String diagrams for higher mathematics with `wiggle.py`

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Outline

(1) String diagrams for higher mathematics

(2) with `wiggle.py`

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~~(1a) all about associativity, unitality, etc.~~

(2) with `wiggle.py`

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(1) String diagrams for higher mathematics

~~(1a) all about associativity, unitality, etc.~~

(2) with `wiggle.py`

(2a) demo

(2b) implementation notes

A monoid

"... and then ..."

$$\cdot \xrightarrow{f} \cdot \xrightarrow{g} \cdot \xrightarrow{h} \cdot$$

Arrow diagram

String diagram

A category

"Multi-coloured monoid"

$$A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D$$

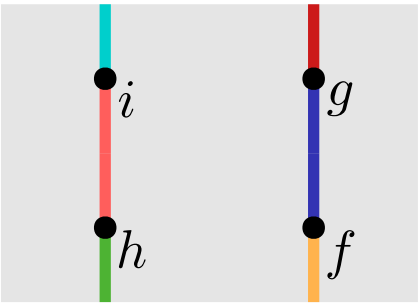
Arrow diagram

String diagram

A monoidal category

"... and then ..., but also..."

$$A \otimes B \xrightarrow{h \otimes f} C \otimes D \xrightarrow{i \otimes g} E \otimes F$$



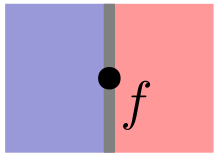
Arrow diagram

String diagram

A bicategory

"multi-coloured monoidal category"

$$\begin{tikzcd} n \arrow[r, bend right] \arrow[r, bend left] \arrow[r, phantom, "f\Uparrow"] & m \end{tikzcd}$$

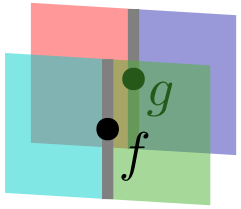


Arrow diagram String diagram

A monoidal bicategory

"... and then ..., but also..., meanwhile..."

$$\begin{tikzcd} m \otimes o \arrow[r, bend right] \arrow[r, bend left] \arrow[r, phantom, "f \otimes g \Uparrow"] & l \otimes n \end{tikzcd}$$

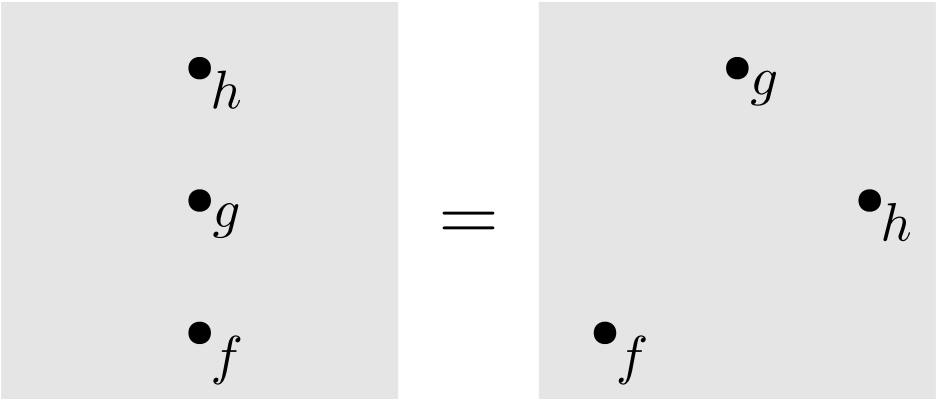


Arrow diagram

String diagram

Commutativity

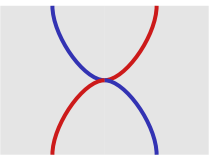
from “and then” to “and”



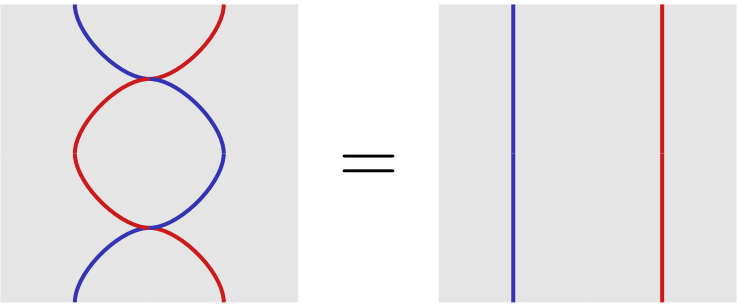
Commutative monoids

Higher commutativity

Equations become morphisms:

