Autonomization of monoidal categories

Antonin Delpeuch

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SYCO 3

UNIVERSITY OF OXFORD
DEPARTMENT OF COMPUTER SCIENCE

Outline

1. Pregroup grammars and compositional semantics
2. Free yourselves from the strings of tensors!
3. Examples of applications

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Context

Pregroup grammars (Lambek, 1993, Lambek, 1999)

the	film	that	Emily	directed
$np \cdot n^l$	n	$n^r \cdot n \cdot np^{ll} \cdot s^l$	np	$np^r \cdot s \cdot np^l$

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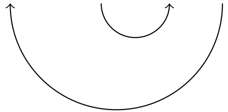

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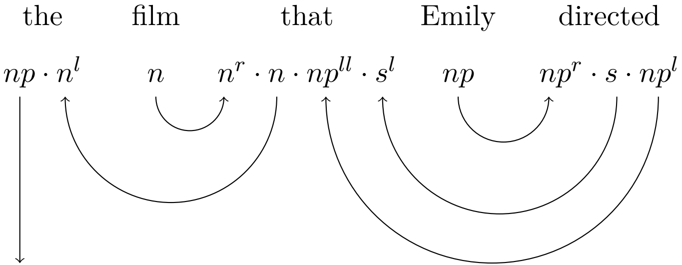


Autonomous (or rigid) categories

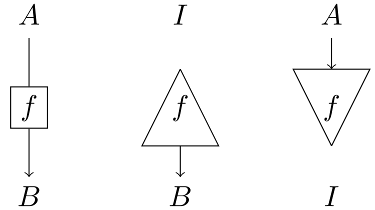
- Objects (= types)
  - are closed under $\_ \otimes \_$ (product of types), $\_^l$ and $\_^r$ (adjoints).
  - contain basic types, and I, neutral for $\otimes$.
- Arrows (= type reductions) between two objects
  - can be composed with $\circ$ (sequential composition) and $\otimes$ (parallel composition) ;
  - contain $1_A : A \to A$ (identity of A) and

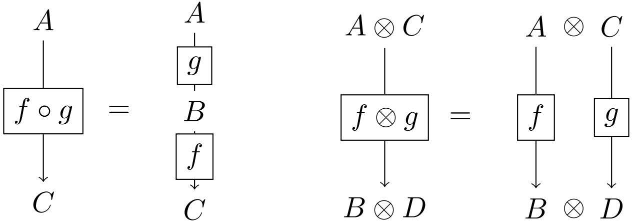
$$\epsilon^l : A^l \otimes A \to I \qquad \epsilon^r : A \otimes A^r \to I$$

$$\eta^l : I \to A \otimes A^l \qquad \eta^r : I \to A^r \otimes A$$

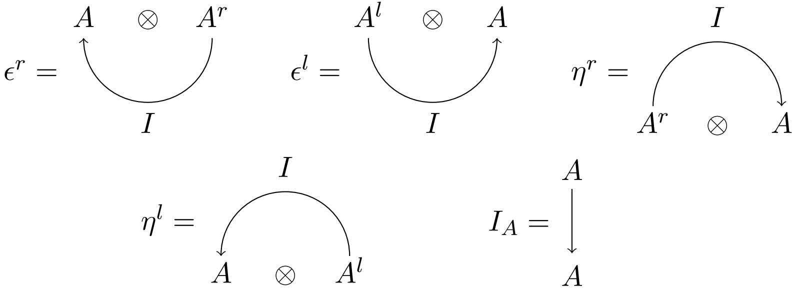
and such that some equations hold.

