Higher categories for quantum many-body physics

Pieter Claeys
Dresden

Austen Lamacraft
Cambridge

Jamie Vicary
Cambridge

Tenth Symposium on Compositional Structures (SYCO 10)
University of Edinburgh
20 December 2022

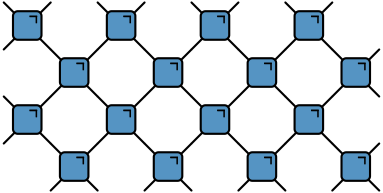
Modelling quantum many-body systems

How should we model the dynamics of a 1d chain of interacting quantum systems?

Modelling quantum many-body systems

How should we model the dynamics of a 1d chain of interacting quantum systems?

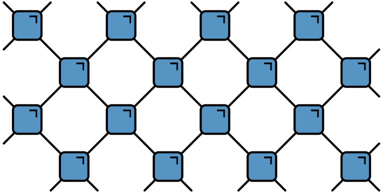
Brickwork circuits give a minimal local dynamical model, built from 2-site unitary gates:



Modelling quantum many-body systems

How should we model the dynamics of a 1d chain of interacting quantum systems?

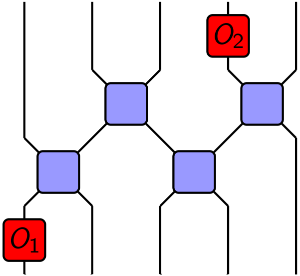
Brickwork circuits give a minimal local dynamical model, built from 2-site unitary gates:



We want to analyze emergent properties of these circuits, and relate them to real systems.

Correlations outside the light cone

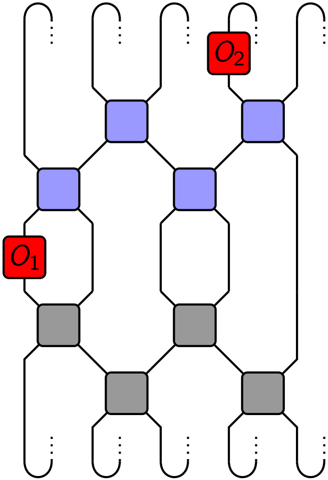
Let's verify this property: trivial correlations for measurements outside the light cone.



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

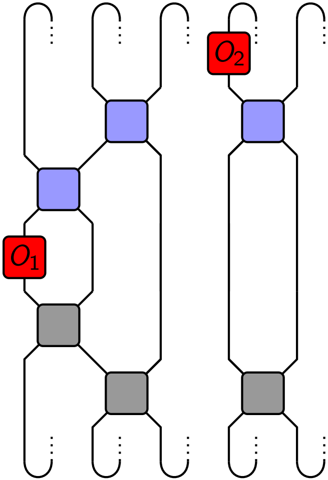
$$\mathrm{Tr}(U^\dagger O_1 U O_2) \quad =$$



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

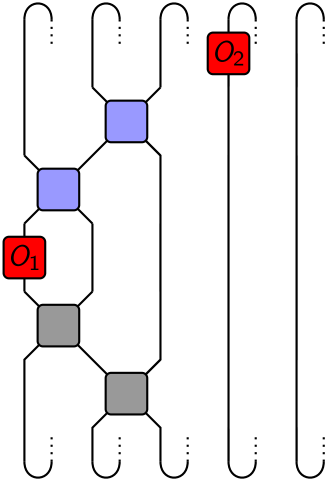
$$\mathrm{Tr}(U^\dagger O_1 U O_2) \quad =$$



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

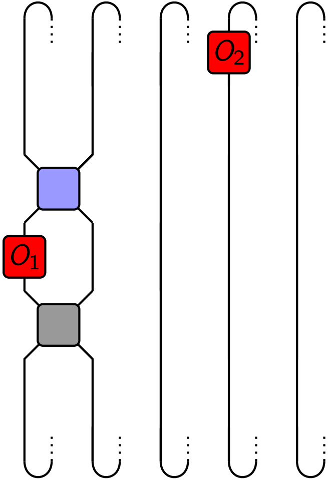
$$\mathrm{Tr}(U^\dagger O_1 U O_2) \quad =$$



