

F* and Meta-F*

Formal Verification,
Language Extensibility, and
Proof automation

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Microsoft®
Research

As applied in Project Everest



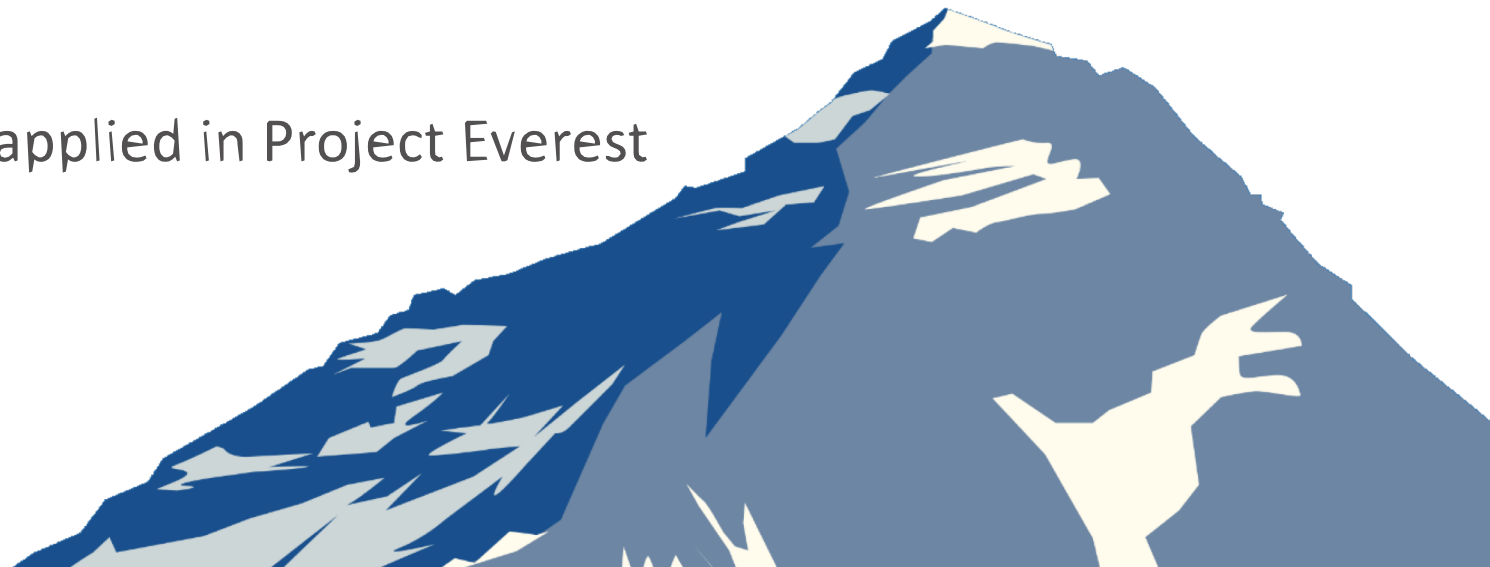
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<https://fstar-lang.github.io>

<https://project-everest.github.io/>



Goals of this lecture

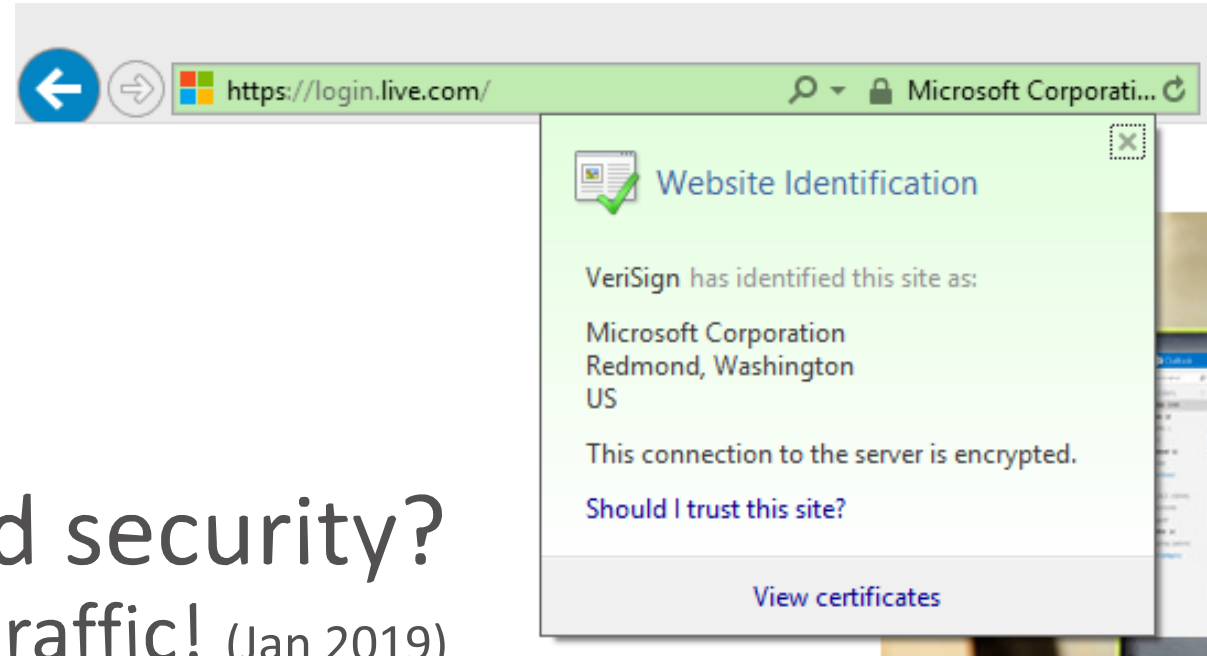
- **Project Everest:** TLS verification at scale, using...
- **F*:** dependently-typed programming (context)
- **Meta-F*:** bridging interactive and automated theorem proving

Background on Project Everest



TLS: Transport Layer Security

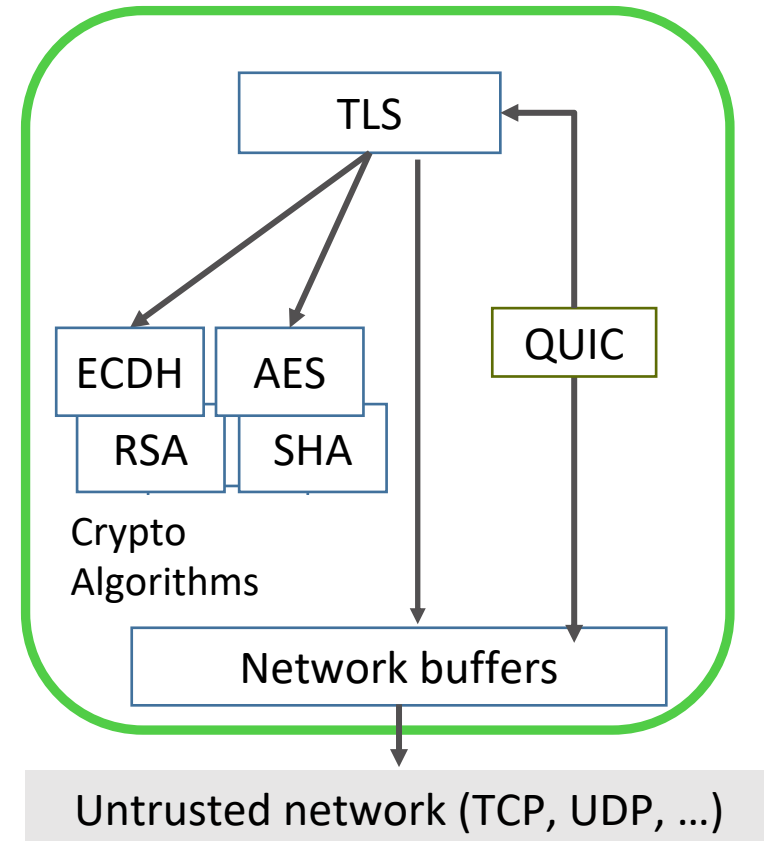
- Most widely deployed security?
Now 73% of the internet traffic! (Jan 2019)
- Web, cloud, email, VoIP, 802.1x, VPNs, ...