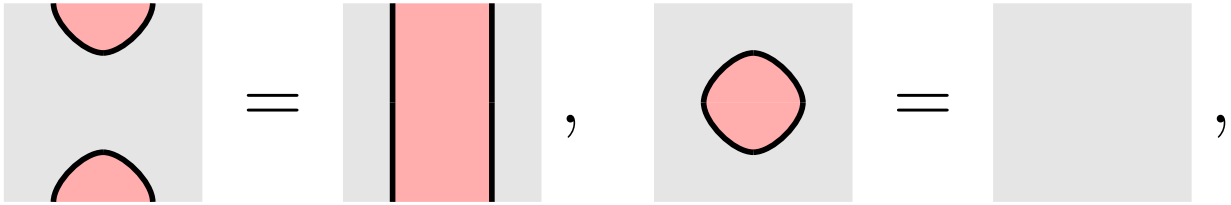


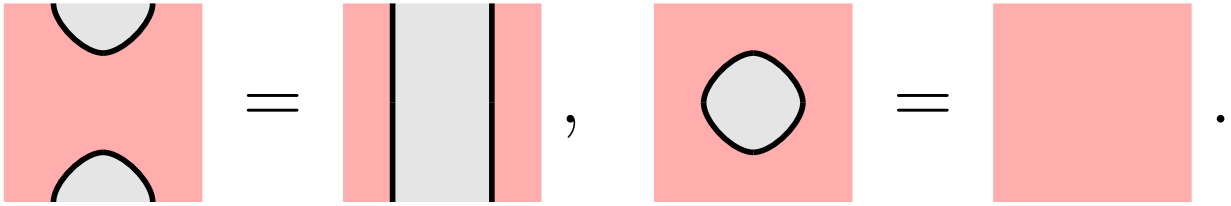
That satisfy further equations:



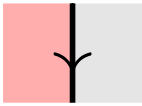
Higher equality

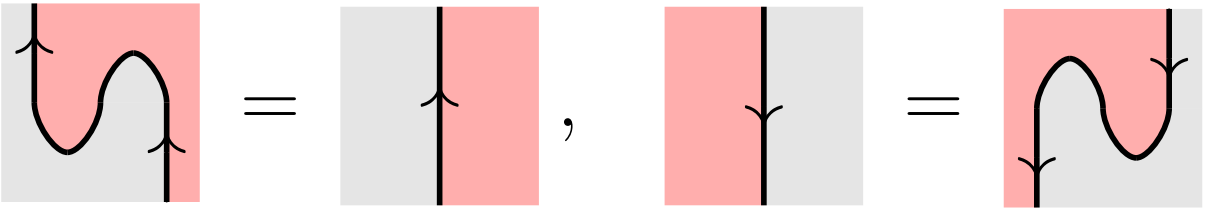
In a given 2-category, an *equivalence* between objects  and is a pair of 1-cells  ,  and isomorphisms  ,  :





