

homotopy.io: a proof assistant for finitely-presented globular
 n -categories
SYCO 12

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Structure of this talk

- ▶ Key features of homotopy.io
- ▶ n -dimensional string diagrams
- ▶ Implementation-focused tour of the foundations of homotopy.io
- ▶ homotopy.io demo

homotopy.io

- ▶ Web-browser-based graphical proof assistant written in Rust and compiled to WebAssembly. Access at <https://beta.homotopy.io>.
- ▶ Renders 2D geometry as interactive SVGs, and 3D and 4D geometry via WebGL.
- ▶ Export diagrams in a variety of formats, including TikZ, SVG, and STLs for 3D printing.
- ▶ Provides a rich set of tools for manipulating diagrams, and generating higher-dimensional structure.
- ▶ Supports fully-coherent invertible generators.
- ▶ Save and publish your proofs and share them with others by URL.

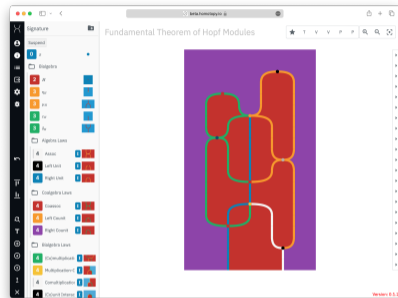


Figure 1: homotopy.io interface

n -dimensional string diagrams

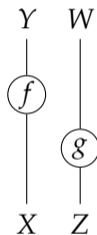


Figure 2: 2D string diagram representing $f \otimes id(W) \circ id(X) \otimes g$

n -dimensional string diagrams

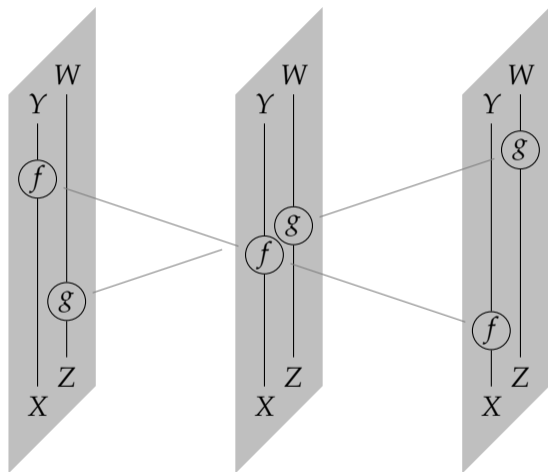


Figure 3: 3D string diagram of interchange law $f \otimes id(W) \circ id(X) \otimes g \cong id(Y) \otimes g \circ f \otimes id(Z)$, as 2D slices

n -dimensional string diagrams

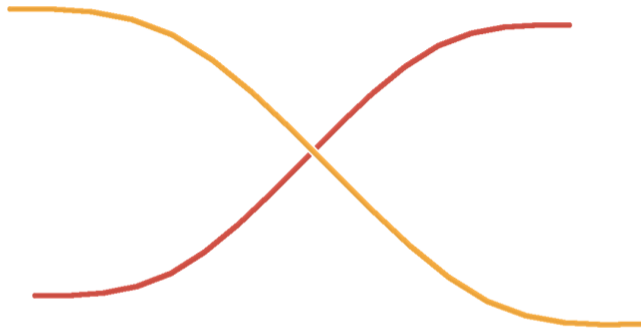


Figure 4: 3D string diagram of interchange law as 3D geometry

Recursive encoding of diagrams and rewrites

```
type frame = int
type generator = { dimension: int; id: int }
type rewrite =
  | Rewrite0Identity
  | Rewrite0 of { source: generator; target: generator; label: frame }
  | RewriteN of { cones: cone list }
and cone = {
  index: int;
  source: cospan list;
  target: cospan;
  slices: rewrite list;
}
and cospan = { forward: rewrite; backward: rewrite }
type diagram =
  | Diagram0 of generator
  | DiagramN of { source: diagram; cospans: cospan list }
```