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

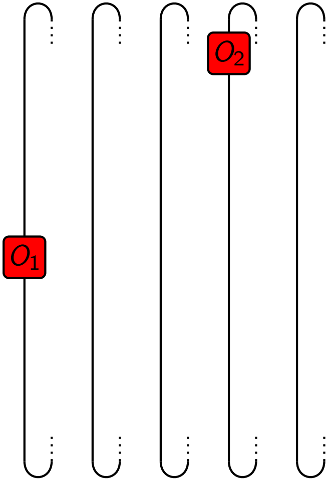
$$\mathrm{Tr}(U^\dagger O_1 U O_2) \quad =$$



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

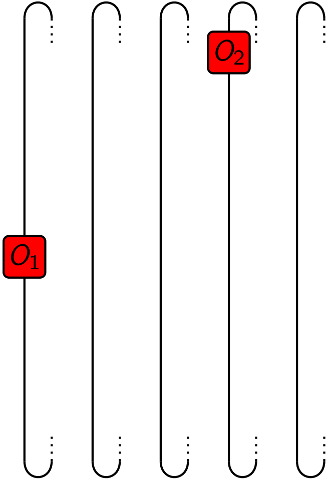
$$\mathrm{Tr}(U^\dagger O_1 U O_2) \quad =$$



Correlations outside the light cone

Let's verify this property: trivial correlations for measurements outside the light cone.

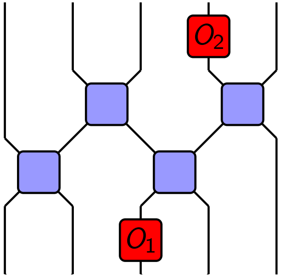
$$\mathrm{Tr}(U^\dagger O_1 U O_2) =$$



This factorizes, so the correlation is trivial.

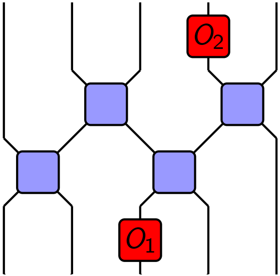
Correlation inside light cone

For measurements *inside* the light cone, the correlations can take any value.

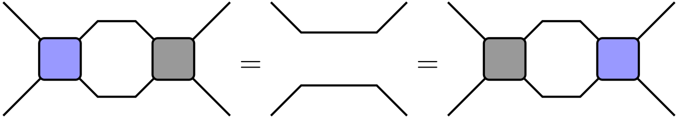


Correlation inside light cone

For measurements *inside* the light cone, the correlations can take any value.

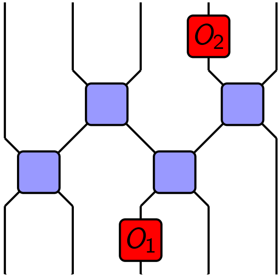


But now suppose the gates are also *horizontally unitarity*:

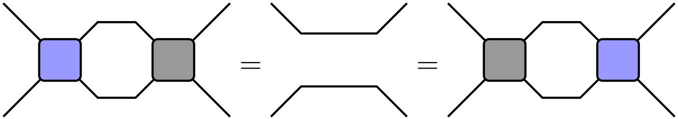


Correlation inside light cone

For measurements *inside* the light cone, the correlations can take any value.



But now suppose the gates are also *horizontally unitarity*:



Then applying a space-time symmetry, the previous proof applies.

So correlations inside the light cone are now also trivial!

Dual unitary brickwork circuits

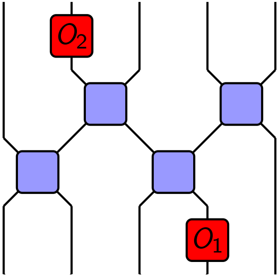
A *dual unitary* is a 2-site gate which is unitary, and also horizontally unitary.