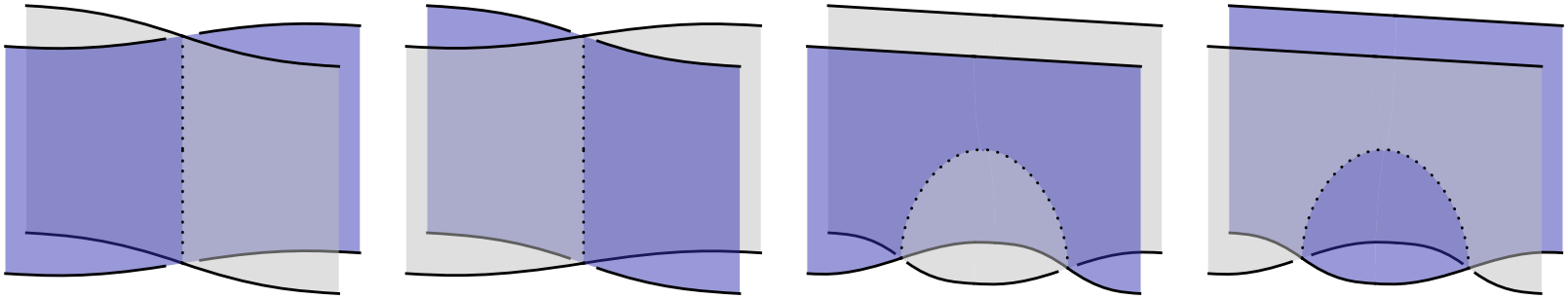
Adjointness

An *adjoint pair* of 1-cells is a *left adjoint*  and a *right adjoint*  such that there exists 2-cells  and  called the *unit* and *counit* respectively, with



Higher commutativity

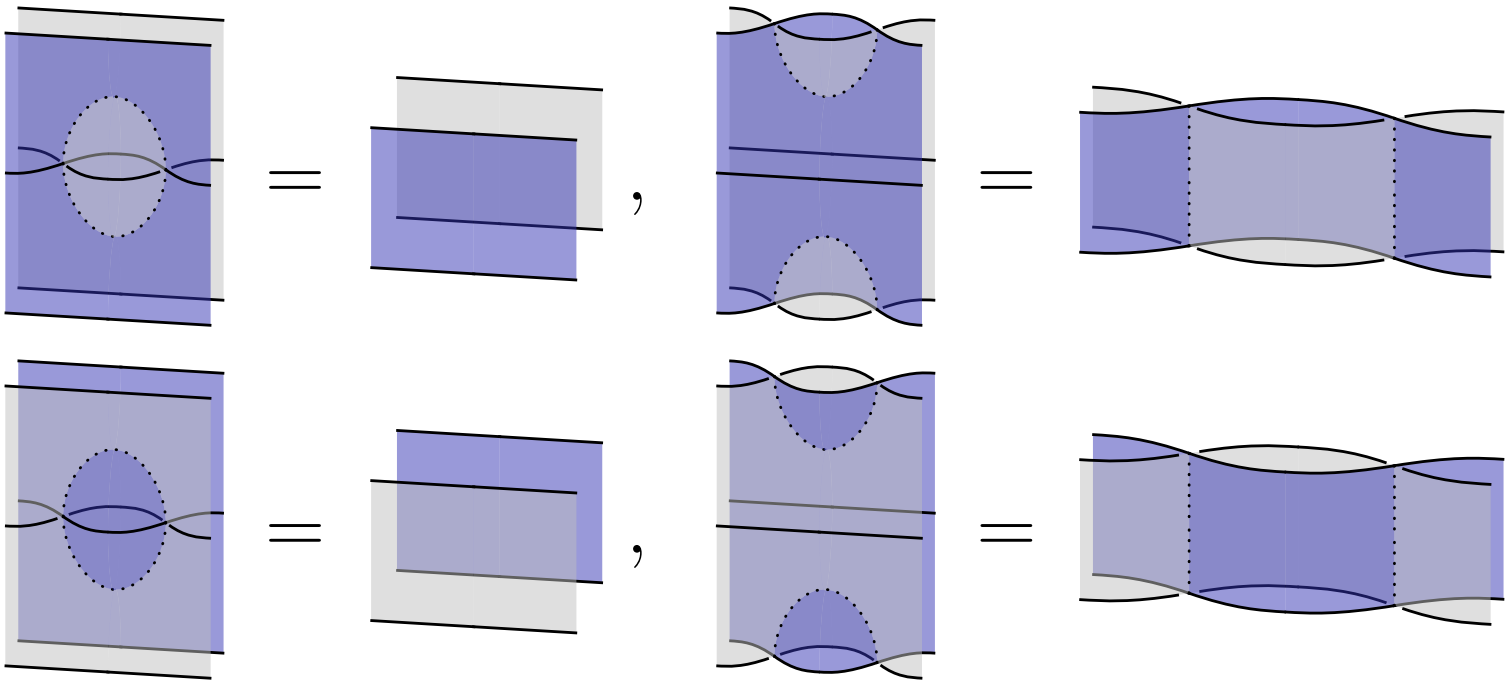
Braided monoidal bicategory has:



& these form an adjoint equivalence...

