Situated Transition Systems

(ACT 2021)

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Span(RGraph) $\leftrightsquigarrow$ transition systems $\bar{w}$ boundary

$$\begin{tikzcd}
 & R \arrow[dl, "\delta_0"'] \arrow[dr, "\delta_1"] & \\
U & & V
\end{tikzcd}$$

- Edges of U, V are <u>events</u>.

- R is the transition system.

- δ_0, δ_1 map transitions to boundary events.

Composition $\leftrightsquigarrow$ Consistent events

Pullback ensures
- consistency along shared boundary

$$\begin{tikzcd}
 & & S \circ R \arrow[dl] \arrow[dr] \arrow[dd, phantom, "\vee", very near start] & & \\
 & R \arrow[dl] \arrow[dr] & & S \arrow[dl] \arrow[dr] & \\
U & & V & & W
\end{tikzcd}$$

tensor product $\leftrightsquigarrow$ Independent Systems

$$\begin{tikzcd}
 & S \times R \arrow[dl, "\delta_0 \times \delta_0"'] \arrow[dr, "\delta_1 \times \delta_1"] & \\
U \times U' & & V \times V'
\end{tikzcd}$$

- trivial edges allow asynchronous execution

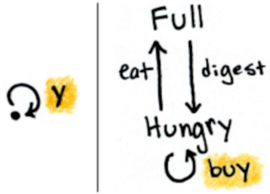
$$M = $$ 

$$\mathsf{Gear} : M \to M \quad = $$ 

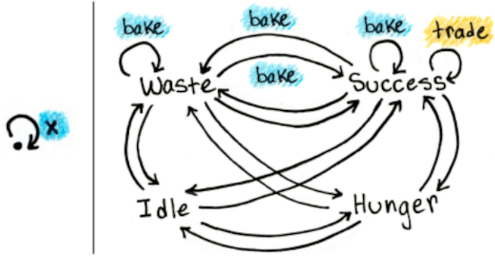
$$\mathsf{Gear} \circ \mathsf{Gear} : M \to M \quad = $$ 

$\text{Baker} : U \to V$

$\text{Eater} : V \to 1$



$\text{Eater} \circ \text{Baker} : U \to 1$



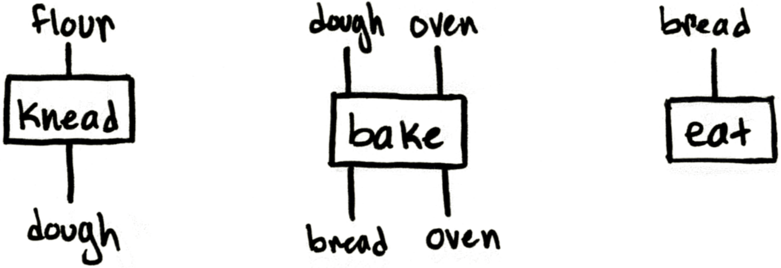
Waste = (Open, Full)
Success = (Open, Hungry)
Idle = (Closed, Full)
Hunger = (Closed, Hungry)

Resource Theories ⟷ Symmetric Monoidal Categories

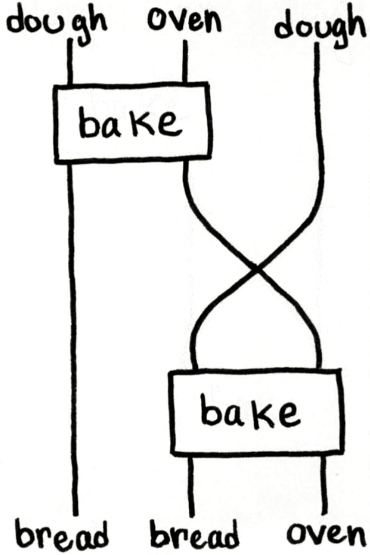
atomic objects:

{bread, dough, flour, oven}

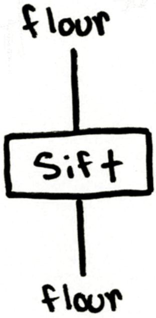
generating morphisms:

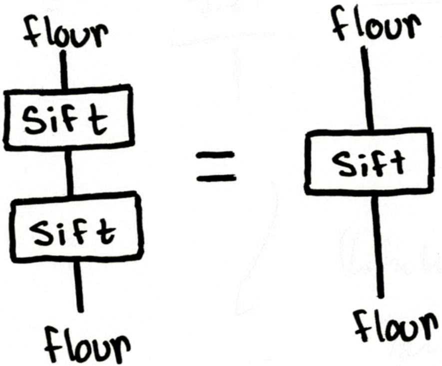


Resource Transformations ⟷ String Diagrams

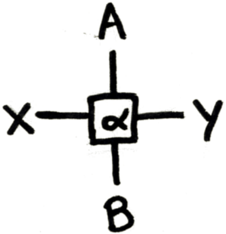


Same Effect ⟷ Equal as Morphisms

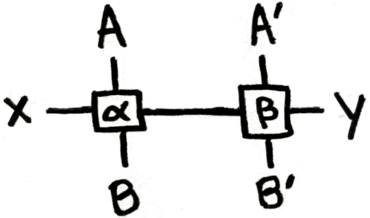


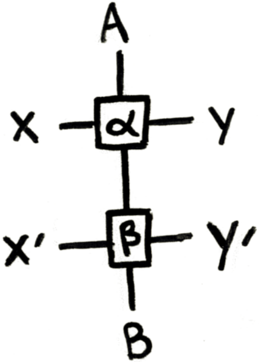


Single object double categories

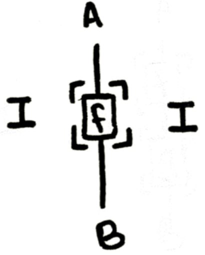


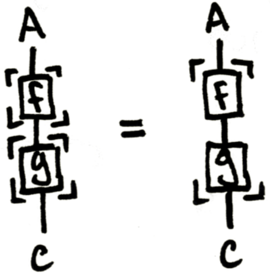
- Horizontal & vertical edge categories are monoids

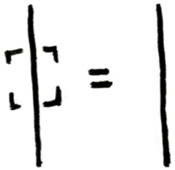


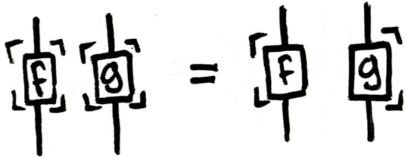


Build such a double category from $\mathbb{A}$ ← our resource theory



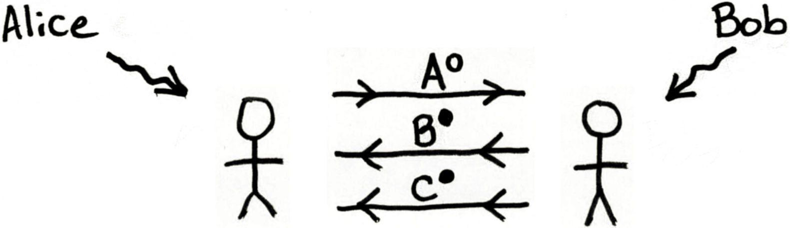






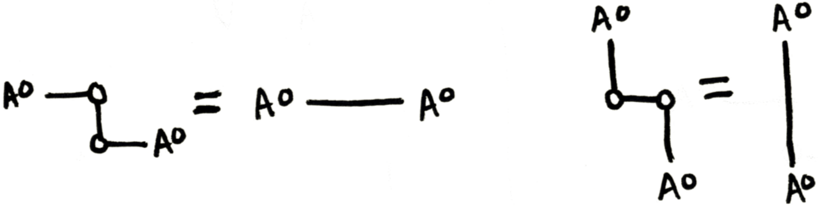
Vertical Edge Monoid $\leftrightsquigarrow$ Exchanges

