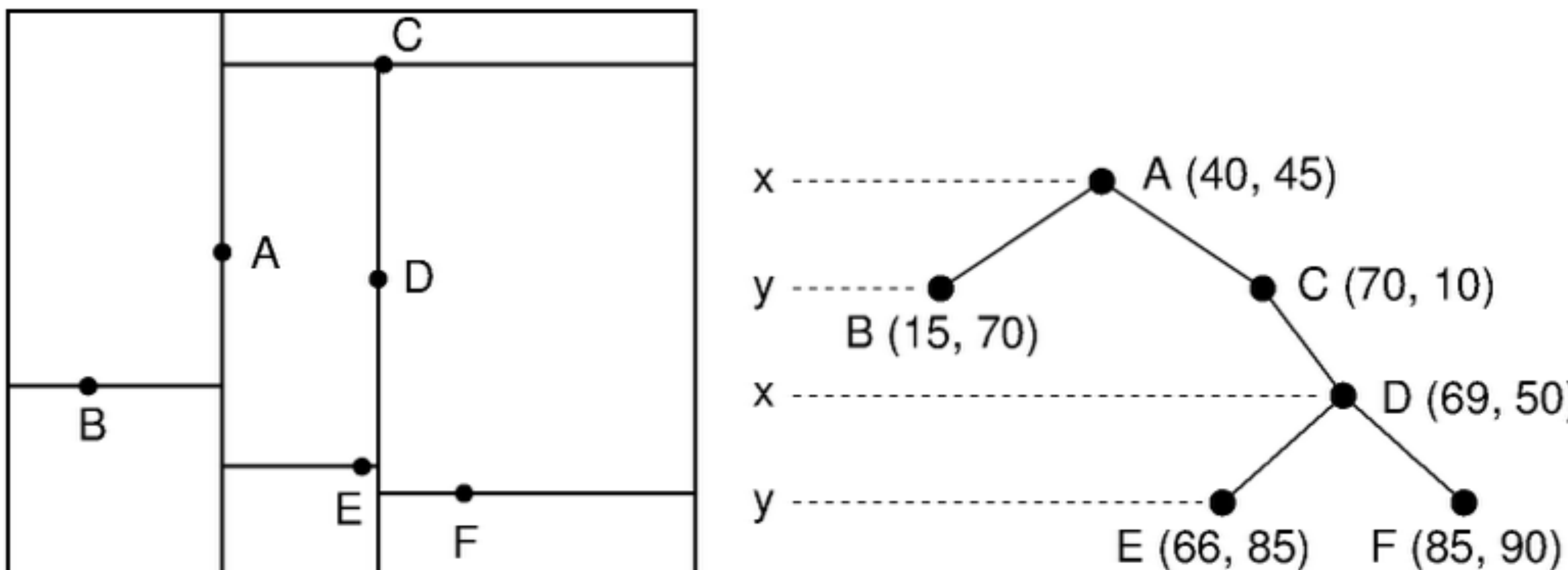
Free categories a la Lambek

- General principle: Morphisms in free categories are proof trees modulo commuting conversions
- For monoidal categories: Noncommutative linear logic of tensor
- So: we have an equivalence of categories between string diagrams (modulo isotopy) and proof trees (modulo commuting conversions)

