ST: Set Theory in HOL

Extensionality

$$\forall s\ t.\ (s = t) = (\forall x.\ x \in s = x \in t)$$

Empty set

$$\exists s.\ \forall x.\ \neg(x \in s)$$

Union

$$\forall s.\ \exists t.\ \forall x.\ x \in t = (\exists u.\ x \in u \wedge u \in s)$$

Power sets

$$\forall s.\ \exists t.\ \forall x.\ x \in t = x \subseteq s$$

Separation

$$\forall p\ s.\ \exists t.\ \forall x.\ x \in t = x \in s \wedge p\ x$$

Foundation

$$\forall s.\ \neg(s = \emptyset) \Rightarrow \exists x.\ x \in s \wedge (x \cap s = \emptyset)$$

Replacement

$$\forall f\ s.\ \exists t.\ \forall y.\ y \in t = \exists x.\ x \in s \wedge (y = f\ x)$$

Infinity

$$\exists s.\ \emptyset \in s \wedge \forall x.\ x \in s \Rightarrow (x \cup \{x\}) \in s$$