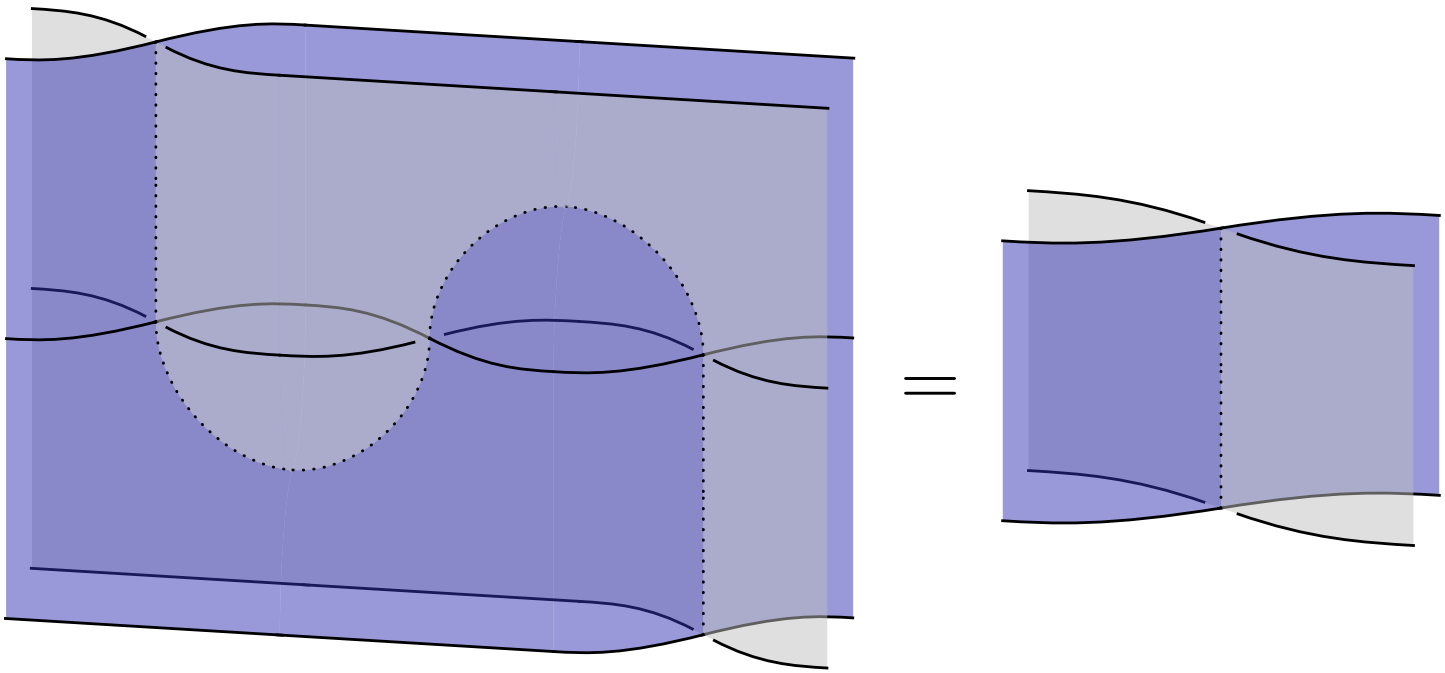
Higher commutativity

Equivalence:



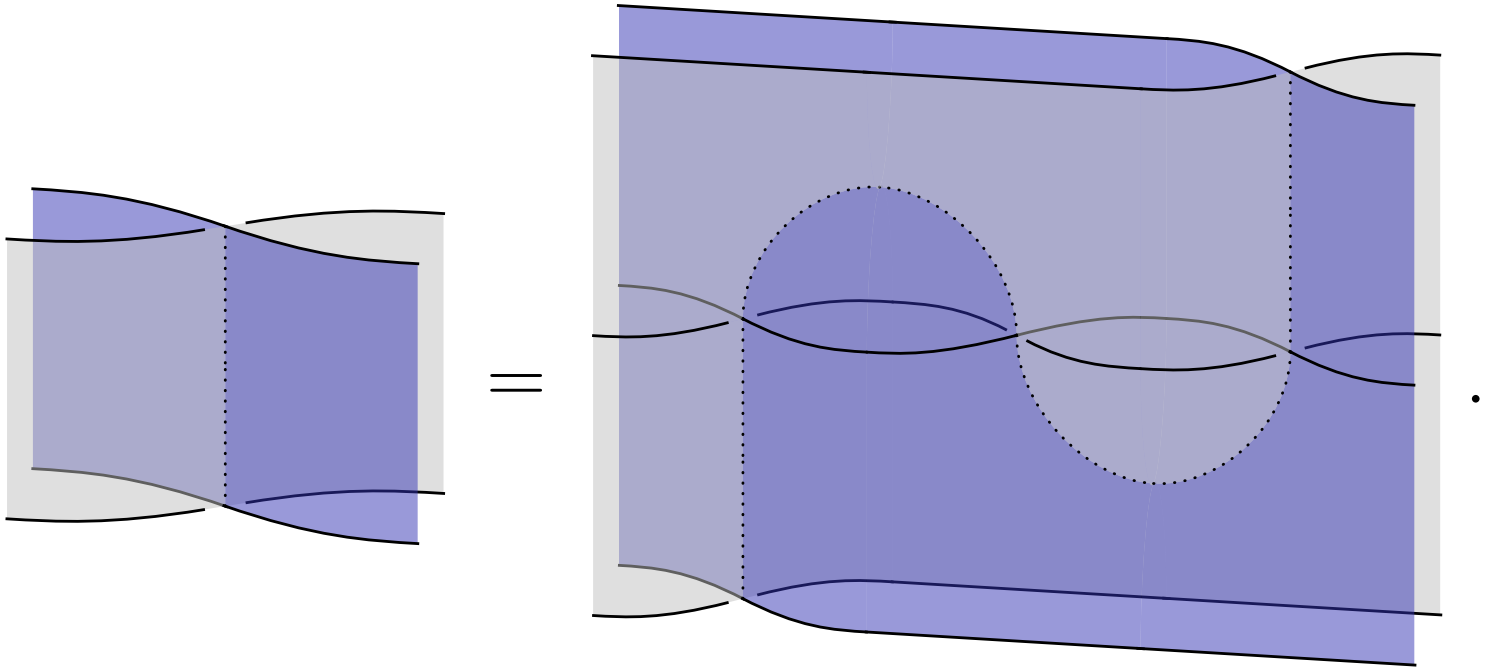
Higher commutativity

Adjointness:



Higher commutativity

Adjointness:



demo of `wiggle.py`

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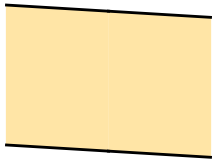
```
from huygens.namespace import color
from huygens.wiggle import Cell0, Cell1, Cell2
yellow = color.rgba(1.0, 0.80, 0.3, 0.5)
Cell0("m", fill=yellow)
```



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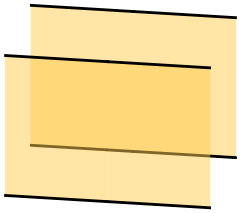
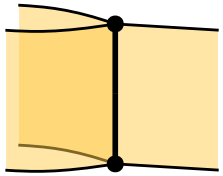
m = Cell0("m", fill=yellow)
m@m
```

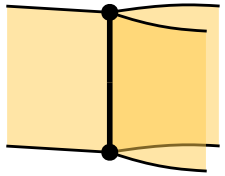


demo of `wiggle.py`

```
m = Cell0("m", fill=yellow)
Cell1(m@m, m)
```

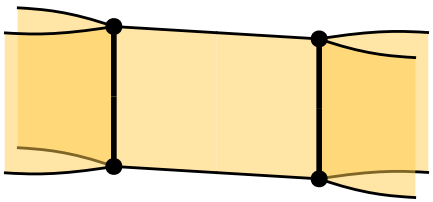


```
Cell1(m, m@m)
```

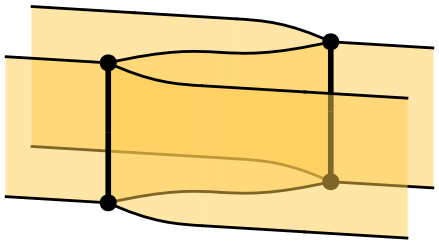


demo of `wiggle.py`