Dual unitary brickwork circuits

A *dual unitary* is a 2-site gate which is unitary, and also horizontally unitary.

For these gates, the only nontrivial correlations are therefore *on the lightcone*.

Information must travel "at light speed".

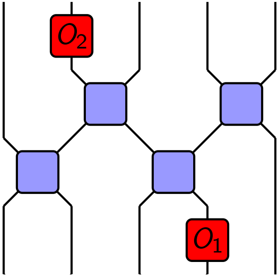


Dual unitary brickwork circuits

A *dual unitary* is a 2-site gate which is unitary, and also horizontally unitary.

For these gates, the only nontrivial correlations are therefore *on the lightcone*.

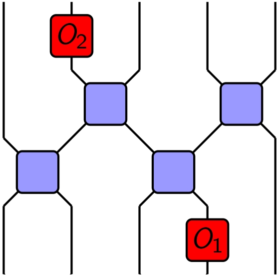
Information must travel "at light speed".



Dual unitarity is currently popular in the physics community (10,000 papers per year!)

Dual unitary brickwork circuits

A *dual unitary* is a 2-site gate which is unitary, and also horizontally unitary.



For these gates, the only nontrivial correlations are therefore *on the lightcone*.

Information must travel "at light speed".

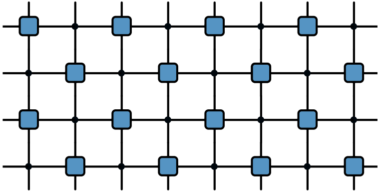
Dual unitarity is currently popular in the physics community (10,000 papers per year!)

As toy models of many-body quantum systems, they have many cool, unusual properties:

- **Exact solvability.** Single-site correlation functions can be efficiently computed.
- **Maximal entanglement velocity.** Entanglement spreads at fastest possible rate.
- **Maximally chaotic.** Ergodic behaviour with same statistics as random matrix models.

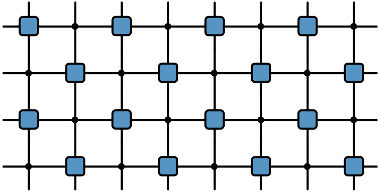
Dual unitary clockwork circuits

Tomaz Prosen recently introduced a different model,
built from 1-site 2-controlled gates:



Dual unitary clockwork circuits

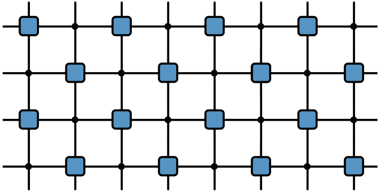
Tomaz Prosen recently introduced a different model,
built from 1-site 2-controlled gates:



He calls them *interaction round-a-face*, but we call them *clockwork circuits*.

Dual unitary clockwork circuits

Tomaz Prosen recently introduced a different model,
built from 1-site 2-controlled gates:

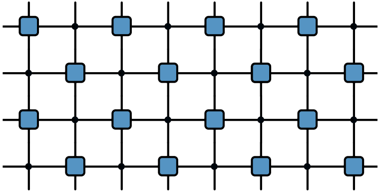


He calls them *interaction round-a-face*, but we call them *clockwork circuits*.

Prosen gives a new definition of *dual unitarity* for these circuits.

Dual unitary clockwork circuits

Tomaz Prosen recently introduced a different model,
built from 1-site 2-controlled gates:



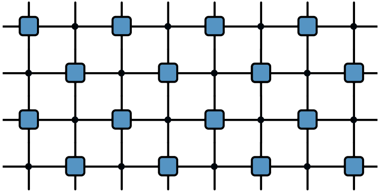
He calls them *interaction round-a-face*, but we call them *clockwork circuits*.

Prosen gives a new definition of *dual unitarity* for these circuits.

He then shows they share all the good properties of dual unitary brickwork circuits!