Representation

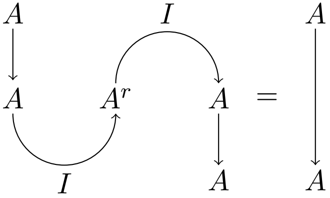


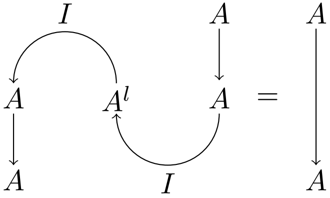


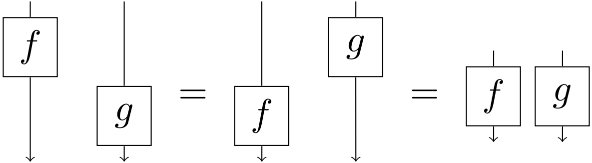
ϵ and η



Some equalities



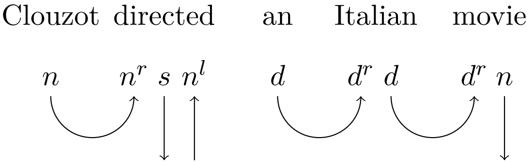




Pregroup reductions as arrows

Clouzot	directed	an	Italian	movie
n	$n^r\ s\ n^l$	d	$d^r\ d$	$d^r\ n$

Pregroup reductions as arrows



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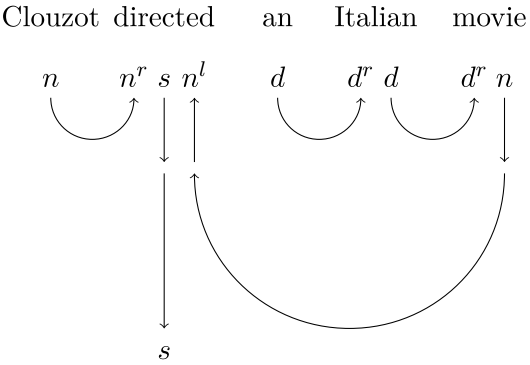
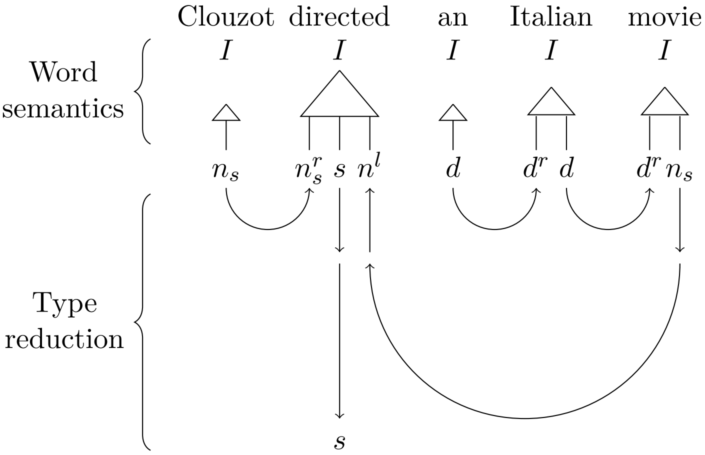


Compositional semantics



Motto: Type reduction $\circ$ Word meanings = Sentence meaning

Distributional Compositional Categorial model

DisCoCat (Coecke, Sadrzadeh, and Clark, 2011) : use $(\mathbf{Vect}, \otimes, I)$, finite dimensional vector spaces over $\mathbb{R}$ and linear maps between them.

$$= \begin{pmatrix} 0.73 \\ -2.3 \\ 0.1 \\ 1.4 \end{pmatrix}$$

$$= \begin{pmatrix} -0.3 & 3.9 & -2.1 & 0.4 \\ -2.3 & 2.2 & 1.5 & -1.6 \\ 0.1 & 0.3 & -3.8 & 1.2 \\ 1.4 & 3.4 & 0.1 & 3.2 \end{pmatrix}$$

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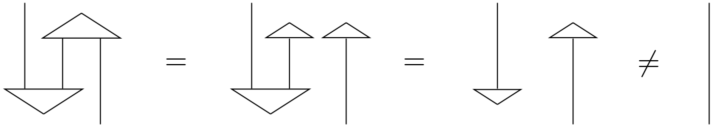
The dimension of a word representation is **exponential** in the length of the grammatical type.

Why should we use the tensor product?

The direct sum $\oplus$ is cartesian, so it cannot have cups and caps:



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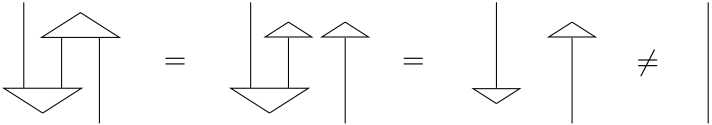

General belief in the community: "sticking with the categorical framework [...] forces us to stay within the world of linear maps" (Wijnholds and Sadrzadeh, 2018).