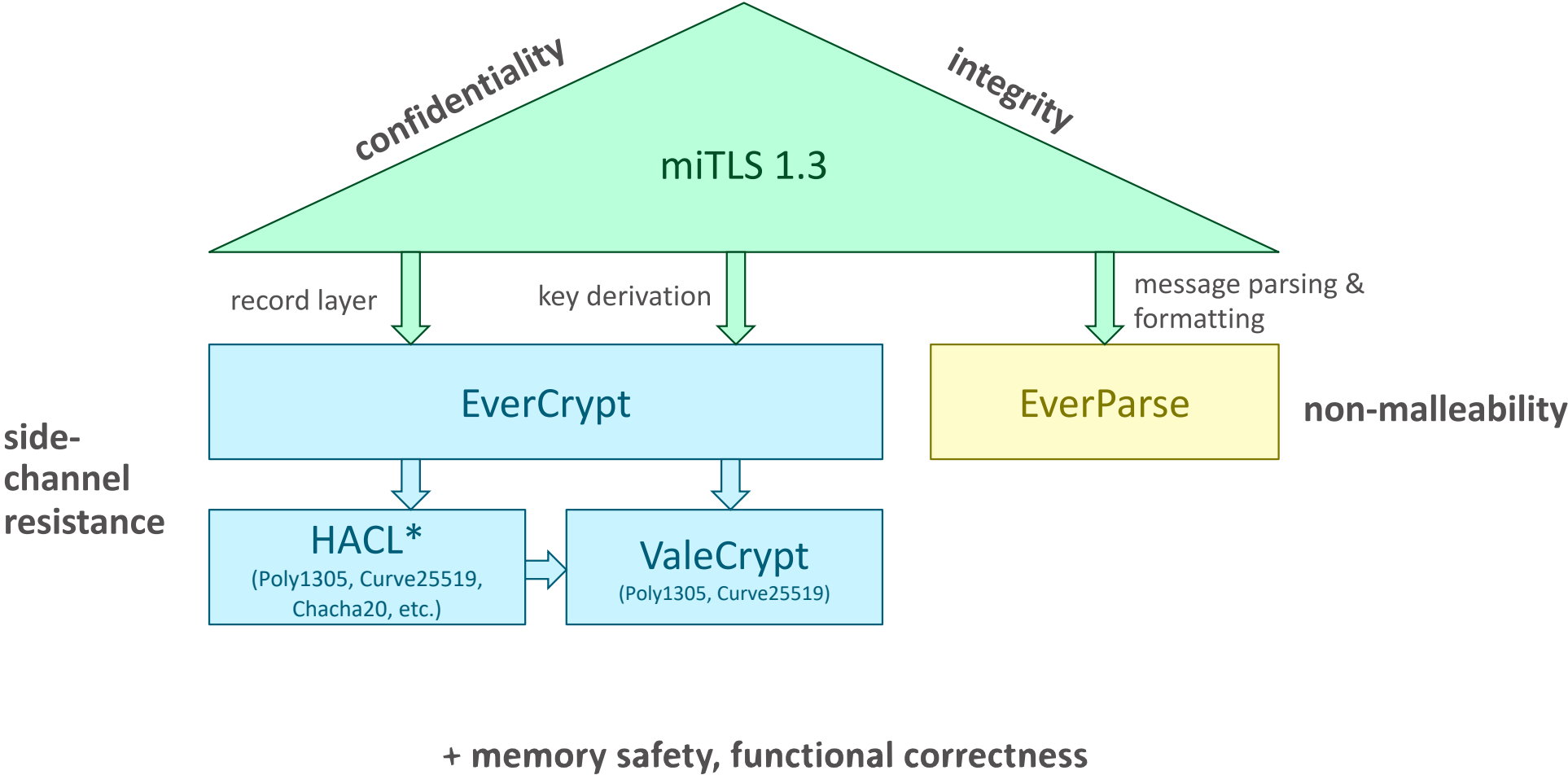
Project Everest

Verified Secure Components in the TLS Ecosystem

- Strong verified security
- Widespread deployment
- Trustworthy, usable tools
- Growing expertise in high-assurance software development
- Open source

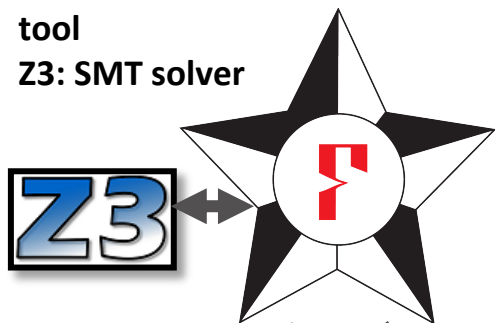


Everest: a verification pyramid (600,000 kloc and counting)



Verification Tools and Methodology

F*: A general purpose
programming
language
and verification
tool
Z3: SMT solver



Math spec in F*

poly1305_mac computes a
polynomial in $GF(2^{130-5})$,
storing the result in `tag`,
and not modifying anything
else

```
val poly1305_mac: tag:nbytes 16 →
  len:u32 →
  msg:nbytes len{disjoint tag msg} →
  key:nbytes 32 {disjoint msg key ∧ disjoint tag key} →
  ST unit
(requires (λ h → msg ∈ h ∧ key ∈ h ∧ tag ∈ h))
(ensures (λ h0 _ h1 →
  let r=Spec.clamp h0.[sub key 0 16] in
  let s=h0.[sub key 16 16] in
  modifies {tag} h0 h1 ∧
  h1.[tag] == Spec.mac_1305 (encode_bytes h0.[msg]) r s))
```

kreMLin

Compiler from
(a subset of)
F* to C

C

ASM

Efficient ASM
implementation
overhead

```
procedure{:quick}{:public}{:exportSpecs} Poly1305(
  ghost ctx_b:buffer64,
  void inp_b:buffer64,
  ghost len_in:nat64,
  ghost finish_in:nat64)
poly1305_mac(uint8_t *tag, uint32_t len, uint8_t *msg, uint8_t
*key)
{
  ctx @= rdi; inp @= rsi; len @= rdx; finish @= rcx;
  h0 @= r14; h1 @= rbx; h2 @= rbp;
  ctx_in := (if will then rdx else ctx);
  uint64_t tmp = 0;
  if !acc then inp;
  n := 0x1_0000_0000_0000_0000;
  uint64_t r = tmp + (uint32_t)5;
  p := n * n * 4 * 5;
  modify uint8_t s[16] = { 0 };
  crypto_symmetric_poly1305_poly1305_init(r, s, key);
  ensure crypto_symmetric_poly1305_poly1305_process(msg, len, acc,
r); finish_in == 0 ==>
  crypto_symmetric_poly1305_poly1305_finish(tag, acc, s);
}
```

Everest in Action, so far

Production deployments of Everest Verified Cryptography



WinQUIC: Delivered Everest TLS 1.3 and crypto stack to Windows Networking, in the latest Windows developer previews



Mozilla NSS runs Everest verified crypto for several core algorithms



Everest verified crypto in the Linux kernel via WireGuard secure VPN (soon)

So what is this F* thing anyway?

A programming language

A proof assistant

A program verification tool

Two camps of program verification tools

Interactive proof assistants

Coq,
Isabelle,
Agda,
Lean, PVS, ...

CompCert,
seL4,
Bedrock,
4 colors

air

gap

Semi-automated verifiers of imperative programs

Dafny,
FramaC,
Why3

Verve,
IronClad,
miTLS
Vale

- **In the left corner:** Very expressive dependently-typed logics, but only purely functional programming
- **In the right:** effectful programming, SMT-based automation, but only first-order logic

F*: Bridging the gap

- **Functional programming language with effects**
 - Like OCaml, Haskell, F#, ...
 - Compiles to OCaml or F#
 - A subset of F* compiled to C (with manual control over memory management)
- **With an expressive core dependent type theory**
 - Like Coq, Agda, Lean, ...
- **Semi-automated verification using SMT**
 - Like Dafny, Vcc, Liquid Haskell, ...
- **In-language extensibility and proof automation using metaprograms**

A first taste

- Write ML-like code

```
let rec factorial n =  
  if n = 0 then 1  
  else n * factorial (n - 1)
```

- Give it a specification, claiming that `factorial` is a total function from non-negative to positive integers.

```
val factorial: n:int{n >= 0} -> Tot (i:int{i >= 1})
```

- Ask F^* to check it

```
fstar factorial.fst  
Verified module: Factorial  
All verification conditions discharged successfully
```

```
fstar factorial.fst  
Verified module: Factorial  
All verification conditions discharged successfully
```

F* builds a typing derivation of the form:

$$\Gamma_{\text{prelude}} \vdash \text{let factorial } n = e : t \Leftarrow \phi$$

- In a context Γ_{prelude} including definitions of F* primitives
- The program `let factorial n = e` has type t , given the validity of a logical formula ϕ
- ϕ is passed to **Z3** (an automated theorem prover/SMT solver) to check for validity
- If the check succeeds, then, from the metatheory of F*, the program is safe at type t