Example 2D diagram encoding

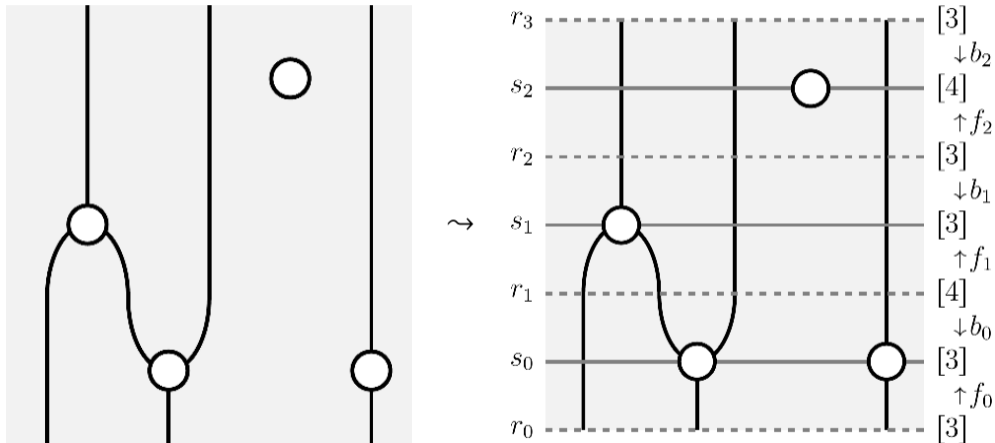


Figure 5: Encoding a 2D string diagram (Reutter and Vicary 2019)

Example 2D diagram encoding

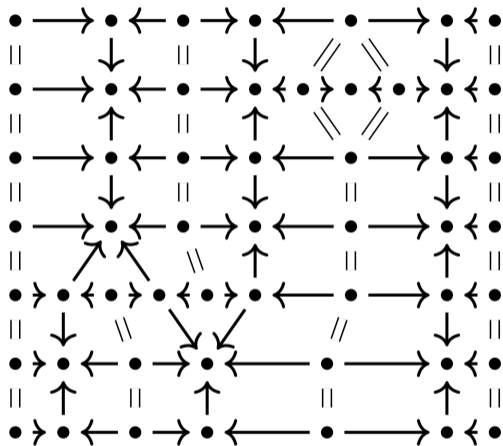


Figure 6: Figure 5 wireframe, with explicit rewrites

Contraction

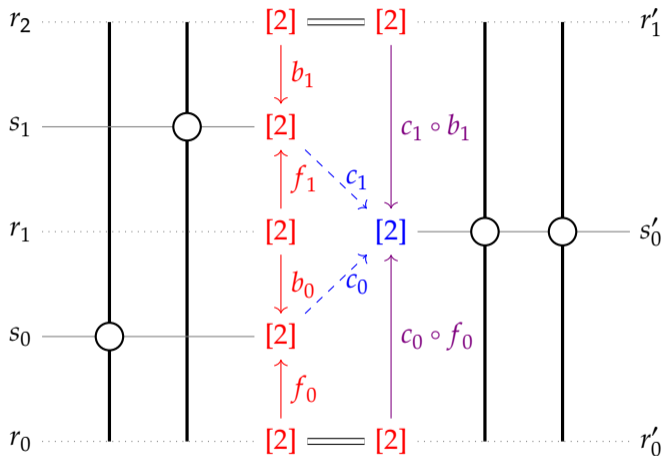






Figure 7: Contracting the two-bead diagram

Demo

References I

-  Heidemann, Lukas, David Reutter and Jamie Vicary (18th May 2022). 'Zigzag normalisation for associative n -categories'. In: *Proceedings of the 37th Annual ACM/IEEE Symposium on Logic in Computer Science*. 37th Annual ACM/IEEE Symposium on Logic in Computer Science. Haifa Israel: ACM. DOI: 10.1145/3531130.3533352. arXiv: 2205.08952 [cs, math].
-  Hu, Nick, Calin Tataru and Jamie Vicary (2024). 'Coherent invertibility in associative n -categories'. In: 39th Annual ACM/IEEE Symposium on Logic in Computer Science. Tallinn, Estonia. Submitted.
-  Reutter, David and Jamie Vicary (11th Feb. 2019). 'High-level methods for homotopy construction in associative n -categories'. In: 2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS). Vancouver, Canada: IEEE. DOI: 10.1109/LICS.2019.8785895. arXiv: 1902.03831.
-  Tataru, Calin and Jamie Vicary (11th May 2023). *A Layout Algorithm for Higher-Dimensional String Diagrams*. DOI: 10.48550/arXiv.2305.06938. arXiv: 2305.06938 [math]. preprint.

References II

-  Tataru, Calin and Jamie Vicary (30th Jan. 2024). *The Theory and Applications of Anticolimits*. DOI: [10.48550/arXiv.2401.17076](https://doi.org/10.48550/arXiv.2401.17076). arXiv: 2401.17076 [math]. preprint.