Dual unitary clockwork circuits

Tomaz Prosen recently introduced a different model,
built from 1-site 2-controlled gates:



He calls them *interaction round-a-face*, but we call them *clockwork circuits*.

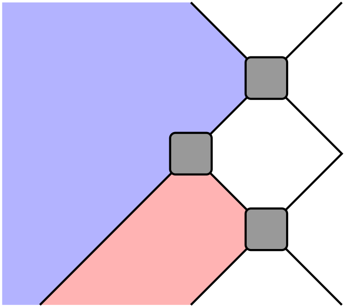
Prosen gives a new definition of *dual unitarity* for these circuits.

He then shows they share all the good properties of dual unitary brickwork circuits!

This is surprising — their structure is very different. **How can we understand this?**

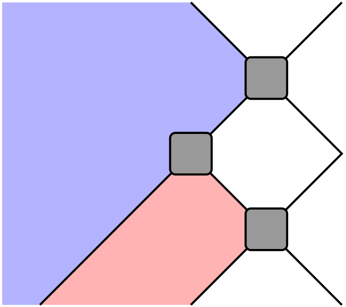
Shaded tensor networks

A *shaded tensor network* is a planar string diagram, where some of the regions are shaded.



Shaded tensor networks

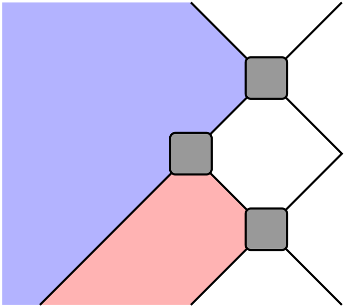
A *shaded tensor network* is a planar string diagram, where some of the regions are shaded.



It is a string diagram for a subcategory of the **2Hilb**, the 2-category of 2–Hilbert spaces.

Shaded tensor networks

A *shaded tensor network* is a planar string diagram, where some of the regions are shaded.

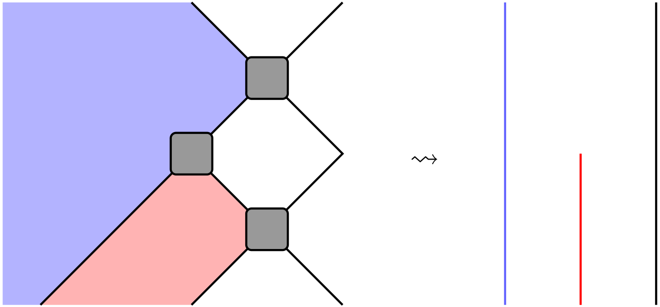


It is a string diagram for a subcategory of the **2Hilb**, the 2-category of 2–Hilbert spaces.

To express the linear data of a shaded tensor network in ordinary circuit notation:

Shaded tensor networks

A *shaded tensor network* is a planar string diagram, where some of the regions are shaded.



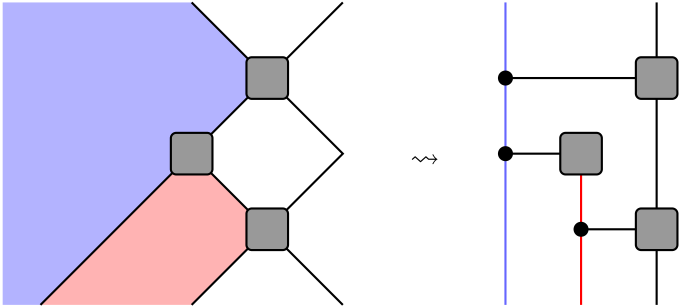
It is a string diagram for a subcategory of the **2Hilb**, the 2-category of 2–Hilbert spaces.

To express the linear data of a shaded tensor network in ordinary circuit notation:

- Regions become wires

Shaded tensor networks

A *shaded tensor network* is a planar string diagram, where some of the regions are shaded.



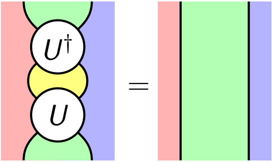
It is a string diagram for a subcategory of the **2Hilb**, the 2-category of 2–Hilbert spaces.