The functional core of F*

- Recursive functions

```
val factorial : int -> int
let rec factorial n = (if n = 0 then 1 else n * (factorial (n - 1)))
```

- Inductive datatypes (immutable) and pattern matching

```
type list (a:Type) =
  | Nil : list a
  | Cons : hd:a -> tl:list a -> list a

val map : ('a -> 'b) -> list 'a -> list 'b
let rec map f x = match x with
  | [] -> []
  | h :: t -> f h :: map f t
```

- Lambdas (unnamed, first-class functions)

```
map (fun x -> x + 42) [1;2;3]
```

Refinement types

```
type nat = x:int{x>=0}
```

- Refinements introduced by type annotations (code unchanged)

```
val factorial : nat -> nat  
let rec factorial n = (if n = 0 then 1 else n * (factorial (n - 1)))
```

- Logical obligations discharged by SMT (simplified)

```
n >= 0, n <> 0 |= n - 1 >= 0  
n >= 0, n <> 0, factorial (n - 1) >= 0 |= n * (factorial (n - 1)) >= 0
```

- Refinements eliminated by **subtyping**: `nat <: int`

```
let i : int = factorial 42  
let f : x:nat{x>0} -> int = factorial
```

Beyond pure code: weakest preconditions

- Reflects the semantics of an effectful (stateful) construct logically
- Reduces program verification to verifying the validity of a logical formula (sent to SMT)
- A weakest precondition is a **predicate transformer** from post-condition to precondition
- One WP per language construct

Beyond pure code: weakest preconditions

Examples of weakest preconditions:

- $WP(\text{return } e) Q = Q e$
- $WP(\text{bind } e1 \text{ as } x \text{ in } e2) Q = WP e1 (\lambda x \rightarrow WP e2 Q x)$

For any effectful expression, compute the WP.

Top-level functions are checked against initial precondition and desired post-condition.

Beyond Pure Code

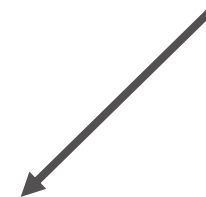
Effects

- Programmers model effects with monads
 - $\text{st } a = s \rightarrow a * s$
- F^* derives a WP calculus for use with that monad
 - Also known as *Dijkstra* monads
 - $\text{st_post } a = a * s \rightarrow \text{prop}$
 - $\text{st_pre} = s \rightarrow \text{prop}$
 - $\text{wp_st } a = \text{st_post } a \rightarrow \text{st_pre}$
- Along with a computation type for effectful terms
 - Computations indexed by their own WPs: $\text{STATE } a \text{ (wp : wp_st } a)$

Effectful programs with Hoare-style Specifications

```
let invert (r:ref int)
  : STExn unit
  (requires fun h0 -> True)
  (ensures fun h0 v h1 -> no_exn v ==> h0.[r] <> 0 /\ h1.[r] = 1 / h0.[r])
=
  let x = !r in
  if x = 0 then
    raise Division_by_zero
  else r := 1 / x
```

only pure code in pre
and post-conditions



Effectful programs with Hoare-style Specifications

```
let invert (r:ref int)
  : STExn unit
  (requires fun h0 -> h0.[r] <> 0)
  (ensures fun h0 v h1 -> no_exn v /\ h1.[r] = 1 / h0.[r])
=
  let x = !r in
  if x = 0 then
    raise Division_by_zero
  else r := 1 / x
```

Exploiting Expressiveness & Extensibility

Low*: A subset of F* that compiles to C

- Embed within F* a CompCert C-like memory model
 - Low*: An effect for stateful programs manipulating a C-like memory model
 - Programs in the Low* effect are extracted to C by KreMLin, F*'s C backend
- Separate memory region for heaps and stacks
 - A region is a heterogeneous partial map
 - `region = addr -> option (a:Type & a)`
- With libraries modeling mutable arrays, pointers, structs, unions

Low* to C

And to support compilation to C, in nearly 1-1 correspondence, for auditability of our generated code

Designed to allow manipulating a C-like view of memory

Low*

COMPILED

C

```
1 let chacha20
2   (len: uint32{len ≤ blocklen}) (output: bytes{len = output.length})
3   (key: keyBytes) (nonce: nonceBytes{disjoint [output; key; nonce]}) (counter: uint32) :
4     Stack unit (requires (λ m0 → output ∈ m0 ∧ key ∈ m0 ∧ nonce ∈ m0)) Erased
5               (ensures (λ m0 _m1 → modifies1 output m0 m1)) = specification
6   push_frame ();
7   let state = Buffer.create 0ul 32ul in
8   let block = Buffer.sub state 16ul 16ul in
9   chacha20_init block key nonce counter;
10  chacha20_update output state len;
11  pop_frame ()
```

Stack allocation

Pointer arithmetic

```

let encrypt (#i:id) (e:writer i) (ad:bytes) (l:plainLen) (p:plain i l)
: ST (cipher i l)
  (requires (λ h0 →
    l ≤ max_TLSPlaintext_fragment_length ∧
    sel h0 (ctr e.counter) < max_ctr))
  (ensures (λ h0 c h1 →
    modifies [e.log_region] h0 h1 ∧
    h1 `HS.contains` (ctr e.counter) ∧
    sel h1 (ctr e.counter) == sel h0 (ctr e.counter) + 1 ∧
    (authId i ⇒
      let log = ilog e.log in
      let ent = Entry l c p in
      let n = Seq.length (sel h0 log) in
      h1 `HS.contains` log ∧
      witnessed (at_least n ent log) ∧
      sel h1 log == snoc (sel h0 log) ent))) =

let h0 = get() in
let ctr = ctr e.counter in
HST.recall ctr; //lemma
let text = if safeId i then create_ l 0z else repr i l p in
let n = !ctr in
lemma_repr_bytes_values n; //lemma
let nb = bytes_of_int (AEAD.noncelen i) n in
let iv = AEAD.create_nonce e.aead nb in
lemma_repr_bytes_values (length text); //lemma
let c = AEAD.encrypt e.aead iv ad text in
if authId i then
  begin
    let ilog = ilog e.log in
    HST.recall ilog; //lemma
    let ictr: ideal_ctr e.region i ilog = e.counter in
    testify_seqn ictr;
    write_at_end ilog (Entry l c p);
    HST.recall ictr; //lemma
    increment_seqn ictr;
    HST.recall ictr //lemma
  end
else
  ctr := n + 1;
c

```



Dependently typed specs in the style of Hoare Type Theory / Coq



Dafny-ish proofs mostly automated by SMT, with a few well-chosen user-supplied lemmas

But SMT-based proofs can go awry

- E.g., when using theories like non-linear arithmetic

```
let lemma_carry_limb_unrolled (a0 a1 a2:nat) : Lemma
  (requires T)
  (ensures (a0 % p44 + p44 * ((a1 + a0 / p44) % p44) + p88 * (a2 + ((a1 + a0 / p44) / p44))
    = a0 + p44 * a1 + p88 * a2)) =
  let open FStar.Math.Lemmas in
  let z = a0 % p44 + p44 * ((a1 + a0 / p44) % p44) + p88 * (a2 + ((a1 + a0 / p44) / p44)) in
  distributivity_add_right p88 a2 ((a1 + a0 / p44) / p44); (* argh! *)
  pow2_plus 44 44;
  lemma_div_mod (a1 + a0 / p44) p44;
  distributivity_add_right p44 ((a1 + a0 / p44) % p44) (p44 * ((a1 + a0 / p44) / p44)); (* argh! *)
  assert (p44 * ((a1 + a0 / p44) % p44) + p88 * ((a1 + a0 / p44) / p44) = p44 * (a1 + a0 / p44) );
  distributivity_add_right p44 a1 (a0 / p44); (* argh! *)
  lemma_div_mod a0 p44
```

