# A layout algorithm for higher-dimensional string diagrams

Calin Tataru

University of Cambridge

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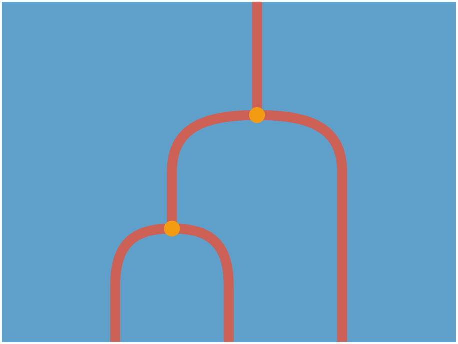
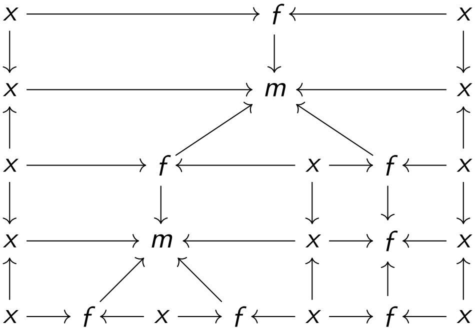
# Homotopy.io

- *Homotopy.io* is a proof assistant for higher category theory.
- It lets you build terms in finitely-presented $n$-categories.
- Terms have a direct geometrical representation.

# Example of 2-diagram



# Corresponding zigzag diagram



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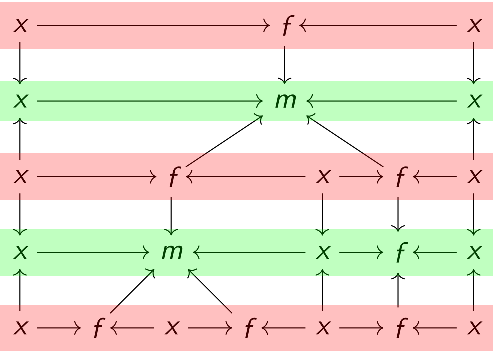
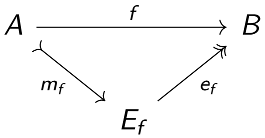


# Mono-epi factorization

**Poset** admits a mono-epi factorization system:




where $E_f = A \sqcup (B \setminus f[A])$, $m_f$ is the canonical inclusion, $e_f = [f, \mathrm{id}]$.

# Injectification

## Definition

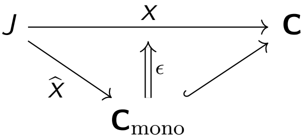
Given a diagram in a category $\mathbf{C}$,

$$X : J \to \mathbf{C}$$

an *injectification* is defined to be a diagram

$$\widehat{X} : J \to \mathbf{C}_{\mathrm{mono}}$$

equipped with a pointwise epi natural transformation



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- If not, $\widehat{X}$ and $\epsilon$ must already be defined on $J{\downarrow}i$.

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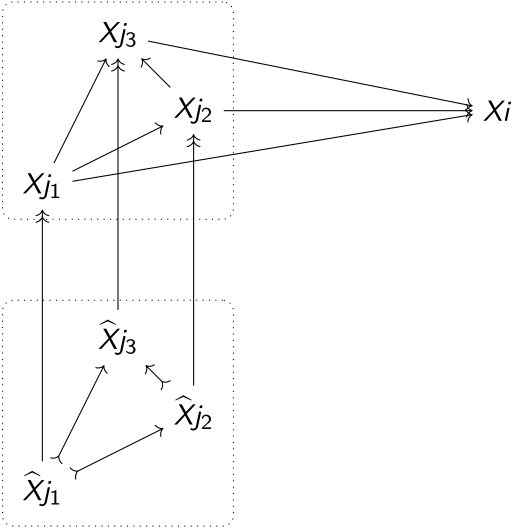

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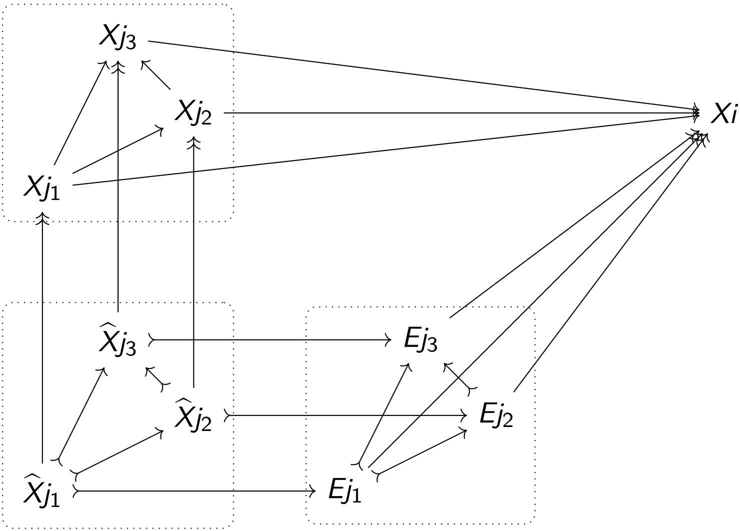