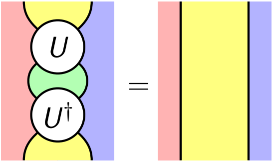
To express the linear data of a shaded tensor network in ordinary circuit notation:

- Regions become wires
- Vertices are *controlled* by the wires of adjacent regions

Biunitarity

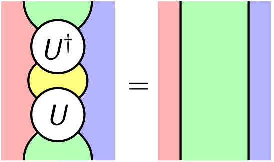
In the world of shaded tensor networks, a 4-valent vertex U can be vertically unitary:

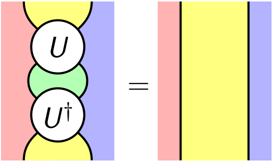




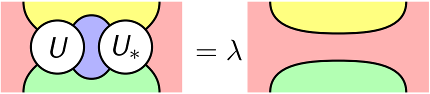
Biunitarity

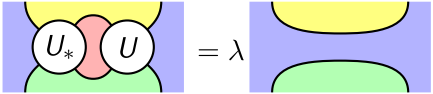
In the world of shaded tensor networks, a 4-valent vertex U can be vertically unitary:





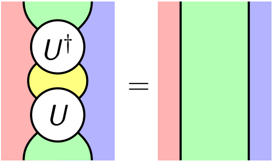
It can also be *horizontally unitary*:

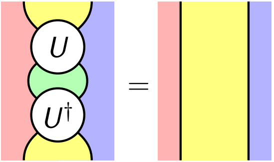




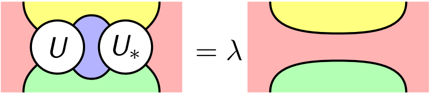
Biunitarity

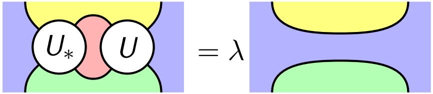
In the world of shaded tensor networks, a 4-valent vertex U can be vertically unitary:





It can also be *horizontally unitary*:





A 4-valent map is *biunitary* when it is vertically and horizontally unitary.

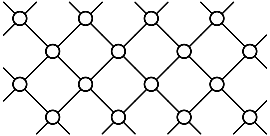
Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

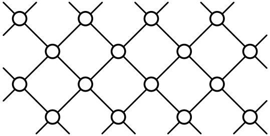
In the unshaded case, these are just ordinary dual unitary brickwork circuits:



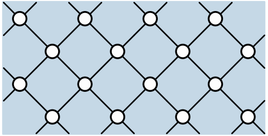
Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

In the unshaded case, these are just ordinary dual unitary brickwork circuits:



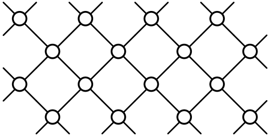
Let's see what happens in the shaded case.



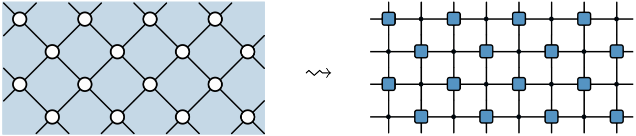
Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

In the unshaded case, these are just ordinary dual unitary brickwork circuits:



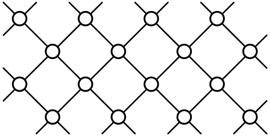
Let's see what happens in the shaded case. We get Prosen's circuits!



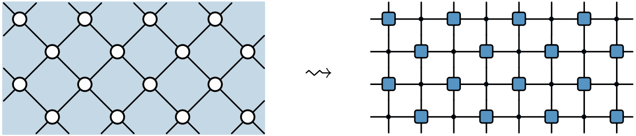
Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

In the unshaded case, these are just ordinary dual unitary brickwork circuits:



Let's see what happens in the shaded case. We get Prosen's circuits!