k-d trees

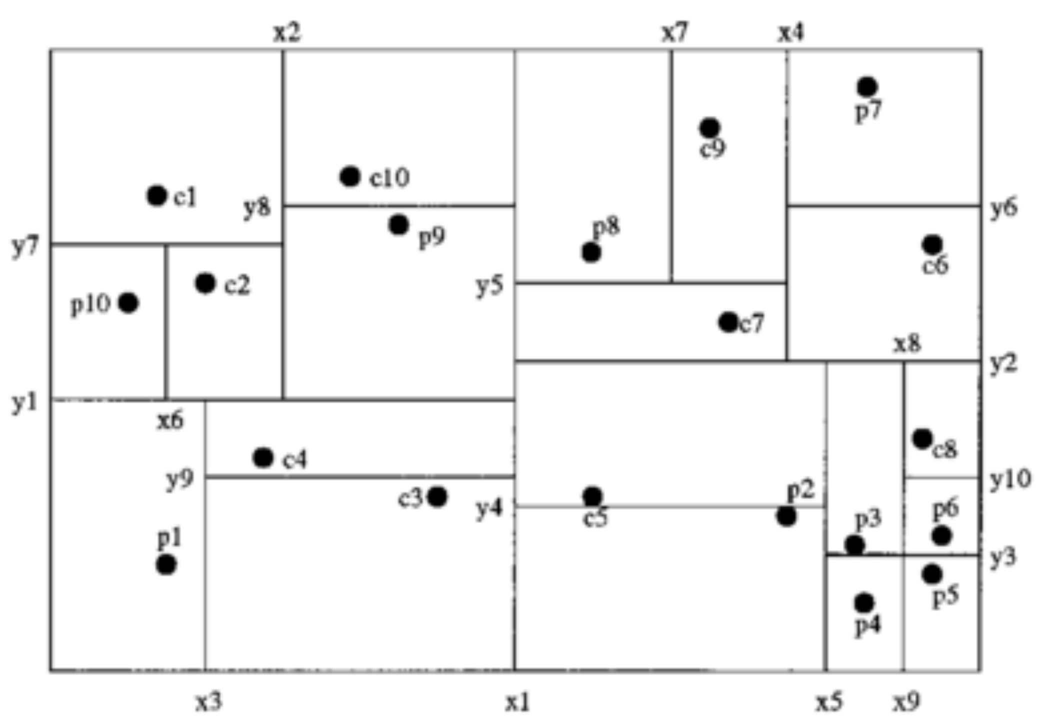
- A data structure from computational geometry
- Special case of binary space partition trees
- Closes the gap between topology and logic