Forced to write very detailed proof terms when SMT fails

And can be at a low level of abstraction

- Lots of boilerplate to define parsers/formatters and prove them mutually inverse

emacs@NSWAMY-SURFBK

File Edit Options Buffers Tools Help

```
let cipherSuiteBytes cs =
  match cs with
  | UnknownCipherSuite b1 b2 → twobytes (b1,b2)
  | NullCipherSuite → twobytes ( 0x00z, 0x00z )

  | CipherSuite13 AES128_GCM SHA256 → twobytes ( 0x13z, 0x01z )
  | CipherSuite13 AES256_GCM SHA384 → twobytes ( 0x13z, 0x02z )
  | CipherSuite13 CHACHA20_POLY1305 SHA256 → twobytes ( 0x13z, 0x03z )
  | CipherSuite13 AES128_CCM SHA256 → twobytes ( 0x13z, 0x04z )
  | CipherSuite13 AES128_CCM8 SHA256 → twobytes ( 0x13z, 0x05z )

  | CipherSuite Kex_RSA None (MACOnly MD5) → twobytes ( 0x00z, 0x01z )
  | CipherSuite Kex_RSA None (MACOnly SHA1) → twobytes ( 0x00z, 0x02z )
  | CipherSuite Kex_RSA None (MACOnly SHA256) → twobytes ( 0x00z, 0x3Bz )
  | CipherSuite Kex_RSA None(MtE (Stream RC4_128) MD5) → twobytes ( 0x00z, 0x04z )
  | CipherSuite Kex_RSA None(MtE (Stream RC4_128) SHA1) → twobytes ( 0x00z, 0x05z )

  | CipherSuite Kex_RSA None(MtE (Block TDES_EDE_CBC) SHA1) → twobytes ( 0x00z, 0x0Az )
  | CipherSuite Kex_RSA None(MtE (Block AES128_CBC) SHA1) → twobytes ( 0x00z, 0x2Fz )
  | CipherSuite Kex_RSA None(MtE (Block AES256_CBC) SHA1) → twobytes ( 0x00z, 0x35z )
  | CipherSuite Kex_RSA None(MtE (Block AES128_CBC) SHA256) → twobytes ( 0x00z, 0x3Cz )
  | CipherSuite Kex_RSA None(MtE (Block AES256_CBC) SHA256) → twobytes ( 0x00z, 0x3Dz )

  (*****)
  | CipherSuite Kex_DH (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1) → twobytes ( 0x00z, 0x0Dz )
  | CipherSuite Kex_DH (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1) → twobytes ( 0x00z, 0x10z )
  | CipherSuite Kex_DHE (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1) → twobytes ( 0x00z, 0x13z )
  | CipherSuite Kex_DHE (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1) → twobytes ( 0x00z, 0x16z )

  | CipherSuite Kex_DH (Some DSA) (MtE (Block AES128_CBC) SHA1) → twobytes ( 0x00z, 0x30z )
  | CipherSuite Kex_DH (Some RSASIG) (MtE (Block AES128_CBC) SHA1) → twobytes ( 0x00z, 0x31z )
  | CipherSuite Kex_DHE (Some DSA) (MtE (Block AES128_CBC) SHA1) → twobytes ( 0x00z, 0x32z )
  | CipherSuite Kex_DHE (Some RSASIG) (MtE (Block AES128_CBC) SHA1) → twobytes ( 0x00z, 0x33z )

let parseCipherSuite b =
  match b.[0u1], b.[1u1] with
  | ( 0x00z, 0x00z ) → Correct(NullCipherSuite)

  | ( 0x13z, 0x01z ) → Correct (CipherSuite13 AES128_GCM SHA256)
  | ( 0x13z, 0x02z ) → Correct (CipherSuite13 AES256_GCM SHA384)
  | ( 0x13z, 0x03z ) → Correct (CipherSuite13 CHACHA20_POLY1305 SHA256)
  | ( 0x13z, 0x04z ) → Correct (CipherSuite13 AES128_CCM SHA256)
  | ( 0x13z, 0x05z ) → Correct (CipherSuite13 AES128_CCM8 SHA256)

  | ( 0x00z, 0x01z ) → Correct(CipherSuite Kex_RSA None (MACOnly MD5))
  | ( 0x00z, 0x02z ) → Correct(CipherSuite Kex_RSA None (MACOnly SHA1))
  | ( 0x00z, 0x3Bz ) → Correct(CipherSuite Kex_RSA None (MACOnly SHA256))
  | ( 0x00z, 0x04z ) → Correct(CipherSuite Kex_RSA None (MtE (Stream RC4_128) MD5))
  | ( 0x00z, 0x05z ) → Correct(CipherSuite Kex_RSA None (MtE (Stream RC4_128) SHA1))

  | ( 0x00z, 0x0Az ) → Correct(CipherSuite Kex_RSA None (MtE (Block TDES_EDE_CBC) SHA1))
  | ( 0x00z, 0x2Fz ) → Correct(CipherSuite Kex_RSA None (MtE (Block AES128_CBC) SHA1))
  | ( 0x00z, 0x35z ) → Correct(CipherSuite Kex_RSA None (MtE (Block AES256_CBC) SHA1))
  | ( 0x00z, 0x3Cz ) → Correct(CipherSuite Kex_RSA None (MtE (Block AES128_CBC) SHA256))
  | ( 0x00z, 0x3Dz ) → Correct(CipherSuite Kex_RSA None (MtE (Block AES256_CBC) SHA256))

  (*****)
  | ( 0x00z, 0x0Dz ) → Correct(CipherSuite Kex_DH (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1))
  | ( 0x00z, 0x10z ) → Correct(CipherSuite Kex_DH (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1))
  | ( 0x00z, 0x13z ) → Correct(CipherSuite Kex_DHE (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1))
  | ( 0x00z, 0x16z ) → Correct(CipherSuite Kex_DHE (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1))

  | ( 0x00z, 0x30z ) → Correct(CipherSuite Kex_DH (Some DSA) (MtE (Block AES128_CBC) SHA1))
  | ( 0x00z, 0x31z ) → Correct(CipherSuite Kex_DH (Some RSASIG) (MtE (Block AES128_CBC) SHA1))
  | ( 0x00z, 0x32z ) → Correct(CipherSuite Kex_DHE (Some DSA) (MtE (Block AES128_CBC) SHA1))
  | ( 0x00z, 0x33z ) → Correct(CipherSuite Kex_DHE (Some RSASIG) (MtE (Block AES128_CBC) SHA1))

1\*- p1.fst Top L18 (Fundamental -2)
```

```
let inverse_cipherSuite (x: cipherSuite)
: Lemma
  (requires (¬ (UnknownCipherSuite? x)))
  (ensures (let y = cipherSuiteBytes x in
    Some? y ⇒ parseCipherSuite (Some?.v y) = Correct x))
= ...
```

```
let inverse_cipherSuite' (x: lbytes 2)
: Lemma
  (ensures (let y = parseCipherSuite x in
    Correct? y /\ not (UnknownCipherSuite? (Correct?._0 y)) ==>
    Some? (cipherSuiteBytes (Correct?._0 y)) /\
    x == Some?.v (cipherSuiteBytes (Correct?._0 y)))) =
...

```

Automated (implicit?) theorem proving

- Unlike Coq, no direct access to the context
- Cannot stop, examine hypotheses and goals
- Proof-irrelevant theory: the SMT solver does not return a proof witness
- Non-modular: one big WP computed for each function

Enter Metaprogramming

Domain-specific languages, ad hoc proof automation, extensibility

Allow programmers to:

- Customize how programs are typechecked, producing domain-specific VCs
- Metaprogram programs and their proofs
- Define their own equivalence preserving program transformations
- ...

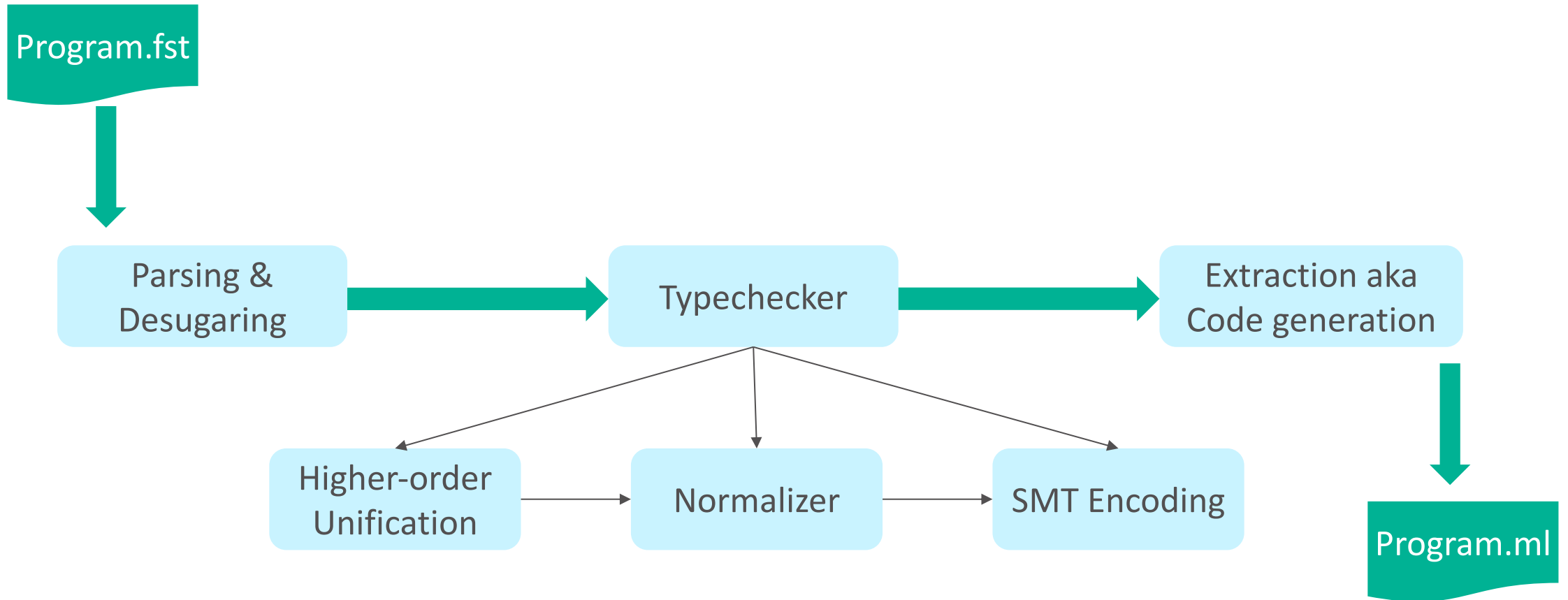
Domain-specific languages, ad hoc proof automation, extensibility