We Understand $A^\circ \otimes B^\bullet \otimes C^\bullet$ as in:

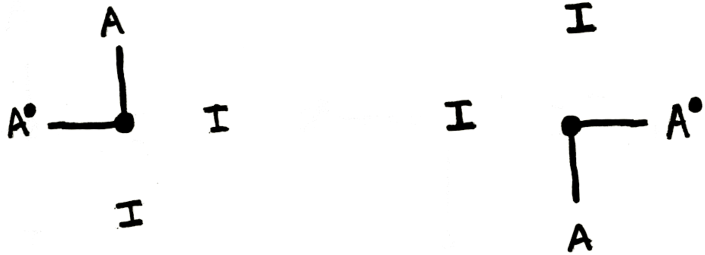


Add <u>conjoints</u> for each $A \in \mathbb{A}_0$. i.e., cells

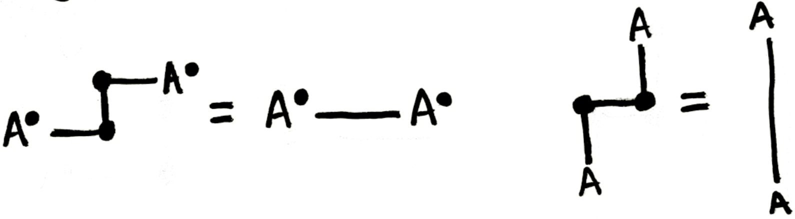
Satisfying:



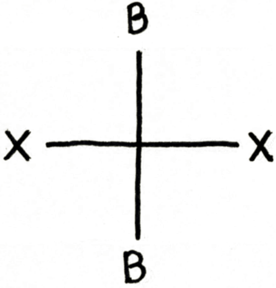
Add **Companions** for each $A \in \mathbb{A}_0$. i.e., cells

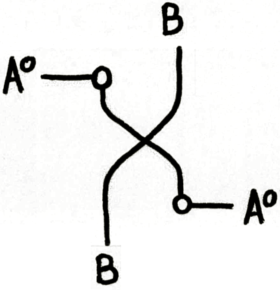


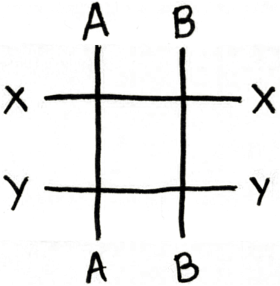
Satisfying:



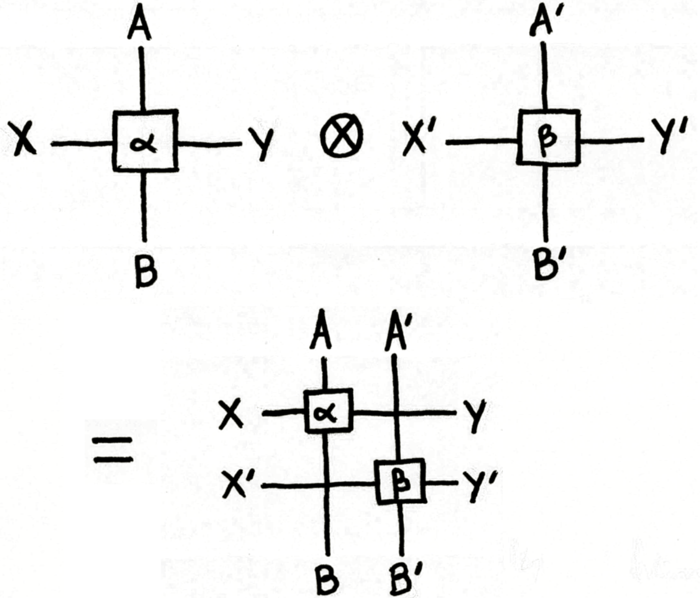
Crossing cells arise from corner structure:



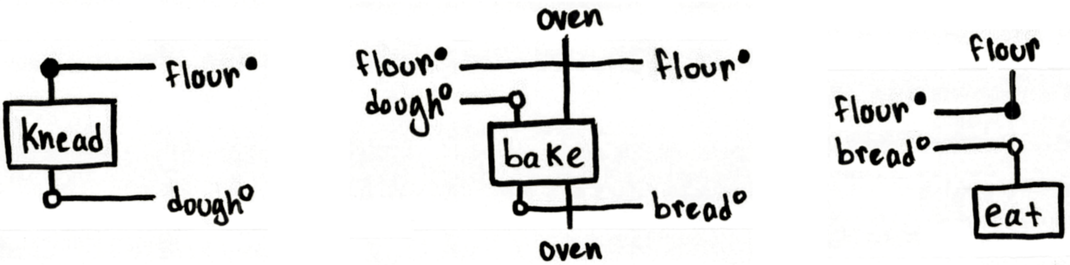




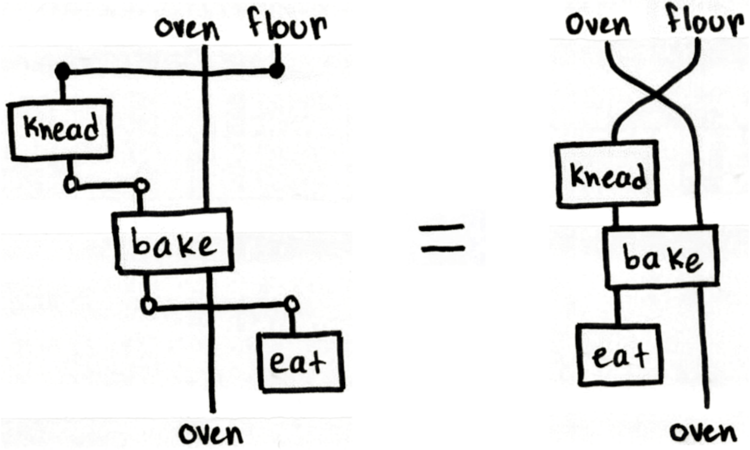
Crossing Cells $\longrightarrow$ Monoidal Double Category



Now Cells ⟷ Concurrent Resource Transformations

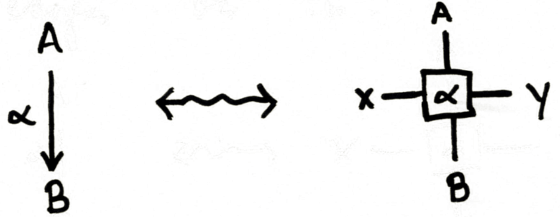


Horizontal Composition ⟷ Interaction



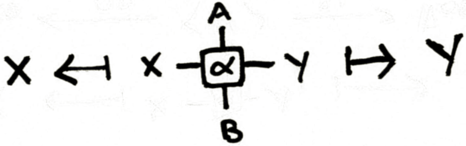
Write $\mathbb{A}^{\circ\bullet}$ for the (vertical edge) monoid of exchanges

Write $\langle \mathbb{A} \rangle$ for the graph with vertices $\mathbb{A}_0$ and edges as in:



Then there is a span of RGraphs

$$\mathbb{A}^{\circ\bullet} \xleftarrow{\delta_0} \langle \mathbb{A} \rangle \xrightarrow{\delta_1} \mathbb{A}^{\circ\bullet}$$



An _$\mathbb{A}$-situated boundary_ (U, φ_U) consists of a reflexive graph U and $\varphi_U : U \longrightarrow \mathbb{A}^{\circ\circ}$ in RGraph.

An _$\mathbb{A}$-situated transition system_ (R, φ_R) consists of a Span(RGraph) $U \leftarrow R \rightarrow V$ and $\varphi_R : R \longrightarrow \langle \mathbb{A} \rangle$ such that

$$\begin{tikzcd}
U \arrow[d, "\varphi_U"'] & R \arrow[l] \arrow[r] \arrow[d, "\varphi_R"] & V \arrow[d, "\varphi_V"] \\
\mathbb{A}^{\circ\circ} & \langle \mathbb{A} \rangle \arrow[l, "\delta_0"] \arrow[r, "\delta_1"'] & \mathbb{A}^{\circ\circ}
\end{tikzcd}$$

in RGraph.