```
m = Cell0("m", fill=yellow)
mm_m = Cell1(m@m, m)
m_mm = Cell1(m, m@m)
mm_m << m_mm
```

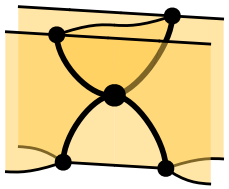


```
(m_mm @ m) << (m @ mm_m)
```



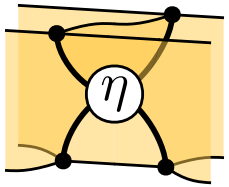
demo of `wiggle.py`

```
m = Cell0("m", fill=yellow)
mm_m = Cell1(m@m, m)
m_mm = Cell1(m, m@m)
src = mm_m << m_mm
tgt = (m_mm @ m) << (m @ mm_m)
Cell2(tgt, src)
```



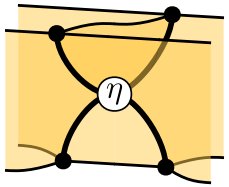
demo of `wiggle.py`

```
...
from huygens.namespace import *
p = path.circle(0, 0, 0.2)
cvs = Canvas()
cvs.fill(p, [white]).stroke(p).
cvs.text(0, 0, r"$\eta$", st_center)
Cell2(tgt, src, pip_cvs=cvs)
```



demo of `wiggle.py`

```
...
from huygens.namespace import *
p = path.circle(0, 0, 0.2)
cvs = Canvas().scale(0.6)
cvs.fill(p, [white]).stroke(p).
cvs.text(0, 0, r"$\eta$", st_center)
Cell2(tgt, src, pip_cvs=cvs)
```



demo of `wiggle.py`

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from huygens.namespace import *
p = path.circle(0, 0, 0.2)
cvs = Canvas().scale(0.6)
cvs.fill(p, [white]).stroke(p).
cvs.text(0, 0, r"$\eta$", st_center)
m @ Cell2(tgt, src, pip_cvs=cvs)
```
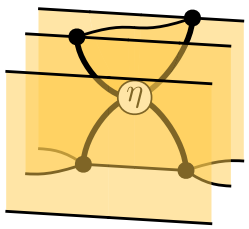
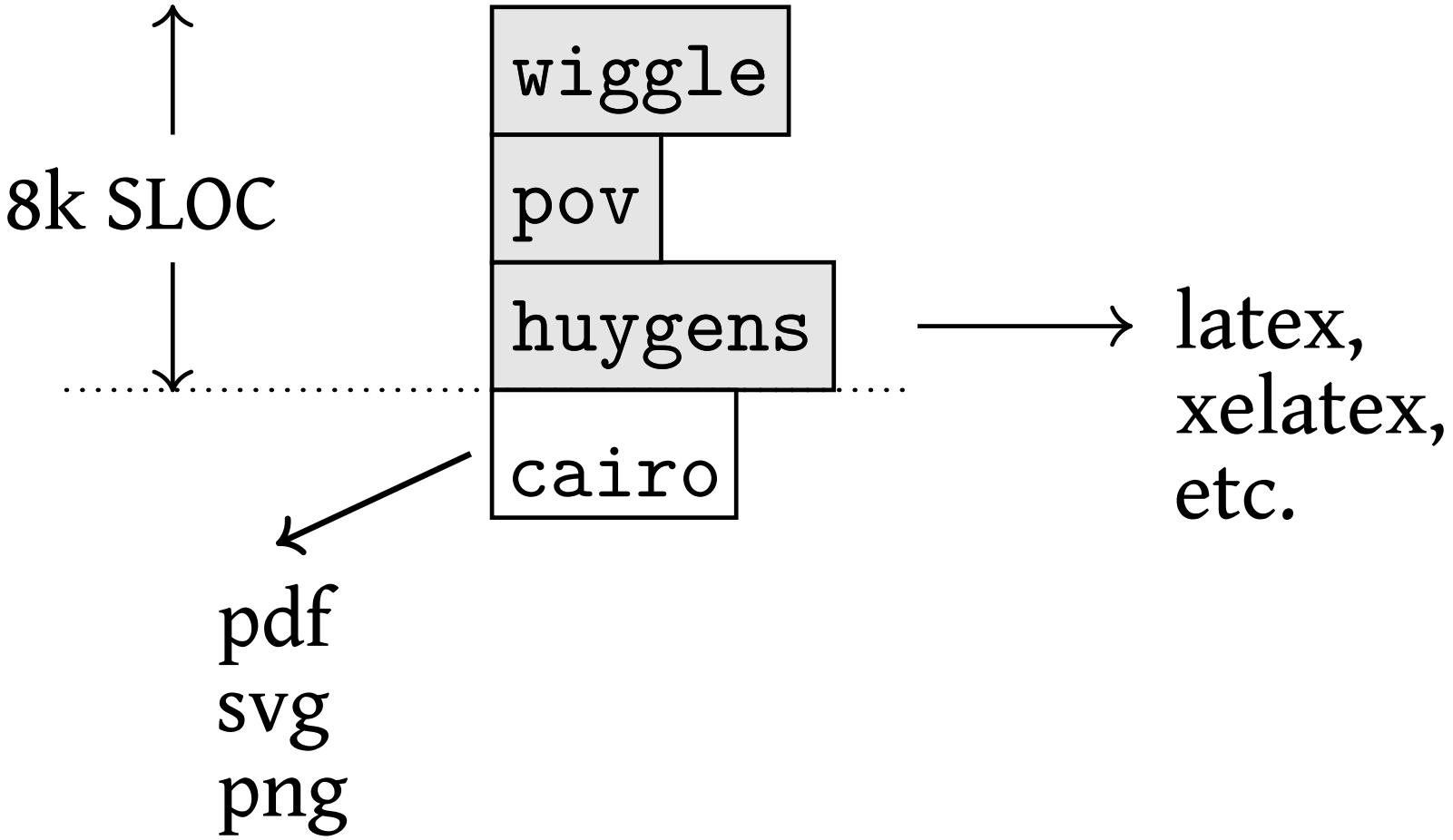



