Functions Inside ST

- The notation:

  $\mathtt{Fn}\ \mathtt{x} \in \mathtt{X}.\ t[\mathtt{x}]$

  is like λ-abstraction inside ST:

  $\{\langle \mathtt{x}, \mathtt{y}\rangle \in \mathtt{X} \times \mathsf{Image}(\lambda \mathtt{x}.t[\mathtt{x}])\mathtt{X} \mid \mathtt{y} = t[\mathtt{x}]\}$

- $\mathtt{X} \longrightarrow \mathtt{Y}$ is the set of all set-functions from to $\mathtt{X}$
  - N.B. the symbol $\longrightarrow$ is overloaded!

- Function application

  $\mathtt{f} \diamond \mathtt{x} = \varepsilon \mathtt{y}.\ \langle \mathtt{x}, \mathtt{y}\rangle \in \mathtt{f}$

- Set function version of β-conversion

  $\mathtt{y} \in \mathtt{X} \vdash (\mathtt{Fn}\ \mathtt{x} \in \mathtt{X}.\ t[\mathtt{x}]) \diamond \mathtt{y} = t[\mathtt{y}]$

- Note HOL types:

  $(\mathtt{Fn}\ \mathtt{x} \in \mathtt{X}.\ t[\mathtt{x}]) : V$

  $(\lambda \mathtt{x}.\ t[\mathtt{x}]) : V \rightarrow V$