ε-operator and Axiom of Choice

- In HOL $\varepsilon x.\ P\ x$ is a term such that

  $$\forall P.\ (\exists x.\ P\ x) \Rightarrow P(\varepsilon x.\ P\ x)$$

- Hence:

  $$\forall s.\ \neg(s = \emptyset) \Rightarrow (\varepsilon x.\ x \in s) \in s$$

  which shows:

  $$\exists f{:}V{\to}V.\ \forall s.\ \neg(s = \emptyset) \Rightarrow f\ s \in s$$

- Thus Global Choice is a theorem