By treating the F^* object language as its own
metalanguage

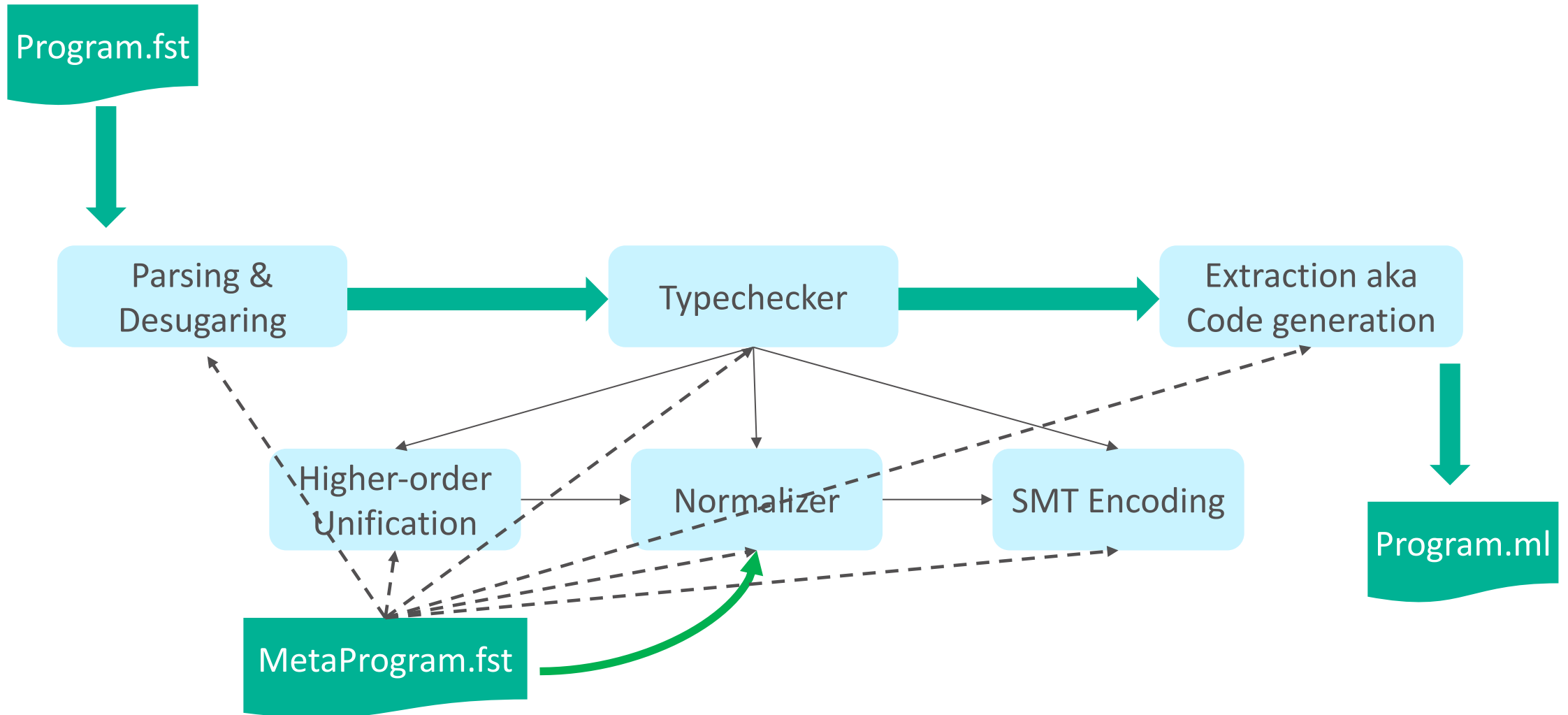
Building on *elaborator reflection* a great idea by

- David Christiansen and Edwin Brady (Idris)
- Leonardo de Moura (Lean)

A passive compiler pipeline



Scripting components with a metaprogram



Scripting a language implementation from within the language

Provide a API to compiler internals for F* (meta)programs to reflect and/or construct

- Syntax of terms
- Typechecking environment
- Typing derivations
- ...

F* compiler runs F* metaprograms to build and typecheck other F* programs

From F^* to Meta- F^* , In three easy steps

1. Metaprogramming as a computational effect of *proof-state transformers*
2. Primitive operations to manipulate proof-states, using trusted compiler primitives
3. Reflecting on syntax: quotation and unquotation

Some terminology

- Meta-programs execute at any stage of the compiler
- The environment (meta-program state) is the compiler state
- When executing within the type-checker:
 - meta-program=tactic
 - environment=proof state

Recovering proof structure from WPs

- Synthesize a WP; get a verification condition
- Intercept it before sending it to the SMT solver
- A new construct: **assert ϕ by t**
- **assert ϕ ; e** generates $\phi \text{ by } t \wedge (\phi \Rightarrow \text{VC } e)$
- Traverse the VC, collecting hypotheses leading to ϕ
- Remove ϕ from the VC
- Run meta-program with hypotheses; goal = ϕ

Proof-state: A collection of typed holes

A hole is a missing program fragment in a context

$$\Gamma \vdash ?_0 : \tau$$

(aka a *goal*)

The proof-state is a collection of pending holes

$$\overline{\{\Gamma_i \vdash ?_i : \tau_i\}}$$

+ some internal persistent state

(e.g., unionfind graph of the unifier)

Metaprograms are proofstate transformers

```
let meta a = proofstate → Dv (either (a × proofstate) error)
```

- Uses an existing F* effect for non-termination: Dv
- The type of the state is an abstract type: proofstate
- error is the type of exceptions

State + Exception + Non-termination monad

Metaprogramming as a user-defined effect

```
let meta a = proofstate → Dv (either (a × proofstate) error)
```

```
(* Monad *)
```

```
let return a x : meta a = λ ps → Inl (x, ps)
```

```
let bind a b (f:meta a) (g: a → meta b) : meta b =  
  λ ps → match f ps with  
    | Inl (x, ps') → g x ps'  
    | Inr err → Inr err
```

```
(* Actions *)
```

```
let get () : meta ps = λ ps → Inl (ps, ps)
```

```
let raise a err : meta a = λ ps → Inr (err, ps)
```

- Standard definitions of return, bind, get, raise
- Exceptions reset the state

Metaprogramming as a user-defined effect

```
let meta a = proofstate → Dv (either (a × proofstate) error)
```

Conspicuously, no **put** action

The proofstate is a compiler internal data structure.