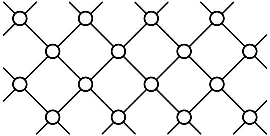


So brickwork and clockwork circuits have a *unified description* using the shaded calculus.

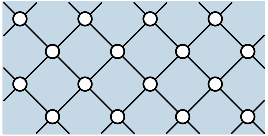
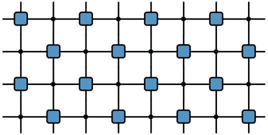
Homogeneous biunitary circuits

Now let's look at *biunitary circuits*, with all shading the same.

In the unshaded case, these are just ordinary dual unitary brickwork circuits:



Let's see what happens in the shaded case. We get Prosen's circuits!

 $\rightsquigarrow$ 

So brickwork and clockwork circuits have a *unified description* using the shaded calculus.

This also recovers Prosen's definition of dual unitarity for clockwork circuits.

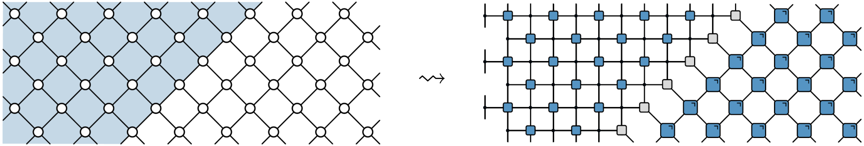
Biunitary circuits — dynamical boundary

Our more general language lets us try new things. What happens if we drop homogeneity?

Biunitary circuits — dynamical boundary

Our more general language lets us try new things. What happens if we drop homogeneity?

The simplest possible structure is a *dynamical boundary*:

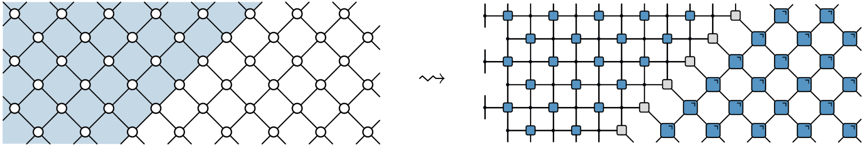


This boundary moves left-to-right, separating clockwork and brickwork circuits.

Biunitary circuits — dynamical boundary

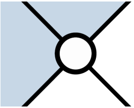
Our more general language lets us try new things. What happens if we drop homogeneity?

The simplest possible structure is a *dynamical boundary*:



This boundary moves left-to-right, separating clockwork and brickwork circuits.

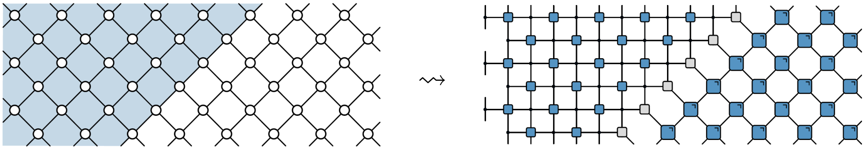
At the boundary we require a new sort of vertex, with two shaded and two unshaded regions.



Biunitary circuits — dynamical boundary

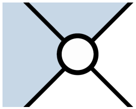
Our more general language lets us try new things. What happens if we drop homogeneity?

The simplest possible structure is a *dynamical boundary*:



This boundary moves left-to-right, separating clockwork and brickwork circuits.

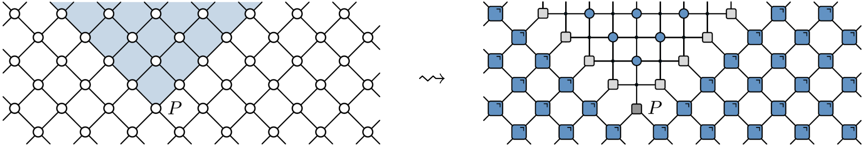
At the boundary we require a new sort of vertex, with two shaded and two unshaded regions.



We can classify these. They are exactly *quantum Latin squares*: grids of elements of a Hilbert space, with every row and column giving an orthonormal basis.

