# Groups in ST

- Textbook definition of a group:

```
ST_Group G • ∼ ı =
  (• ∈ G → G → G) ∧
  (∼ ∈ G → G) ∧
  (ı ∈ G) ∧
  (∀x y z ∈ G.
      x • (y • z) = (x • y) • z) ∧
  (∀x ∈ G. x • ∼x = ı) ∧
  (∀x ∈ G. x • ı = x)
```

- Messy to prove:

```
⊢ ST_Group G • ∼ ı
    ⇒ ∀x ∈ G. ∼x • x = ı

⊢ ST_Group G • ∼ ı
    ⇒ ∀x ∈ G. ı • x = x
```

- Because one needs to prove

```
⊢ x ∈ G ∧ • ∈ G→(G→G) ∧ ∼ ∈ G→G
    ⇒ ∼x • (x • ∼x) ∈ G
```

- HOL version handles this by typechecking