A metaprogram can read it but cannot modify it arbitrarily (meta-result)

```
(return)  
let get () : meta ps = λ ps → Inl (ps, ps)  
let raise a err : meta a = λ ps → Inr (err, ps)  
  
(* Ask F* to derive an effect from meta *)  
new_effect { META : a:Type → Effect  
            with repr = meta; bind; return; raise; get }
```

F* extended with meta-programs

- A new effect
 - regular **let** and implicit return
 - WP semantics derived *automatically* (Dijkstra Monads for free)
- Running it
 - automatically translated back to a functional implementation (state-passing)
 - executed on the F* normalizer (general reduction machinery for pure programs)

Abstraction for efficient execution

- F* meta-programs are deeply-embedded in the normalizer
- Need to embed/de-embed

- ```
let rec eval (t: term) = match t with
 | Tm_App (Tm_Fvar "addition", e1, e2) ->
 match eval e1, eval e2 with
 | Tm_constant c1, Tm_constant c2 ->
 Tm_constant (c1 + c2)
 | e1, e2 -> Tm_App (Tm_Fvar "addition", e1, e2)
 | Tm_App (Tm_Builtin lid, e1) ->
 // how to represent the state e1?
 | Tm_App (f, x) -> subst ...
```

# Abstraction for efficient execution

- The proof state is **abstract** from the point of view of meta-programs
- No need to **embed** and **de-embed**
- Jump back and forth:
  - outside of the normalizer into the type-checker code when a primitive action is called
  - from the type-checker code back into the (suspended) computation in the normalizer when a primitive action returns

## Step 2

# Primitive operations on proofstate

- Every typing rule (read backwards) provided as a proofstate primitive

$$\frac{\Gamma, x:t_1 \vdash ?_1 : t_2}{\Gamma \vdash ?_0 : (x:t_1 \rightarrow t_2)} \text{ [TC-Abs]} \quad \text{where } ?_0 := \lambda x:t_1. ?_1$$

```
intro : unit -> Meta binder
```

```
intro () (Γ ⊢ ?0 : (x:t1 → t2) :: rest) =
```

```
 Inl (x, (Γ, x:t1 ⊢ ?1 : t2 :: rest))
```

```
 where fresh x and ?0 := fun (x:t1) -> ?1
```

```
intro () _ = raise (Failure “Goal is not an arrow”)
```

## Step 3

# Reflecting on syntax

- Locally nameless abstract syntax of F\* terms provided as a datatype to metaprograms

- Quotation: Builds the syntax of a term  
``(0 + 1) : term`

- Unquotation
  - Typechecks syntax at a given type

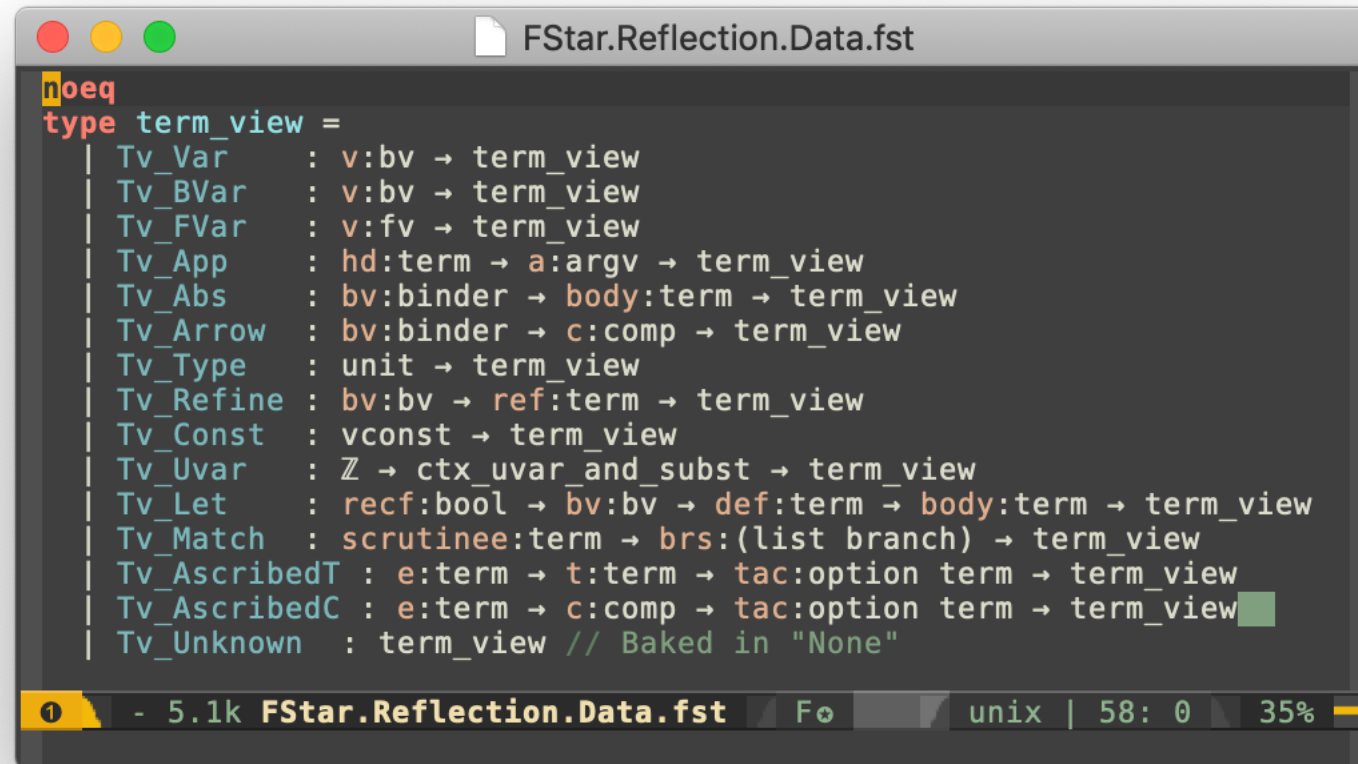
```
type term =
| T_Var : v:bv → term
| T_BVar : v:bv → term
| T_FVar : v:fv → term
| T_App : hd:term → a:argv → term
| T_Abs : bv:binder → body:term → term
| T_Arrow : bv:binder → c:comp → term
| T_Type : unit → term
| T_Refine : bv:bv → ref:term → term
| T_Const : vconst → term
| T_Uvar : ℤ → ctx_uvar_and_subst → term
| T_Let : recf:bool → bv:bv → def:term → body:term → term
| T_Match : scrutinee:term → brs:(list branch) → term
| T_AscribedT : e:term → t:term → tac:option term → term
| T_AscribedC : e:term → c:comp → tac:option term → term
```

```
val unquote: a:Type -> x:term -> Meta a
```

### Step 3

# Reflecting on syntax (but, better)

- Directly reflecting term is inefficient; need to convert potentially large terms
- Instead, use term views to unroll only the head constructor (old idea)
- Apply the same optimization with type term being abstract (no embedding)



```
FStar.Reflection.Data.fst

noeq
type term_view =
 | Tv_Var : v:bv → term_view
 | Tv_BVar : v:bv → term_view
 | Tv_FVar : v:fv → term_view
 | Tv_App : hd:term → a:argv → term_view
 | Tv_Abs : bv:binder → body:term → term_view
 | Tv_Arrow : bv:binder → c:comp → term_view
 | Tv_Type : unit → term_view
 | Tv_Refine : bv:bv → ref:term → term_view
 | Tv_Const : vconst → term_view
 | Tv_Uvar : Z → ctx_uvar_and_subst → term_view
 | Tv_Let : recf:bool → bv:bv → def:term → body:term → term_view
 | Tv_Match : scrutinee:term → brs:(list branch) → term_view
 | Tv_AscribedT : e:term → t:term → tac:option term → term_view
 | Tv_AscribedC : e:term → c:comp → tac:option term → term_view
 | Tv_Unknown : term_view // Baked in "None"
```

1 - 5.1k FStar.Reflection.Data.fst Fo unix | 58: 0 35%

# Putting it together

Entrypoint: Decorate an implicit with a metaprogram  
(program synthesis demo)

```
let id : (a:Type) -> a -> a =
 _ by (
 let a = intro () in [· ⊢ _ : (a:Type) → a → a]
 let x = intro () in [a:Type ⊢ _ : a → a]
 hyp x [a:Type, x:a ⊢ _ : a]
) []
```

