Conceptualizing explanations through category theory

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Introduction

We will consider the representation of temporal processes as objects in a category and a possible way to conceptualize what could count as an explanation. This will be achieved by considering families of dynamical systems and looking which is empirically adequate for the observations we have at hand in this representation [1, 5]. This can be seen as a formalization of the Deductive-Nomological approach [2] for temporal processes as explananda and dynamical systems as explanantia.

The explanandum

The invariant set captures the dynamics of the system, i.e., this set will describe in which part of the state space the system is (most likely) to be found. The Conley-Morse graph summarizes all the dynamics of a system by showing the Conley index of all the chain recurrent sets [3, 4]. For the current purposes it is sufficient to consider the category of Conley-Morse graphs $\mathbf{CMgraphs}$ with as objects Conley-Morse graphs and with as morphisms the identities.

The explanans

As the logical space we consider the set of all local flows φ defined on a open subsets $X \subset \mathbb{R}^n$, constrained by the theory of the target system (assumptions about its composite parts, their ranges and interactions, etc). We define the category $\mathbf{LocFlow}$ to have as objects local flows (with possible constraints) and the morphisms to be topological equivalence. An example would be the local flows one can define by restricting the Hodgkin-Huxley-like models, in order to consider models of the action potential.

The deductive link

The functor $\mathcal{F}\colon \mathbf{LocFlow} \to \mathbf{CMgraphs}$ is then defined as sending objects as

$$\mathcal{F}(\varphi) := \mathbf{CM}(\mathrm{Inv}(X)),$$

which sends a local flow to the finest Conley-Morse graph of its maximally invariant set and sending it to the CM graph with one Morse set that has Conley index which is $\mathbb{Z}$ at level -1 and is zero otherwise if the finest Morse decomposition does not exist. Finally, the morphisms of are sent to the identity morphisms.

Explanation

An explanation for a temporal process represented by the Conley-Morse graph Y (as inferred from measurements) exists if for its hypothesized models $\mathcal{A}$ we have $\mathcal{F}(\mathcal{A}) \cap Y \neq \emptyset$. The set of all descriptive models of a temporal process which has Conley-Morse graph Y is $\mathcal{F}^{-1}(Y)$.

In a construction with topological conjugacy as the morphisms in the category of local flows, we can identify the predictions of bifurcations of empirically adequate models.

References

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