Theories as Theorems

- From the previous transparency:

  Definition: $\forall x:\alpha.\ \mathsf{f}\ x = x$

  Theorems: $\vdash\ \mathsf{f}\ 0 = 0$

  $\vdash\ \forall x:\alpha.\ \ \mathsf{f}(\mathsf{f}\ \ x) = x$

- Translated to set theory:

  Definition: $\forall \alpha\ \ x.\ \ x{\in}\alpha\ \ \Rightarrow\ \ (|\mathsf{f}|\ \ \alpha){\diamond}x\ = x$

  Theorems:

  $\vdash\ \ |0|{\in}|num|\ \ \Rightarrow\ \ (|\mathsf{f}|\ \ |num|){\diamond}|0|\ = 0$

  $\vdash\ \ \forall \alpha\ \ x.\ \ x{\in}\alpha\ \ \Rightarrow\ \ (|\mathsf{f}|\ \ \alpha){\diamond}((|\mathsf{f}|\ \ \alpha){\diamond}x)\ = x$

- As a single theorem:

$$
\begin{array}{l}
\vdash\ \ \forall f : V \to V. \\
\qquad (\forall \alpha\ \ x.\ \ x{\in}\alpha\ \ \Rightarrow\ \ (f\ \ \alpha){\diamond}x\ = x) \\
\qquad \Rightarrow \\
\qquad (|0|{\in}|num|\ \ \Rightarrow\ \ (f\ \ |num|){\diamond}|0|\ = |0|\,) \\
\qquad \wedge \\
\qquad \forall \alpha\ \ x.\ \ x{\in}\alpha\ \ \Rightarrow\ \ (f\ \ \alpha){\diamond}((f\ \ \alpha){\diamond}x)\ = x
\end{array}
$$