# And can be at a low level of abstraction

- Lots of boilerplate to define parsers/formatters and prove them mutually inverse

```
emacs@NSWAMY-SURFBK
File Edit Options Buffers Tools Help

let cipherSuiteBytes cs =
 match cs with
 | UnknownCipherSuite b1 b2 → twobytes (b1,b2)
 | NullCipherSuite
 → twobytes (0x00z, 0x00z)

 | CipherSuite13 AES128_GCM SHA256
 | CipherSuite13 AES256_GCM SHA384
 | CipherSuite13 CHACHA20_POLY1305 SHA256
 | CipherSuite13 AES128_CCM SHA256
 | CipherSuite13 AES128_CCM8 SHA256
 → twobytes (0x13z, 0x01z)
 → twobytes (0x13z, 0x02z)
 → twobytes (0x13z, 0x03z)
 → twobytes (0x13z, 0x04z)
 → twobytes (0x13z, 0x05z)

 | CipherSuite Kex_RSA None (MACOnly MD5)
 | CipherSuite Kex_RSA None (MACOnly SHA1)
 | CipherSuite Kex_RSA None (MACOnly SHA256)
 | CipherSuite Kex_RSA None(MtE (Stream RC4_128) MD5)
 | CipherSuite Kex_RSA None(MtE (Stream RC4_128) SHA1)
 → twobytes (0x00z, 0x01z)
 → twobytes (0x00z, 0x02z)
 → twobytes (0x00z, 0x3Bz)
 → twobytes (0x00z, 0x04z)
 → twobytes (0x00z, 0x05z)

 | CipherSuite Kex_RSA None(MtE (Block TDES_EDE_CBC) SHA1)
 | CipherSuite Kex_RSA None(MtE (Block AES128_CBC) SHA1)
 | CipherSuite Kex_RSA None(MtE (Block AES256_CBC) SHA1)
 | CipherSuite Kex_RSA None(MtE (Block AES128_CBC) SHA256)
 | CipherSuite Kex_RSA None(MtE (Block AES256_CBC) SHA256)
 → twobytes (0x00z, 0x0Az)
 → twobytes (0x00z, 0x2Fz)
 → twobytes (0x00z, 0x35z)
 → twobytes (0x00z, 0x3Cz)
 → twobytes (0x00z, 0x3Dz)

 (*****)
 | CipherSuite Kex_DH (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1)
 | CipherSuite Kex_DH (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1)
 | CipherSuite Kex_DHE (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1)
 | CipherSuite Kex_DHE (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1)
 → twobytes (0x00z, 0x0Dz)
 → twobytes (0x00z, 0x10z)
 → twobytes (0x00z, 0x13z)
 → twobytes (0x00z, 0x16z)

 | CipherSuite Kex_DH (Some DSA) (MtE (Block AES128_CBC) SHA1)
 | CipherSuite Kex_DH (Some RSASIG) (MtE (Block AES128_CBC) SHA1)
 → twobytes (0x00z, 0x30z)
 → twobytes (0x00z, 0x31z)

let inverse_cipherSuite (x: cipherSuite)
: Lemma
 (requires (¬ (UnknownCipherSuite? x)))
 (ensures (let y = cipherSuiteBytes x in
 Some? y ⇒ parseCipherSuite (Some?.v y) = Correct x))
= ...

let parseCipherSuite b =
 match b.[0u1], b.[1u1] with
 | (0x00z, 0x00z) → Correct(NullCipherSuite)

 | (0x13z, 0x01z) → Correct (CipherSuite13 AES128_GCM SHA256)
 | (0x13z, 0x02z) → Correct (CipherSuite13 AES256_GCM SHA384)
 | (0x13z, 0x03z) → Correct (CipherSuite13 CHACHA20_POLY1305 SHA256)
 | (0x13z, 0x04z) → Correct (CipherSuite13 AES128_CCM SHA256)
 | (0x13z, 0x05z) → Correct (CipherSuite13 AES128_CCM8 SHA256)

 | (0x00z, 0x01z) → Correct(CipherSuite Kex_RSA None (MACOnly MD5))
 | (0x00z, 0x02z) → Correct(CipherSuite Kex_RSA None (MACOnly SHA1))
 | (0x00z, 0x3Bz) → Correct(CipherSuite Kex_RSA None (MACOnly SHA256))
 | (0x00z, 0x04z) → Correct(CipherSuite Kex_RSA None (MtE (Stream RC4_128) MD5))
 | (0x00z, 0x05z) → Correct(CipherSuite Kex_RSA None (MtE (Stream RC4_128) SHA1))

 | (0x00z, 0x0Az) → Correct(CipherSuite Kex_RSA None (MtE (Block TDES_EDE_CBC) SHA1))
 | (0x00z, 0x2Fz) → Correct(CipherSuite Kex_RSA None (MtE (Block AES128_CBC) SHA1))
 | (0x00z, 0x35z) → Correct(CipherSuite Kex_RSA None (MtE (Block AES256_CBC) SHA1))
 | (0x00z, 0x3Cz) → Correct(CipherSuite Kex_RSA None (MtE (Block AES128_CBC) SHA256))
 | (0x00z, 0x3Dz) → Correct(CipherSuite Kex_RSA None (MtE (Block AES256_CBC) SHA256))

 (*****)
 | (0x00z, 0x0Dz) → Correct(CipherSuite Kex_DH (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1))
 | (0x00z, 0x10z) → Correct(CipherSuite Kex_DH (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1))
 | (0x00z, 0x13z) → Correct(CipherSuite Kex_DHE (Some DSA) (MtE (Block TDES_EDE_CBC) SHA1))
 | (0x00z, 0x16z) → Correct(CipherSuite Kex_DHE (Some RSASIG) (MtE (Block TDES_EDE_CBC) SHA1))

 | (0x00z, 0x30z) → Correct(CipherSuite Kex_DH (Some DSA) (MtE (Block AES128_CBC) SHA1))
 | (0x00z, 0x31z) → Correct(CipherSuite Kex_DH (Some RSASIG) (MtE (Block AES128_CBC) SHA1))
 | (0x00z, 0x32z) → Correct(CipherSuite Kex_DHE (Some DSA) (MtE (Block AES128_CBC) SHA1))

let inverse_cipherSuite' (x: bytes 2)
: Lemma
 (ensures (let y = parseCipherSuite x in
 Correct? y /\ not (UnknownCipherSuite? (Correct?._0 y)) ==>
 Some? (cipherSuiteBytes (Correct?._0 y)) /\
 x == Some?.v (cipherSuiteBytes (Correct?._0 y)))) =
 ...
```

Remember this?

# Metaprogramming mutually inverse parsers and formatters

```
type color = | Red | Blue | Green | Yellow
type palette = nlist 18 (color × u8)
let ps : (p:parser palette & serializer p) =
 _ by (gen_parser_serializer (`palette))
```

Where, the types capture that parser/serializer are mutual inverses

```
let parser (t:Type) = bytes → option t
let serializer (#t:Type) (p:parser t) =
 s : (x:t → b:bytes) {
 (∀ x. p (s x) = Some x) ∧
 (∀ b. match p b with
 | Some x → s x = b
 | _ → ⊥)}
```

# Putting it together

Entrypoint: Decorate an assertion with a metaprogram

```
let f (x:nat) =
 if x > 1 then
 assert (x * x > x)
 by tac;
 ...
```

$$x : \text{nat}, h : (x > 1) \vdash \_ : (x * x > x)$$

- Assertions produce a goal in a context including control flow hypotheses
- Tackled by the metaprogram `tac`

# SMT: Just one of F\*'s tactic primitives

```
val smt: unit -> Meta unit
```

```
let f (x:nat) =
 if x > 1 then
 assert (x * x > x)
 by (smt());
 ...
```

$x: \text{nat}, h: (x > 1) \vdash \_ : (x * x > x)$

- Assertions produce a goal in a context including control flow hypotheses
- Tackled by the metaprogram `tac`

# But SMT-based proofs can go awry

- E.g., when using theories like non-linear arithmetic

```
let lemma_carry_limb_unrolled (a0 a1 a2:nat) : Lemma
 (requires T)
 (ensures (a0 % p44 + p44 * ((a1 + a0 / p44) % p44) + p88 * (a2 + ((a1 + a0 / p44) / p44))
 = a0 + p44 * a1 + p88 * a2)) =
 let open FStar.Math.Lemmas in
 let z = a0 % p44 + p44 * ((a1 + a0 / p44) % p44) + p88 * (a2 + ((a1 + a0 / p44) / p44)) in
 distributivity_add_right p88 a2 ((a1 + a0 / p44) / p44); (* argh! *)
 pow2_plus 44 44;
 lemma_div_mod (a1 + a0 / p44) p44;
 distributivity_add_right p44 ((a1 + a0 / p44) % p44) (p44 * ((a1 + a0 / p44) / p44)); (* argh! *)
 assert (p44 * ((a1 + a0 / p44) % p44) + p88 * ((a1 + a0 / p44) / p44) = p44 * (a1 + a0 / p44));
 distributivity_add_right p44 a1 (a0 / p44); (* argh! *)
 lemma_div_mod a0 p44
```

**Remember this?**

Forced to write very detailed proof terms when SMT fails

# SMT + Tactics for more automated, robust proofs

Key lemma in poly1305, a MAC algorithm,  
doing multiplication in the prime field  $2^{130} - 5$

```
let poly_multiply (n p r h r0 r1 h0 h1 h2 s1 d0 d1 d2 h1 h2 hh : ℤ)
 : Lemma
 (requires p > 0 ∧ r1 ≥ 0 ∧ n > 0 ∧ 4 × (n × n) == p + 5 ∧ r == r1 × n + r0 ∧
 h == h2 × (n × n) + h1 × n + h0 ∧ s1 == r1 + (r1 / 4) ∧ r1 % 4 == 0 ∧
 d0 == h0 × r0 + h1 × s1 ∧ d1 == h2 × r0 + h1 × s1 ∧ d2 == h2 × r0 ∧ hh == d2 × (n × n)
 (ensures (h × r) % p == hh % p) =
 let r14 = r1 / 4 in
 let h_r_expand = (h2 × (n × n) + h1 × n + h0) × r14 in
 let hh_expand = (h2 × r0) × (n × n) + (h0 × r0 + h1 × s1 + h2 × r0 × (5 × r14)) × n + (h0 × r0 + h1 × s1)
 let b = (h2 × n + h1) × r14 in
 modulo_addition_lemma hh_expand p b;
 assert (h_r_expand == hh_expand + b × (n × n))
 by (canon_semiring int_csr;
 smt())
```

- A reflective metaprogram to canonicalize terms in commutative semirings
- Simplifies goal into a form that smt can then solve using linear arithmetic only
- Prior manual proof required 41 steps of explicit rewriting lemmas (!)