Implementation notes

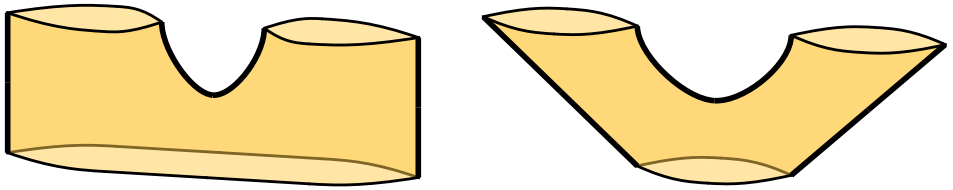


Implementation notes

- reasonable API
  - instance re-use via clones

```
m = Cell0("m", fill=yellow)
m@m
```

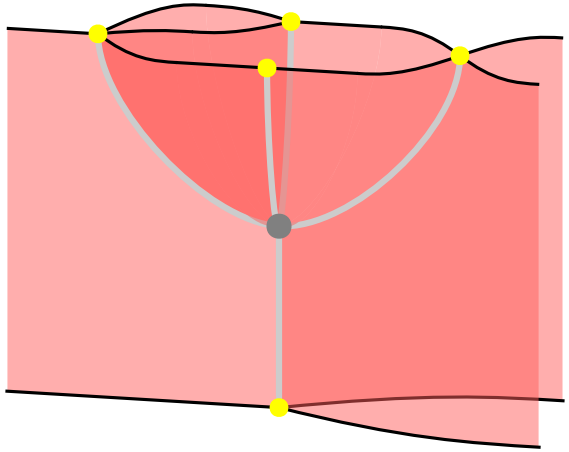
  - linear programming + user constraints