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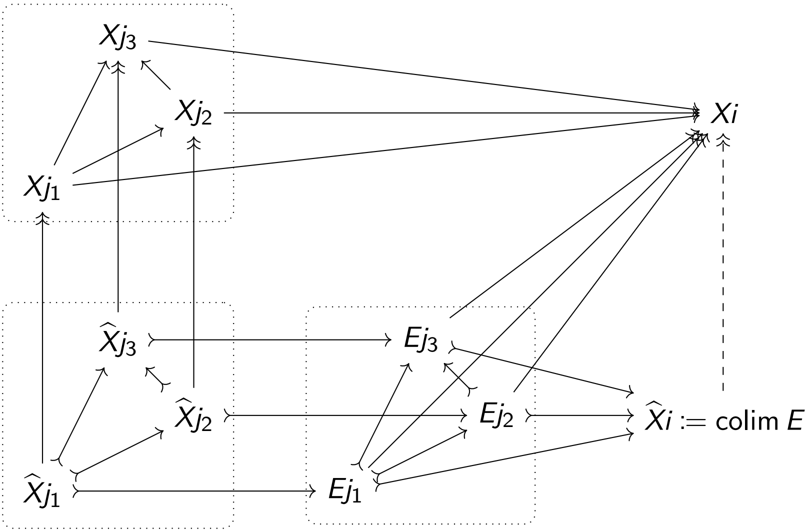
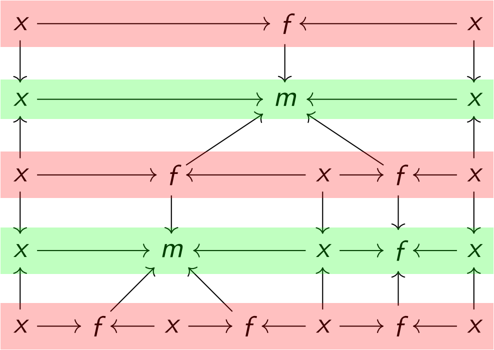


# Layout



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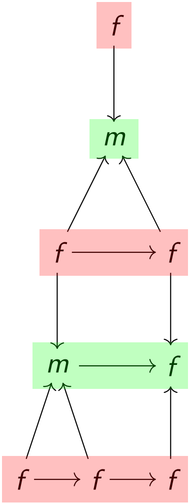

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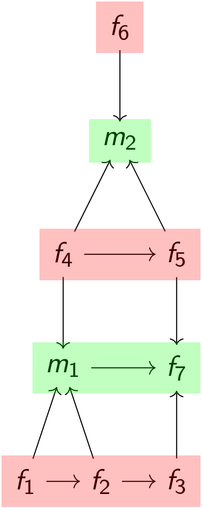

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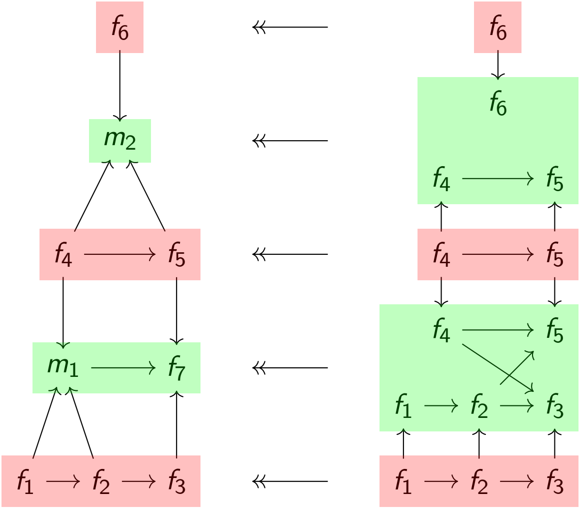

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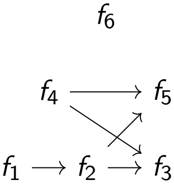

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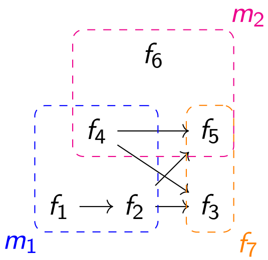

# Distance constraints

$$f_2 - f_1 \geq 1$$
$$f_3 - f_2 \geq 1$$
$$f_3 - f_4 \geq 1$$
$$f_5 - f_2 \geq 1$$
$$f_5 - f_4 \geq 1$$

# Fair averaging constraints (strict)

$$\begin{aligned}
\tfrac{1}{2}(f_1 + f_2) - f_4 &= 0 \\
f_3 - f_5 &= 0 \\
\tfrac{1}{2}(f_4 + f_5) - f_6 &= 0
\end{aligned}$$

# Fair averaging constraints (weak)

$$\begin{array}{ll} \text{minimize} & c_1 + c_2 + c_3 \\ \text{subject to} & \left|\frac{1}{2}(f_1 + f_2) - f_4\right| \leq c_1 \\ & |f_3 - f_5| \leq c_2 \\ & \left|\frac{1}{2}(f_4 + f_5) - f_6\right| \leq c_3 \end{array}$$

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- TikZ export (finally!)