Functions as Sets

- Functions represented by sets of ordered pairs
  - i.e. functions in set theory are sets
- Set-theoretic function application:

$$f \diamond x \;=\; \varepsilon y.\ \langle x, y\rangle \in f$$

  - ε is Hilbert's choice operator
- Set-theoretic function abstraction:

$$\lambda\!\!\lambda x \in X.\ t[x] \;=$$
$$\{\langle x, y\rangle \in X \times\!\!\times \mathsf{Image}(\lambda\, x.t[x])X \;\mid\; y = t[x]\}$$

  - $\times\!\!\times$ is set-theoretic Cartesian Product
  - $\mathsf{Image}\ \mathcal{F}\ X$ is image of set X under $\mathcal{F}$
    - exists via Axiom of Replacement
- Set-theretic version of β-reduction:

$$y \in X \;\Rightarrow\; (\lambda\!\!\lambda x \in X.\ t[x]) \diamond y \;=\; t[y]$$