Theories versus Theorems

- HOL theories can't be encoded as theorems

  `Definition:` $\forall x:\alpha.\ \mathsf{f}\ x\ =\ x$

  `Theorems:` $\vdash\ \mathsf{f}\ 0\ =\ 0$

  $\vdash\ \forall x:\alpha.\ \mathsf{f}(\mathsf{f}\ x)\ =\ x$

  is not equivalent to:

  $$\forall f.\ (\forall x:\alpha.\ f\ x = x)\ \Rightarrow$$
  $$(f\ 0 = 0)\ \wedge\ (\forall x:\alpha.\ f(f\ x) = x)$$

  because variable f is used at different types

- With set theory:
  - theories can be encoded as theorems
  - 'theory interpretation' = specialisation
  - theories abbreviated with definitions