Biunitary circuits — boundary creation

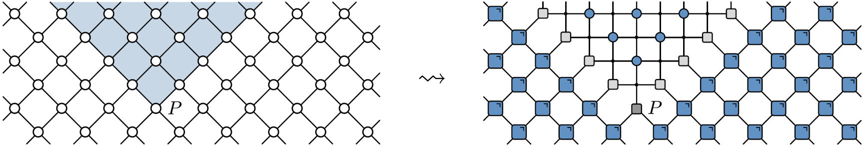
We can create these boundaries dynamically:



Here a new clockwork region is created within an existing brickwork region.

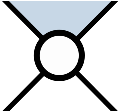
Biunitary circuits — boundary creation

We can create these boundaries dynamically:



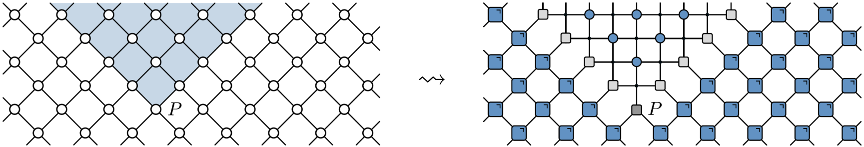
Here a new clockwork region is created within an existing brickwork region.

At point P we encounter another new vertex type, with one shaded region.



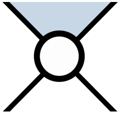
Biunitary circuits — boundary creation

We can create these boundaries dynamically:



Here a new clockwork region is created within an existing brickwork region.

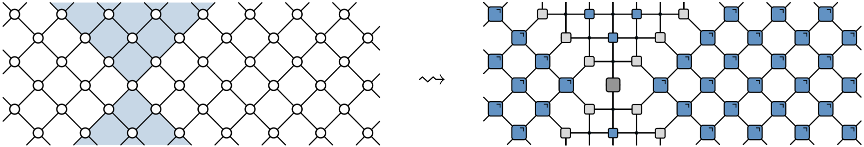
At point P we encounter another new vertex type, with one shaded region.



These are known to correspond to *unitary error bases*, defined as orthogonal and complete families of unitary matrices. (For example, the Pauli matrices.)

Biunitary circuits — boundary reflection

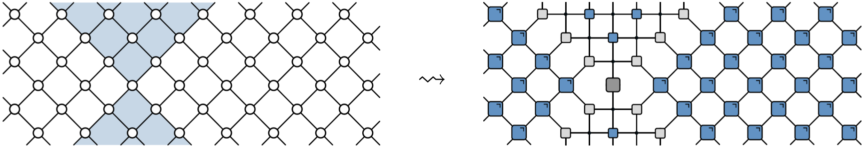
Boundaries can also reflect off each other:



Here a clockwork region contracts to zero width, then expands again.

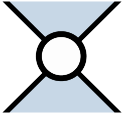
Biunitary circuits — boundary reflection

Boundaries can also reflect off each other:



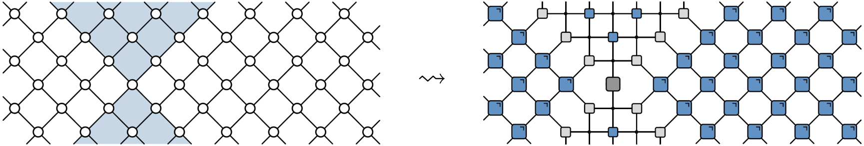
Here a clockwork region contracts to zero width, then expands again.

The central point has a vertex with two non-adjacent shaded regions.



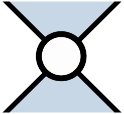
Biunitary circuits — boundary reflection

Boundaries can also reflect off each other:



Here a clockwork region contracts to zero width, then expands again.

The central point has a vertex with two non-adjacent shaded regions.



These are known to correspond to *Hadamard matrices*, unitary matrices where every coefficient has the same absolute value. (This was discovered by Vaughan Jones.)