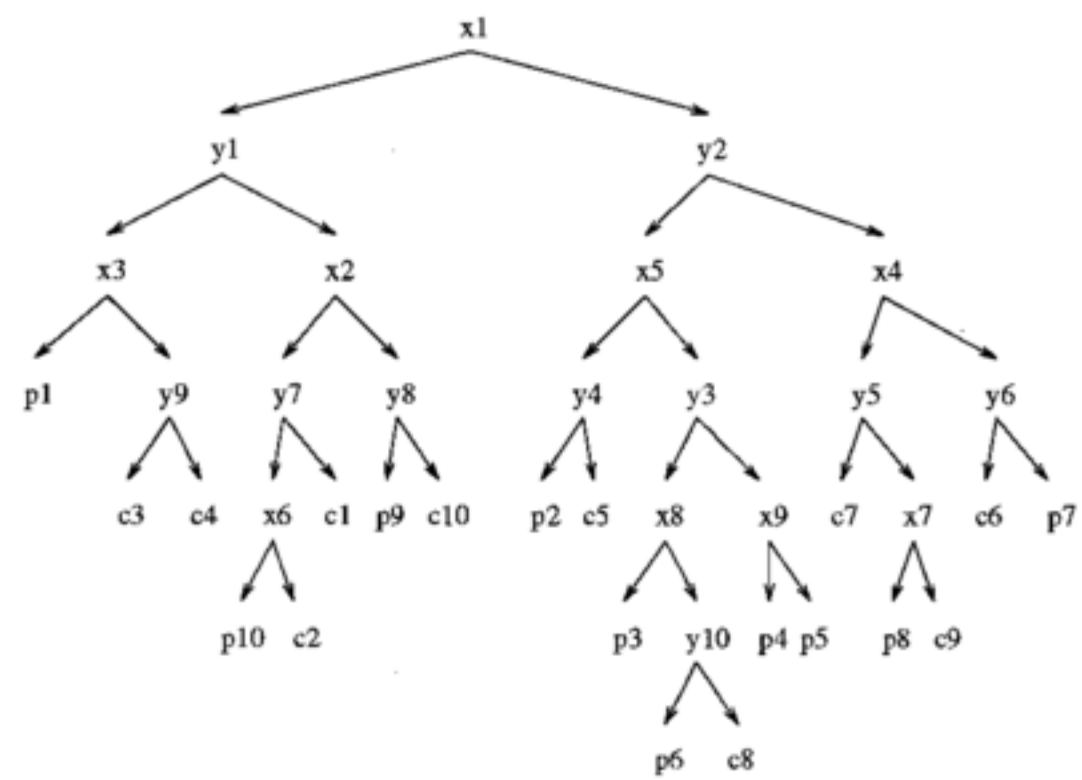
k-d trees by example



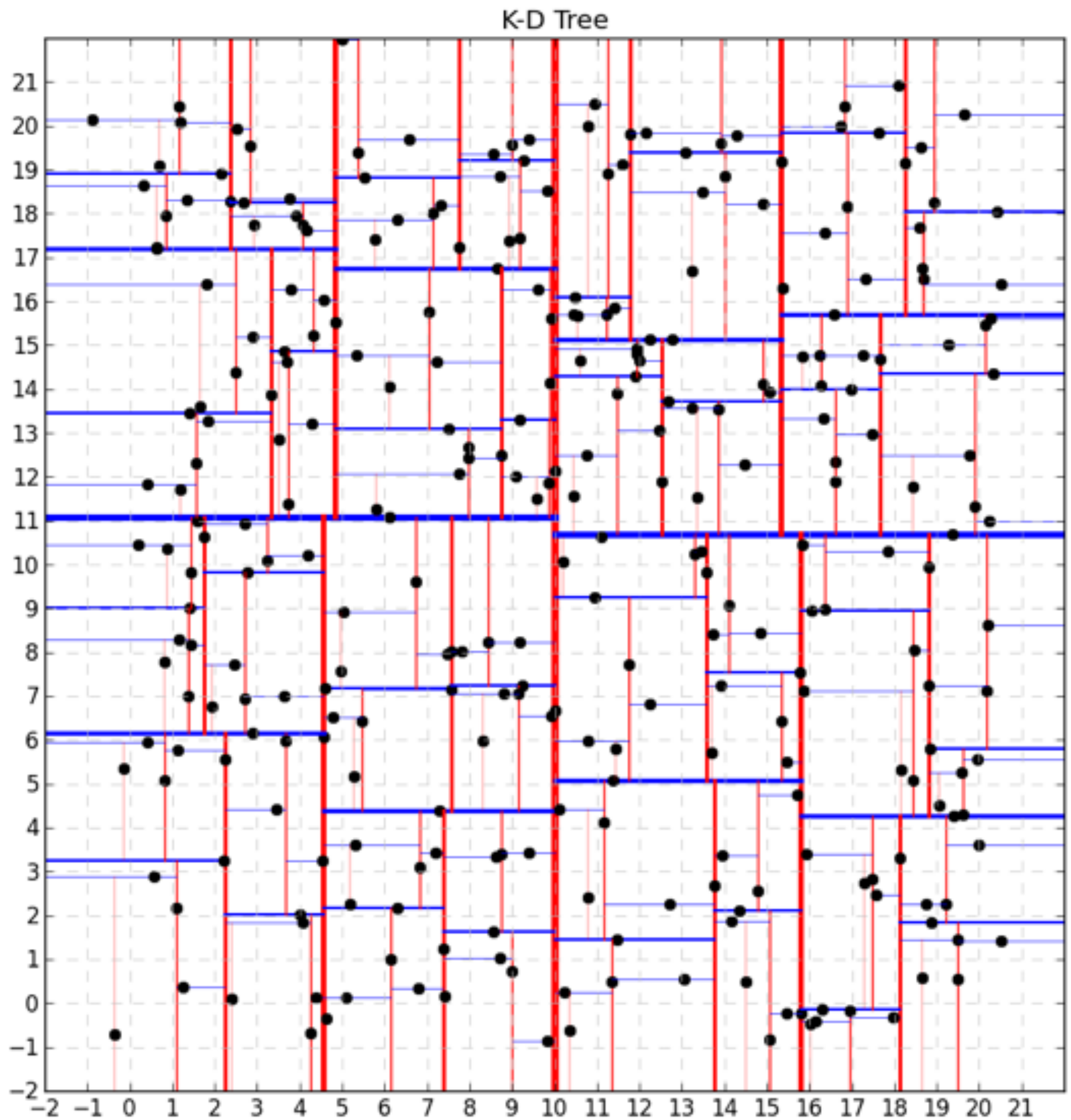
(a) (b)

