Relations and Functions

- Relations:

$$\texttt{X} \leftrightarrow \texttt{Y} = \mathbb{P}(\texttt{X} \times \texttt{Y})$$

- Functions (partial and total):

$$\begin{array}{l}\texttt{X} \nrightarrow \texttt{Y} = \\ \quad \{\texttt{f} \in \texttt{X} \leftrightarrow \texttt{Y} \mid \\ \qquad \forall \texttt{x y1 y2}. \\ \qquad\quad \texttt{x} \mapsto \texttt{y1} \in \texttt{f} \wedge \texttt{x} \mapsto \texttt{y2} \in \texttt{f} \\ \qquad\quad \Rightarrow \\ \qquad\quad (\texttt{y1 = y2})\}\end{array}$$

- Total functions:

$$\begin{array}{l}\texttt{X} \rightarrow \texttt{Y} = \\ \quad \{\texttt{f} \in \texttt{X} \nrightarrow \texttt{Y} \mid \\ \qquad \forall \texttt{x}.\ \texttt{x} \in \texttt{X} \\ \qquad\qquad \Rightarrow \\ \qquad\qquad \exists \texttt{y}.\ \texttt{y} \in \texttt{Y} \wedge \langle \texttt{x}, \texttt{y} \rangle \in \texttt{f}\}\end{array}$$

- Function application

$$\texttt{f} \diamond \texttt{x} = \varepsilon \texttt{y}.\ \texttt{x} \mapsto \texttt{y} \in \texttt{f}$$