The Evolution of Agents

Mohammad Adil Qureshi

A dissertation submitted in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

of the

University of London.

Department of Computer Science

University College London

January 2001

Abstract

Genetic Programming(GP) is a technique that can be used to automatically program computers to perform some required task. The technique is a kind of genetic algorithm in which the representation is a program parse tree instead of a bit-string and the fitness of each parse trees is evaluated by executing the computer program that it represents.

The subject of this thesis is to investigate the use of GP to automatically program multiagent systems. To achieve this goal, we consider the general problems in creating multiagent systems, and show how GP can be used to provide solutions to many of them. Our key contributions are as follows:

We show that it possible to evolve multi-agent systems using GP that:

- exhibit coordinated, coherent behaviour
- communicate explicitly, and in doing so decide what to communicate and how
- can resolve conflicts
- can be integrated into an existing society of agents

We also consider the scalability problems involved in the use of GP, both generally and in particular as a technique for automatically programming agents, and propose solutions to these problems.

Acknowledgements

I would like to thank the department of computer science at UCL for awarding me an EP-SRC PhD quota award which funded this work. I would also like to thank my supervisor Jon Crowcroft for his encouragement and support. Many thanks to William Langdon for introducing me to GP, providing useful pointers and for reviewing some chapters of this thesis. Thanks also to Thomas Haynes for answering questions relating to his research. My thanks to UCL computer science and Java training company JB International for allowing me to use their machines for the compute intensive task of evolving agents.

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Chapter 1

Introduction

Nature has been very successful at ”programming” sophisticated multiagent systems ranging from ”simple” agents such as ants and bees to sophisticated cognitive agents - ourselves. The mechanism used to create these systems is Darwinian evolution. Our hypothesis is that a technique based on Darwinian evolution called Genetic Programming (GP) can be used to automatically program software multiagent systems.

A multiagent system is one in which several interacting, intelligent agents pursue some set of goals or perform some task. The central question in such systems is when and how should which agents interact in order to achieve their goals. Answering this question is difficult and raises a number of issues which are discussed further in the next chapter. We propose that we can use genetic programming to provide a solution to this question and the related issues that it creates, for any given problem domain.

Genetic Programming (GP) [Koz92b] is a relatively new technique that uses the principle of Darwinian evolution [Daw86] to automatically program computers. GP has many advantages over other techniques that have been used in an attempt to automatically program agents such as neural networks and conventional genetic algorithms. The most important is that the representation used by GP is a computer program. It is therefore possible for humans to understand generated solutions and therefore predict their behaviour (compare with neural networks). Furthermore, the generated programs are easily executed on current computer architectures and do not require any specialised environments. A second advantage is that GP has been shown to be very general and has been successfully applied to a wide variety of problems [Koz92b; Koz94b; KDBK99]. Koza claims GP is the most general machine learning technique [Koz92b].

To prove our hypothesis that Genetic Programming can be used to automatically program multiagent systems we attempt to answer the following questions:

- What are the key issues in implementing multiagent systems?

- Is GP capable of providing solutions to them?
- What are the problems in scaling GP to real world multi-agent applications?
- How can we solve these scalability problems?

Answers to the last three questions are supported by results from experiments.

Our key contributions are as follows:

We show that it possible to evolve multiagent systems using GP that:

- exhibit coordinated, coherent behaviour
- communicate explicitly, and in doing so decide what to communicate and how
- can resolve conflicts
- can be integrated into an existing society of agents

We also consider the scalability problems involved in the use of GP, both generally and in particular as a technique for automatically programming agents, and propose solutions to these problems. Related to scalability we discuss the importance of of formal software testing techniques in the development GP technology, and look at the issues involved in the testing of MAS.

Lastly we designed and implemented a state of the art GP system in the Java programming language and made it freely available (complete with source code and documentation) to the GP community in 1997. This system has been supported and kept uptodate and has facilitated GP research by others, which we believe is a key contribution to the GP community.

The layout of this thesis is as follows:

Chapters 2 and 3 introduce Genetic Programming and Multiagent Systems. Together, they set the technical grounding for this thesis. Research combining the two fields is surveyed, and an overview of our Genetic Programming system called GPsys is provided. The pursuit problem was designed as a tested for researching MAS architectures. Haynes and Sandip [HSSW95b; HS96c; HS97] used GP to evolve both heterogeneous and homogeneous agents for this problem. We note however that whilst their work was successful at demonstrating how GP can decompose tasks, conflict resolution was provided automatically by the environment and was not evolved. The main focus of chapter 4 is to show that we can evolve agents that also resolve conflicts using GP. We also further investigate task allocation for the pursuit domain. Communication is an important aspect of multiagent systems, chapter 5 demonstrates that GP can be used to evolve agents that communicate explicitly. We would like to be able add agents to a society of previously-created agents to solve new problems. In chapter 6 We show that GP can be used