  - a bazillion parameters; customizable defaults

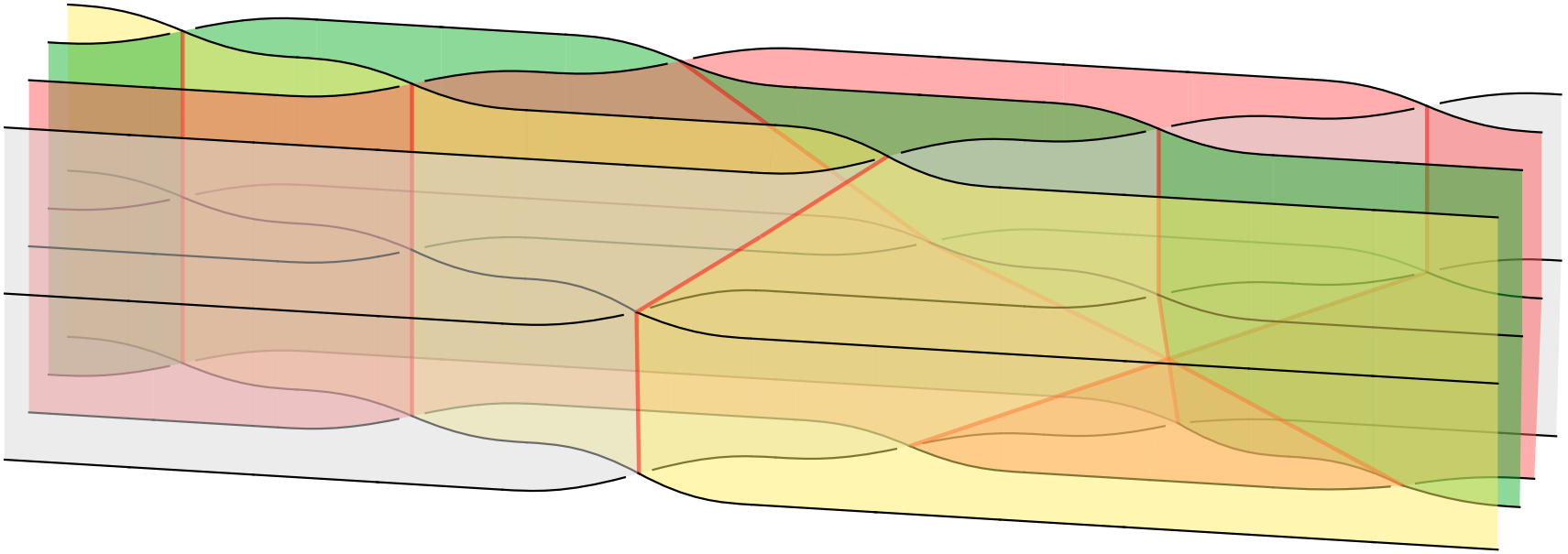
Implementation notes

- vector 3d, not raster 3d
  - alpha blending
  - back to front render order
  - render cones from curvy triangles



Implementation notes

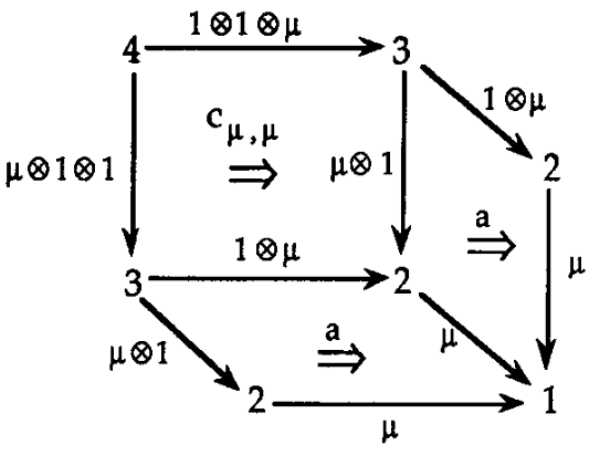
- tree-like data structure
  - lateral, horizontal and vertical composition



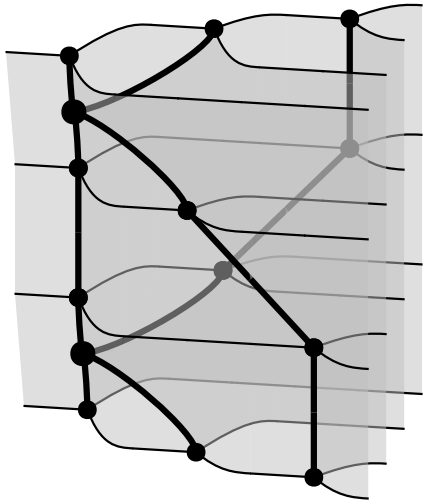
Implementation notes

- API stability
- more documentation
- edge-cases, work-arounds, bugs, ...
  - fixable in principle

The end



How it started



How its going