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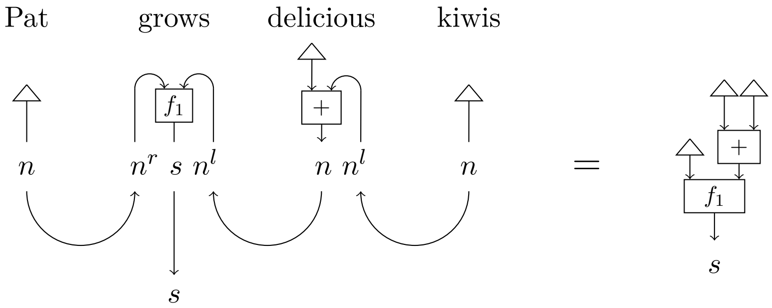
Just cheat and be free!

Our semantic category does not need to have caps and cups: we can **freely add them**.



Trick: caps and cups can be eliminated in any sentence representation.

Constructing free autonomous categories

- Preller and Lambek (2007) construct the free autonomous category generated by a category.
- We need to start from a monoidal category instead. We factorize their construction:

$$\mathbf{Cat} \underset{R'}{\overset{L'}{\underset{\bot}{\rightleftarrows}}} \mathbf{Mon} \underset{R}{\overset{L}{\underset{\bot}{\rightleftarrows}}} \mathbf{Nom}$$

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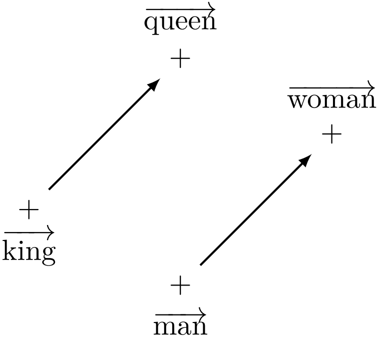
Additive models

Observation by Mikolov et al. (2013):



So, it tempting to define royal$(x) = x + \overrightarrow{queen} - \overrightarrow{woman}$.

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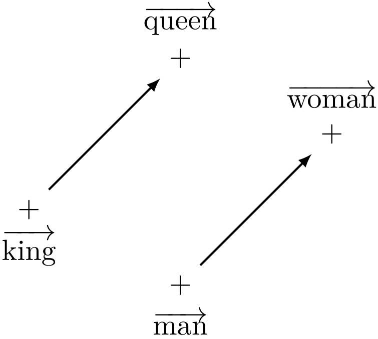
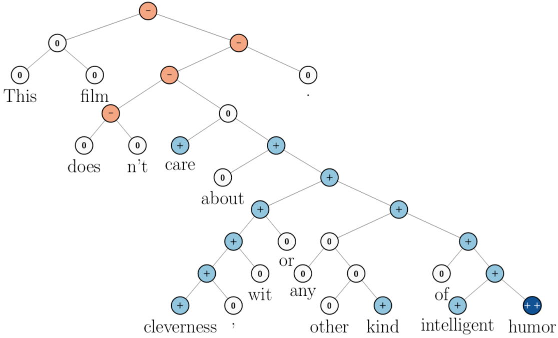


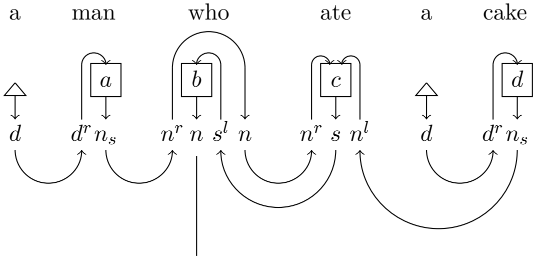
So, it tempting to define $\text{royal}(x) = x + \overrightarrow{queen} - \overrightarrow{woman}$.
That is **forbidden** in $(\textbf{Vect}, \otimes, I)$!

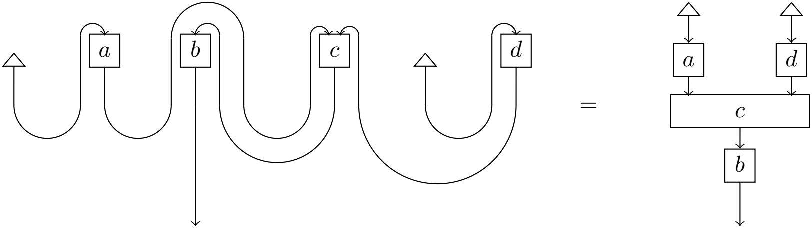
Convolutional neural networks

Socher et al. (2013) combine vectors following a Chomskyian tree:



Lewis (2019) translates this approach to the categorical model, in $(\mathbf{Vect}, \otimes, I)$.


a man who ate a cake
a
b
c
d
d d^r n_s n^r n s^l n n^r s n^l d d^r n_s


a
b
c
d
=
a
d
c
b