# Partial evaluation for Low\* productivity

This is not Low\*:

```
val compress:
 a:sha_alg -> state a -> array u8 -> Stack unit
```

Reason:

```
let state a = function
 | SHA2_224 | SHA2_256 -> array u32
 | SHA2_384 | SHA2_512 -> array u64
```

This *could* be compiled  
as a union. However,  
this is not **idiomatic**.  
Instead, we rely on  
**partial evaluation**:

```
let compress_224 =
 FStar.Tactics.(synth_by_tactic (specialize (compress SHA2_224)
 [% ...]))
let compress_256 =
 FStar.Tactics.(synth_by_tactic (specialize (compress SHA2_256)
 [% ...]))
let compress_384 =
 FStar.Tactics.(synth_by_tactic (specialize (compress SHA2_384)
 [% ...]))
let compress_512 =
 FStar.Tactics.(synth_by_tactic (specialize (compress SHA2_512)
 [% ...]))
```

# Higher-order combinators

- It is frequent to **combine** core operations to generate a **construction** (counter-mode, hash, etc.)

## Example:

- Given any (abstract) *compression function*, one can define a **Merkle-Damgård construction**
- Construction: **folding** the core compression function over multiple **blocks** of input data
- We **do not** want to write this n times...

# An example of a higher-order combinator

```
// Write once; this is not Low*
noextract inline_for_extraction
let mk_compress_blocks (a: hash_alg)
 (compress: compress_st a)
 (s: state a)
 (input: blocks)
 (n: u32 { length input = block_size a * n })
=
 C.Loops.for 0u1 n (fun i ->
 compress s (Buffer.sub input (i * block_size a) (block_size a)))

// Specialize many times; now this is Low*
let compress_blocks_224 = FStar.Tactics.(synth_by_tactic (specialize SHA2_224) [%...])
...
let compress_md5 = FStar.Tactics.(synth_by_tactic (specialize MD5) [%...])
...
```

# Language extensions / DSLs in userland

- Typeclass resolution as a user-provided program
  - a hook to synthesize a term if  $F^*$  fails to infer an implicit argument
  - dedicated syntactic sugar
  - optional arguments
- Syntax for base64 array literals
  - encoded via a string
  - rewritten via a tactic into a suitable  $Low^*$  array declaration
- Deriving
  - printers, marshallng functions, etc.
- More
  - tactics as an  $F^*$  frontend for DSLs

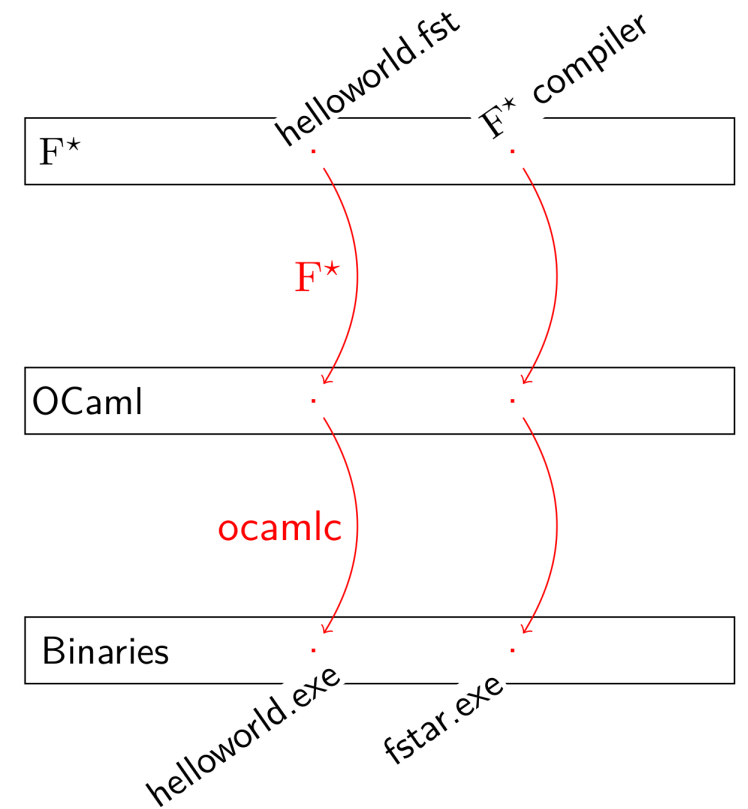
# Executing tactics (reminder)

- Tactics can be **reified** as proof-state-passing programs
- F\* features a **normalizer** (for higher-order unification)
- Upon seeing a tactic, the F\* type-checker **interprets** the tactic program using the normalizer (KAM)
- Tactics are written against an **abstract** state
- Primitive actions are type-checker **primitives**

Works for any metaprogram, really.

# Executing metaprograms natively

- By default, metaprograms are interpreted on F\*'s normalizer
- But, F\* is implemented in F\* and all F\* programs can be compiled to OCaml

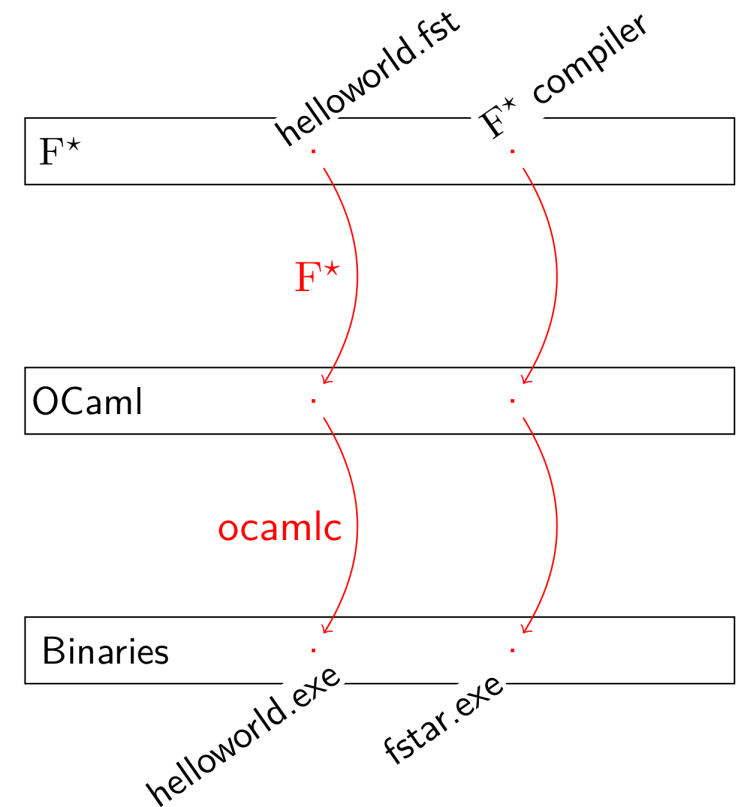


# Dynamic loading of plugins

- Tactic is type-checked then extracted
- Fixed set of primitive type-checker actions; extend it with “dynlink”: dynamic linking to extend the compiler
- Register new primitive actions
- Detect at call-sites and trigger multi-language interop

# Execution across language boundaries

- F\* jumps back and forth between interpreter and native
- Conversions at the boundary: deeply embedded (Tm\_int “42”) to native (42) and conversely.
- 10x performance boost



# Some takeaways

- Freedom of expression
  - Tools for large-scale, full program verification need arbitrary expressive power
- Proof automation, Expressiveness, Control
  - SMT is great, but not a panacea: Eventually hit a complexity/undecidability wall
- Careful combination of tactics and SMT *improve* automation relative to either SMT or tactics alone
- Meta-F\*: Self-scripting a PL implementation with reflective metaprogramming; tactics are just a special case