to evolve agents that work within society of existing agents and learn to interact with them. If we are to make GP a general technique for creating multiagent systems we need to assess its scalability. Chapter 7 looks at scalability issues in GP with a particular emphasis on issues that relate to its application to MAS. The fitness of programs evolved using GP is evaluated by testing them. We believe that the GP community can benefit from formalised software testing techniques developed to test human coded programs in software engineering. Chapter 8 looks at software testing techniques for distributed and multiagent systems. Finally chapter 9 summarises our work, and critically evaluates it. Further areas of research stemming from our work are proposed. The appendices list the best evolved programs for each of the experiments that were performed.

Chapter 2

Genetic Programming

The purpose of this chapter is to introduce genetic programming as a means of automatically generating computer programs. The last part of the chapter provides an overview of a powerful Genetic Programming system that we have designed and implemented.

Some of the most sophisticated programs in existence are not human coded, they evolved naturally. They are to be found inside the cells of living creatures in the form of DNA. These programs were "coded" by the process of natural selection as described by Charles Darwin and are responsible for the highly optimised "machines" that we see around us. It is not surprising therefore that AI community has been active in tying to harness the power of natural evolution to build both hardware and software.

2.1 Genetic Algorithms

John Holland [Hol92] devised a class of algorithms inspired by Darwin's theory of evolution by natural selection called *Genetic Algorithms*. These algorithms abstract the key mechanisms used in natural evolution, namely *selection, reproduction and variation*, and use them for optimisation and search [Gol89]. A population of trial solutions to a given problem are first generated randomly. New solutions are bred from the fittest solutions selected from the old population by applying genetic operators to them. These genetic operators vary the solutions, so that new solutions are generated. The results is a new population of solutions. By repeatedly generating subsequent solutions in the same manner we can theoretically generate progressively better solutions. The best solution obtained so far is tracked during this process. When the best solution meets our termination criteria, or we decide the the maximum number of generations that we permit has been exceeded we terminate the run.

The representation of solutions used in Genetic Algorithms commonly consists of a bit-strings of fixed length. In an engineering optimisation problem for example, these bit-strings might encode a set of design parameters for which we are trying to find an optimal combination of values. The

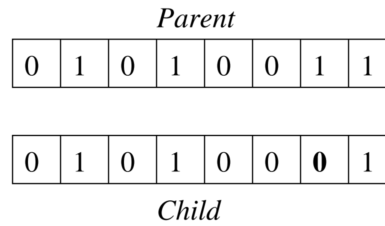


Figure 2.1: GA mutation

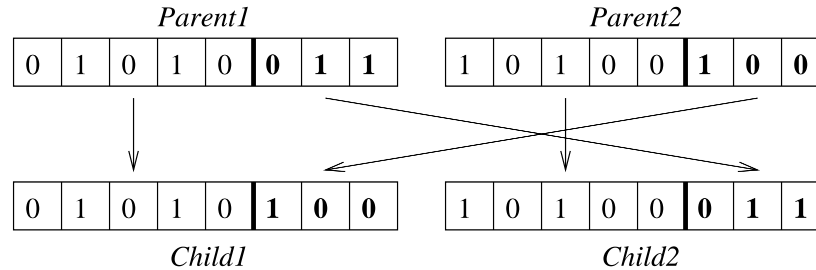


Figure 2.2: GA crossover

initial population consists of a randomly generated set of such bit-strings. The fitness of the bit-string is evaluated by measuring how useful it is. In our example we would decode each bit-string to obtain our parameters, apply them to our model, and evaluate how well our model behaves.

There are two selection schemes that are commonly used, *fitness proportionate selection* and *tournament selection*. In fitness proportionate selection, individuals are selected with probabilities in proportion to their fitness. This requires that the sum of the fitness values of the entire population is known, and makes parallel/distributed implementations difficult. An alternative to fitness proportionate selection is *tournament selection*. In tournament selection a small group of individuals are chosen randomly from the population(typically 7), and ordered by their fitness values. The fittest of these individuals are selected to take part in creating a new individual. In steady state GAs, the individual to be replaced is often the individual with the lowest fitness in the tournament. An advantage with tournament selection is that it is simple to implement, and secondly that by varying the tournament size, one can vary the fitness pressure.

The genetic operators used to create new individuals include *reproduction, mutation and crossover*. The reproduction operator simply creates an exact copy of the selected individual. The mutation operator copies the selected individual and then randomly changes a portion of the bit-string (figure 2.1). The crossover operator takes two individuals and exchanges random portion of the bit-strings of each individual (figure 2.2). This operator potentially creates two new individuals. Often however, only a single individual is created by discarding one of the resultant off-springs.

