Types as Sets

- Set-theoretic type operators:
  - $X \times Y$ – Cartesian product of X and Y
  - $X \twoheadrightarrow Y$ – set of functions from X to Y
  - $\mathsf{List}\ X$ – set of lists over X
- Particular types:

$$\mathsf{true} \in \mathsf{Bool}, \quad \mathsf{false} \in \mathsf{Bool}$$

$$|\wedge| \in \mathsf{Bool} \times \mathsf{Bool} \twoheadrightarrow \mathsf{Bool}$$

$$|+| \in \mathbb{N} \times \mathbb{N} \twoheadrightarrow \mathbb{N}$$

- General typechecking theorems:

$$x \in X \ \wedge \ x \in Y \ \Rightarrow \ \langle x, y \rangle \in X \times Y$$

$$f \in (X \twoheadrightarrow Y) \ \wedge \ x \in X \ \Rightarrow \ f \diamond x \in Y$$

$$(\forall x.\ x \in X \ \Rightarrow \ t[x] \in Y) \ \Rightarrow \ (\lambda x.\ t[x]) \in (X \twoheadrightarrow Y)$$

$$x_1 \in X \wedge \cdots \wedge x_n \in X \ \Rightarrow \ \langle x_1, \ldots, \langle x_n, \varnothing \rangle \ldots \rangle \in \mathsf{List}\ X$$