Group Theory Example (QED)

- Assume

$$\texttt{x} \bullet (\texttt{y} \bullet \texttt{z}) = (\texttt{x} \bullet \texttt{y}) \bullet \texttt{z}$$

$$\texttt{x} \bullet \imath = \texttt{x}$$

$$\texttt{x} \bullet \sim\texttt{x} = \imath$$

- Then

$$\begin{aligned}
\sim\texttt{x} \bullet \texttt{x} &= \sim\texttt{x} \bullet (\texttt{x} \bullet \imath) \\
&= (\sim\texttt{x} \bullet \texttt{x}) \bullet \imath \\
&= (\sim\texttt{x} \bullet \texttt{x}) \bullet (\sim\texttt{x} \bullet \sim(\sim\texttt{x})) \\
&= ((\sim\texttt{x} \bullet \texttt{x}) \bullet \sim\texttt{x}) \bullet \sim(\sim\texttt{x}) \\
&= (\sim\texttt{x} \bullet (\texttt{x} \bullet \sim\texttt{x})) \bullet \sim(\sim\texttt{x}) \\
&= (\sim\texttt{x} \bullet \imath) \bullet \sim(\sim\texttt{x}) \\
&= \sim\texttt{x} \bullet \sim(\sim\ \texttt{x}) \\
&= \imath
\end{aligned}$$

- Hence

$$\begin{aligned}
\imath \bullet \texttt{x} &= (\texttt{x} \bullet \sim\texttt{x}) \bullet \texttt{x} \\
&= \texttt{x} \bullet (\sim\texttt{x} \bullet \texttt{x}) \\
&= \texttt{x} \bullet \imath \\
&= \texttt{x}
\end{aligned}$$