GAs have been successfully applied to a wide variety of problems including engineering, financial, imaging, VLSI circuit layout, gas pipeline control and production scheduling [Dav91], and have been shown to provide near optimal solutions. A mathematical explanation for their success called the Schemata Theorem was proposed by Holland [Hol73], and has been backed by empirical support [Gol89]. The space of bit-strings of a given length can be partitioned into sets, where each set consists of strings with similarities at certain positions in the string. For example consider the set of all 5 bit-strings which begin with 1 and end with 0. We can describe such a string by the *similarity template* or *schema* 1***0 where * represents a wild-card or "don't care" value. There are two properties of schemas that are used by the schemata theorem, *order* and *defining length*. The order of a schema H, denoted $o(H)$ is the number of fixed values in the bit string. For example the string 01**1 has an order of 3. Schemas of higher order are more specific than ones with lower orders. The defining length of a schema H, denoted $\delta(H)$ is the distance between the first and last fixed values. For example, the string 01*0* has a defining length of 3. The schemata theorem makes predictions on the processing of schemas by the GA which can be expressed mathematically as follows:

$$m(H, t+1) \geq m(H, t) \frac{f(H)}{\overline{f}} \left[1 - p_c \frac{\delta H}{l-1} - o(H) p_m \right]$$

where:

- $m(H, t)$ is the number of strings with schema H in the population at time t
- $f(H)$ is the average fitness of the strings with schema H in the population
- $\overline{f}$ is the average fitness of the population
- l is the length of the strings
- p_c is the probability of crossover
- p_m is the probability of mutation

The theorem predicts that short defining length, low order, above average schemas called *building blocks* receive exponentially increasing trials in subsequent generations. A possible explanation of how GAs work is therefore by combining building blocks (or partial solutions) to form strings of high fitness (optimal or near optimal solutions).

2.2 Variable Length and Hierarchical representations

A variable length representation for GAs was first proposed by Smith [Smi80] to create a variant on the classifier systems (evolved production systems) defined by Holland and Reitman [HR78].

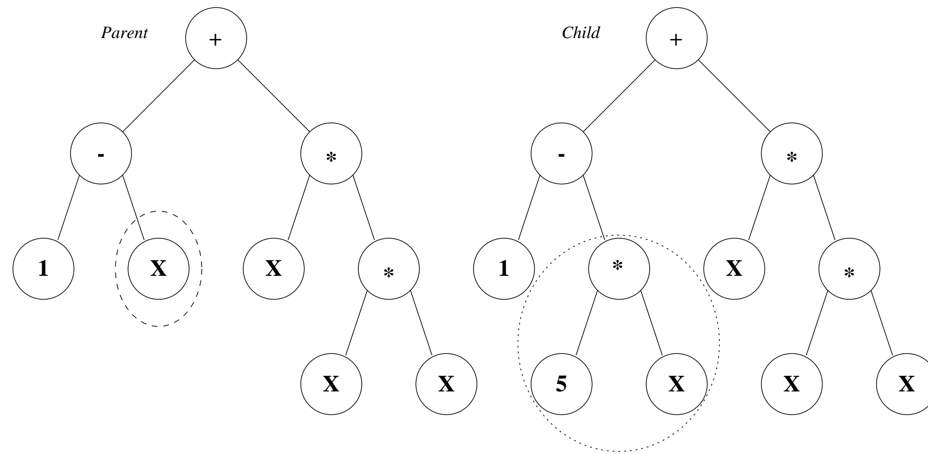


Figure 2.3: GP mutation

The GA was used to evolve complete rule-based programs for playing poker. The crossover operator used by Smith set restrictions on where crossover can be applied in an evolving structure. This inspired Cramer [Cra85] to use a tree based representation to evolve symbolic expressions, using sub-tree crossover. Koza developed a technique for using sub-tree crossover to evolve expressions in the LISP programming language and successfully demonstrated the usefulness of the technique on wide range of problems [Koz92b]. This technique is now known as genetic programming.

2.3 Genetic Programming

Genetic Programming is technique for automatically programming computers. It uses the power of GAs to search the space of possible computer programs to find suitable programs for the desired application [LQ95]. Koza describes it as "the most general machine learning technique", a claim which is substantiated by the large number of applications to which it has been successfully applied [Koz92b; Koz94b; KDBK99]. The representation commonly used in GP is a program parse tree. Program parse trees have the advantage that they represents syntactically correct computer programs, and are easy to manipulate. The branches in the parse tree represent functions which take arguments, the leaves represent zero-argument functions, variables or constants. The fitness of a GP is measured by testing the program that it represents. Fitness is awarded to programs in proportion to how well they perform.

