Higher Order ZF Axioms

Extensionality

$\forall s\ t.\ (s = t) \equiv (\forall x.\ x \in s = x \in t)$

Empty set

$\exists s.\ \forall x.\ \neg(x \in s)$

Union

$\forall s.\ \exists t.\ \forall x.\ x \in t \equiv (\exists u.\ x \in u \wedge u \in s)$

Power sets

$\forall s.\ \exists t.\ \forall x.\ x \in t \equiv x \subseteq s$

Separation

$\forall p\ s.\ \exists t.\ \forall x.\ x \in t \equiv x \in s \wedge p\ x$

Foundation

$\forall s.\ \neg(s = \varnothing) \Rightarrow \exists x.\ x \in s \wedge (x \cap s = \varnothing)$

Replacement

$\forall f\ s.\ \exists t.\ \forall y.\ y \in t \equiv \exists x.\ x \in s \wedge (y = f\ x)$

Infinity

$\exists s.\ \varnothing \in s \wedge \forall x.\ x \in s \Rightarrow (x \cup \{x\}) \in s$