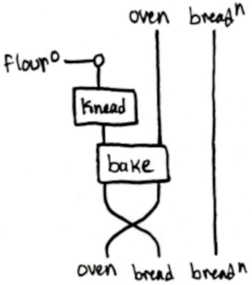
$\mathbb{A}$-situated transition systems form a (Planar) Monoidal category, $S(\mathbb{A})$.

If $\mathbb{A}$ is compact closed, so is $S(\mathbb{A})$.

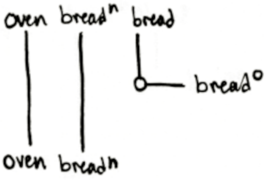
$S(\mathbb{Z})$ is the category Accounts of systems with partita-doppia, introduced by Katis, Sabadini, and Walters in 1998.

Baker : $\text{flour}^\circ \to \text{bread}^\circ$

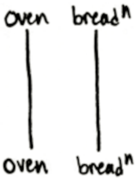
bake_n



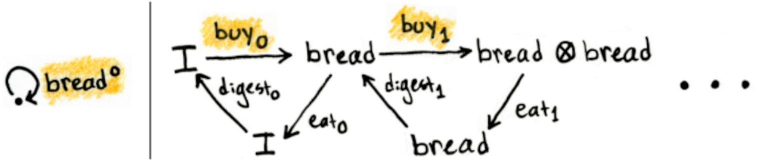
sell_n



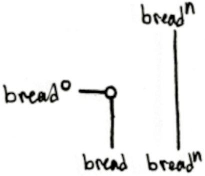
$\text{close}_n = \text{open}_n$



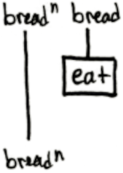
Eater : bread$^\circ \to I$



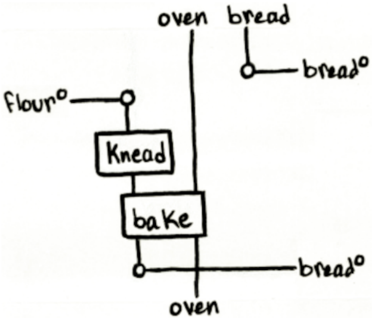
buy$_n$

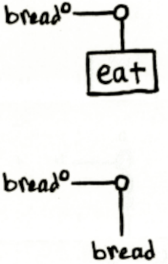


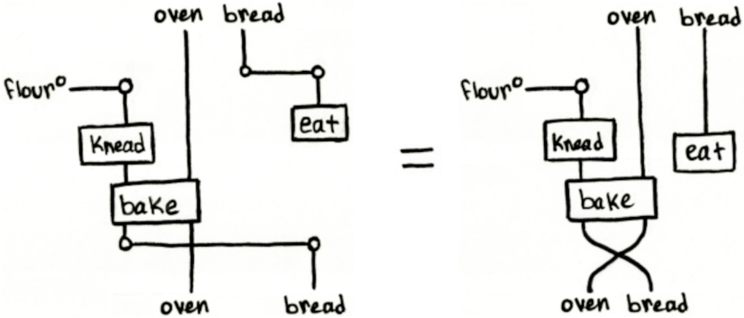
eat$_n$



digest$_n$


oven
bread
bread°
flour°
Knead
bake
bread°
oven


bread°
eat
bread°
bread


oven
bread
flour°
Knead
eat
bake
oven
bread
=
oven
bread
flour°
Knead
eat
bake
oven
bread

Thanks

for

Listening!

P. Katis, N. Sabadini, and R.F.C. Walters. On partita doppia. 1998.

Chad Nester. The Structure of Concurrent Process Histories. COORDINATION 2021.

Chad Nester. Situated Transition Systems. ACT 2021 (to appear).