First Order versus Higher Order

construction of natural numbers

- Define by recursion in logic

$$\mathsf{numV}\ 0 \ = \ \varnothing$$
$$\mathsf{numV}(\mathsf{SUC}\ n) \ = \ (\mathsf{numV}\ n) \ \cup \ \{\mathsf{numV}\ n\}$$

- $\mathsf{numV} : num \to V$ is injective
- Set-theoretic numbers $\mathbb{N}$ are range of numV
- $\mathsf{Vnum} : V \to num$ is inverse of numV
- Define operations on sets via logic:

$$x \oplus y = \mathsf{numV}((\mathsf{Vnum}\ x) + (\mathsf{Vnum}\ y))$$
$$|+| = \{\langle\langle x, y\rangle, z\rangle \in (\mathbb{N} \times \mathbb{N}) \times \mathbb{N} \ \mid \ z = x \oplus y\}$$

- No need for `wfrec`
  - recursion done 'outside' set theory