# Generalised free extensions

Algebraic normalisation and dependent types

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Background

2. Contributions

Open terms Free extensions New Free extensions **Free extensions** modulo semantics of generalised algebras of algebras e.g., monoids, groups, rings dependently-typed operators *Key applications* Staged meta-programming Generalised algebra Applications Free extension Partial evaluation ✓ (Agda) Normalisation by evaluation Categories Semantics Monoidal categories Linear type systems Todo Proof synthesis First order PLs Cartesian categories Todo

3. *Examples* 

#### Purely functional PLs CCCs

Todo

Normalising in a commutative monoid

 $\rightarrow$  (N,+) extended by {x,y,z}  $\leftarrow$ Free variables Concrete monoid

**Syntax**: sums built from a binary operator with unit. 0



• Normal forms: bags of free variables and a *constant*.



$$\begin{array}{c} \longrightarrow \operatorname{Vect}_{\mathbb{R}} \text{ extended by} & f: \mathbb{R}^{2} \to X \\ X & Y: \mathcal{O} \leftarrow g: X \to Y \\ \uparrow & h: Y \to \mathbb{R}^{3} \end{array} \\ \end{array}$$

$$\begin{array}{c} Free \text{ morphisms} & - \end{array}$$

• **Syntax**: well-typed composites.







### Free extensions of algebras

• Simple categorical description in terms of a *universal property*.



## (ii) Generalised free extensions

• Include **types** by generalising the *universal property*.

**Dependent telescope** of **free** variables

### References

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#### Acknowledgements

Supported by an EPSRC Industrial CASE studentship. Further thanks go to Ohad Kammar for an endless supply of insightful comments and Sean K. Moss for some intensely illuminating discussions.