k-d trees by example





k-d trees by example



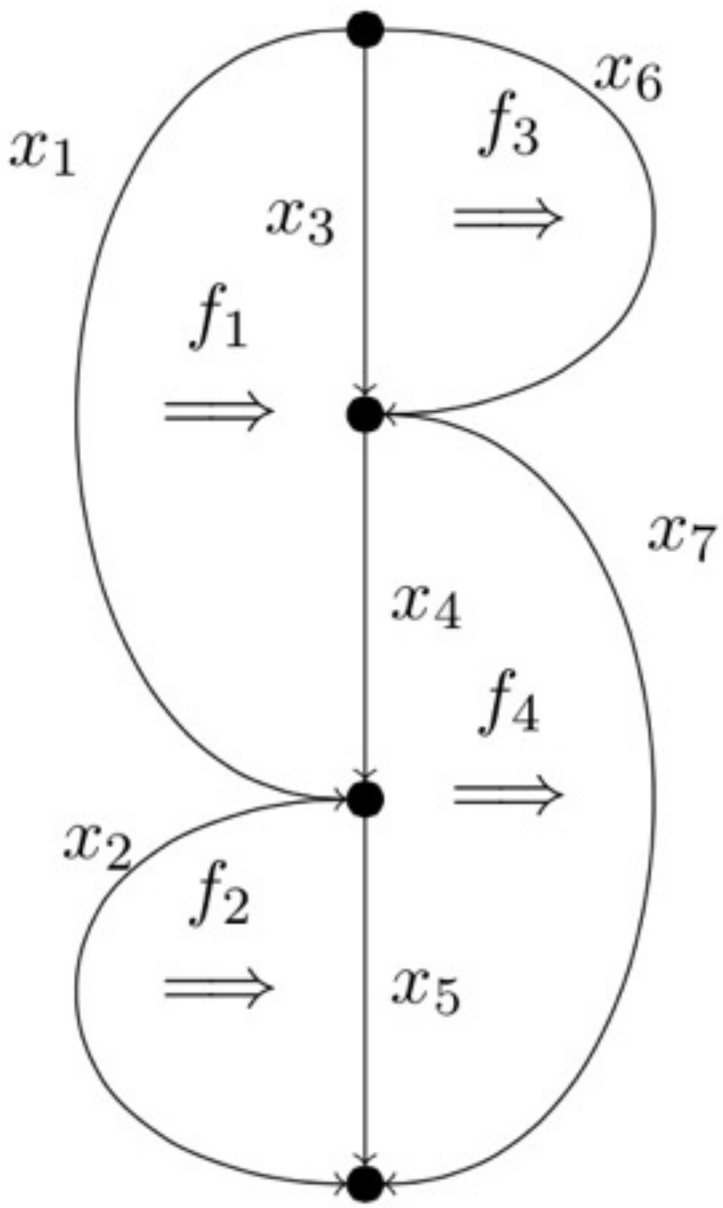
A silly conjecture

Higher category theory is just computational geometry

- People study balancing operations on k-d trees for efficiency reasons

- They ought to be the same as the defining data of a strict n-category

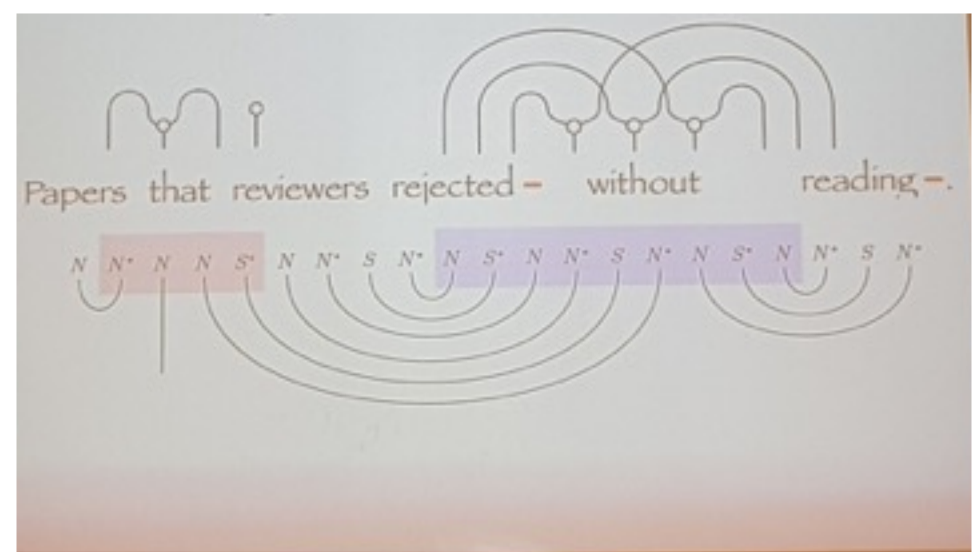
Globular pasting diagrams



Strict monoidal category

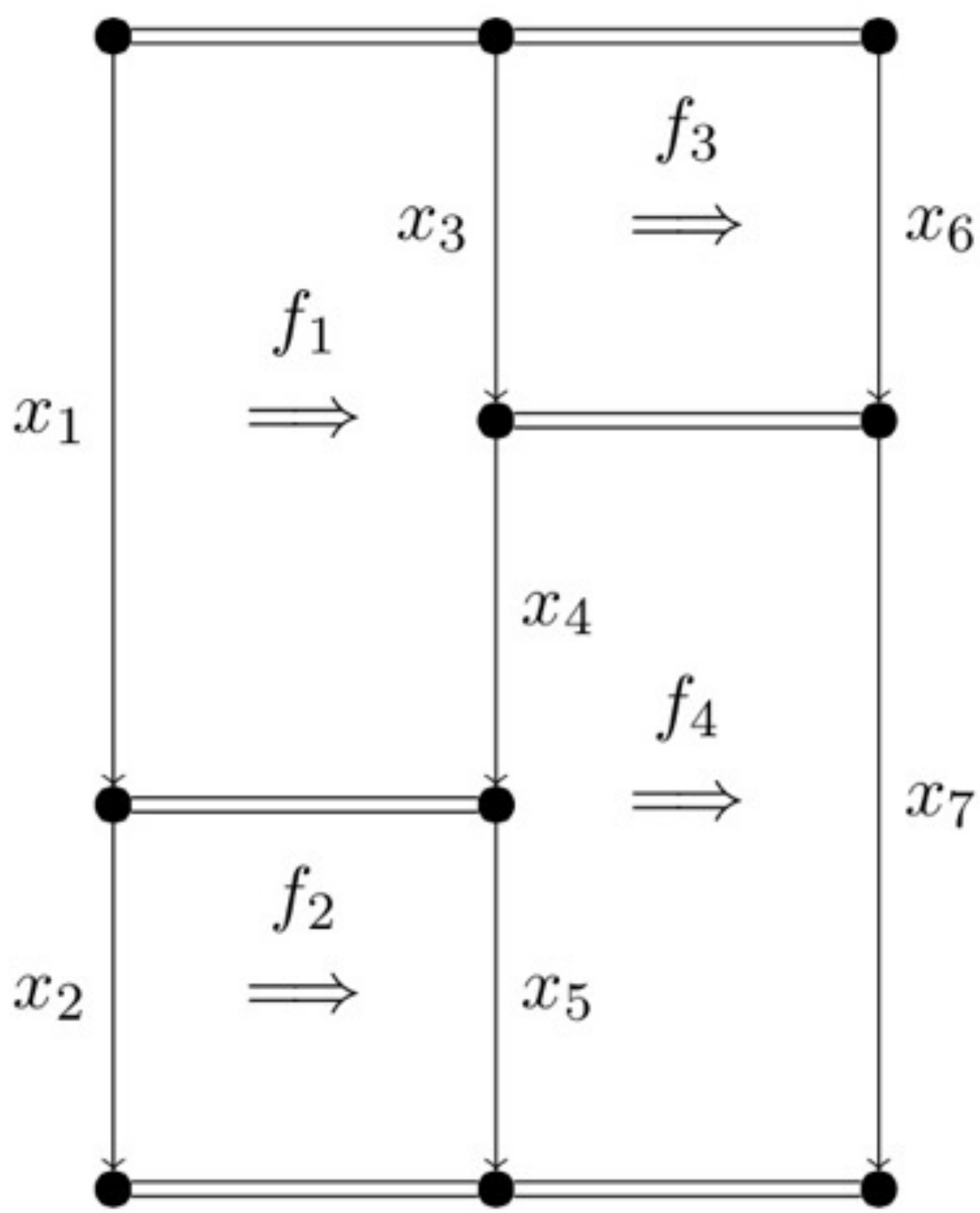
=

1-object 2-category



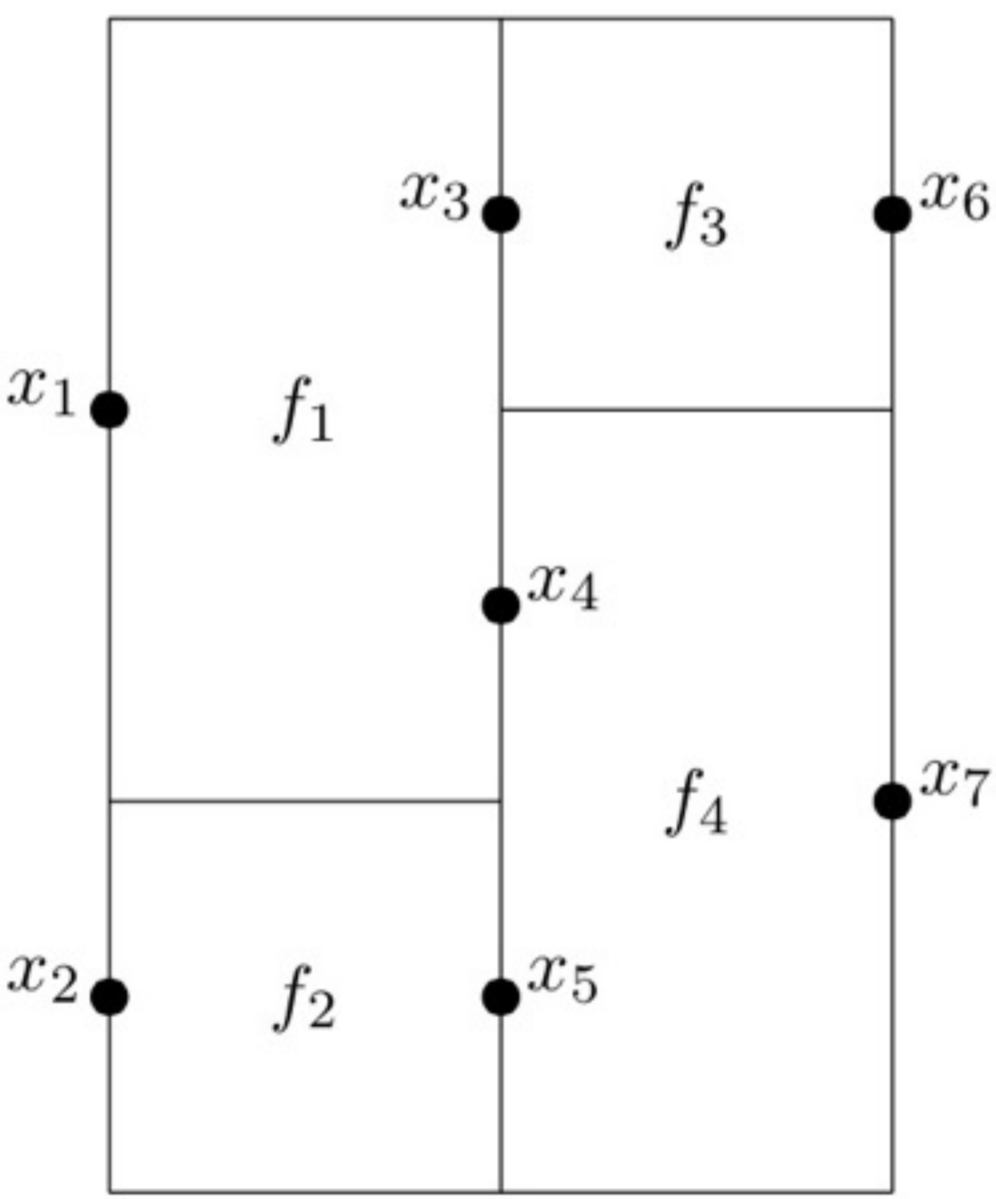
(not suitable for serious work)

Cubical pasting diagrams



Strict monoidal category
=
double category with
1 object and
1 horizontal 1-cell

Brick diagrams



Take an extra Poincaré dual only of the vertical edges

Brick diagrams in SYCO

Tileorders

Theory and Applications of Categories, Vol. 1, No. 7, 1995, pp. 146–155.

A FORBIDDEN-SUBORDER CHARACTERIZATION OF BINARILY-COMPOSABLE DIAGRAMS IN DOUBLE CATEGORIES

ROBERT DAWSON

Transmitted by R. J. Wood

ABSTRACT. Tilings of rectangles with rectangles, and tileorders (the associated double order structures) are useful as "templates" for composition in double categories. In this context, it is particularly relevant to ask which tilings may be joined together, two rectangles at a time, to form one large rectangle. We characterize such tilings via forbidden suborders, in a manner analogous to Kuratowski's characterization of planar graphs.

Conclusion

The following are pretty much the same, more or less:

- String diagrams with a choice of decomposition
- Proof trees for the noncommutative linear logic of tensor
- k-d trees of dimension 2
- Cubical pasting diagrams
- Binarily composable tileorders

Demo time