The reproduction, mutation and crossover operators work with program parse trees instead of bitstrings. Reproduction creates a clone of the selected individual. Mutation works by copying the parent parse tree, selecting a node in the copy at random and then replacing it with a randomly created subtree (figure 2.3). The latter is created using the same process used for generation of initial population. The crossover operator works by copying the parse trees of the parents,

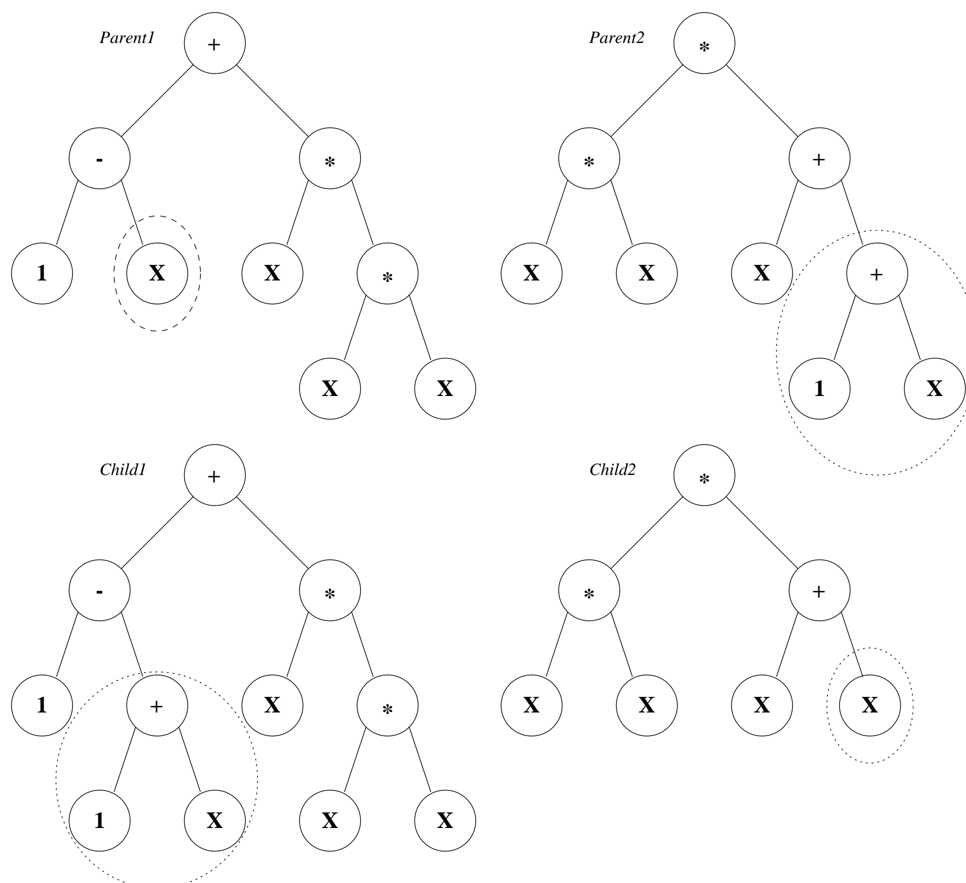


Figure 2.4: GP crossover

selecting random nodes from the copies and swapping them (figure 2.4). Although two children can potentially be created, most GP implementations discard one of them.

The standard GP algorithm is essentially the same as a GA, and is described with pseudo code in table 2.1. The most common selection methods used in GP are fitness proportionate selection and tournament selection, and work in the same way as they do in GAs.

The initial population is created by randomly generating program parse trees. The parse trees are formed by selecting functions and terminals from problem specific sets defined by the user. The specific selection mechanism will be described later in section on run parameters. To allow arbitrary trees to be generated, the functions in the function set must accept arguments chosen from any of the primitives in the function and terminal sets. This property is called *closure*. During tree generation, whenever a function is selected, subtrees for its arguments are also created recursively using the same process. Tree growth is restricted both naturally when the tree has leaves on each outer node, and artificially when a branch of the tree reaches the maximum allowable depth. Other possibilities include when the tree has the maximum allowable nodes, this is typically used with linear representations. To prevent very short trees the root node is usually

Table 2.1: The GP Algorithm

```
initialise population with randomly generated individuals
evaluate the fitness of the individuals in the population
while the termination criteria has not been met
     for p = 1 to size of population
         choose a genetic operator
         select parent(s) from current population based on fitness
         create offspring using selected operator and parent(s)
         place offspring in the new population
         evaluate the fitness of the offspring
     endfor
     the current population becomes the new population
endwhile
```

always chosen from the function set.

2.3.1 The Five Basic Steps

There are 5 preparatory steps required to code a GP application [Koz92b]. These involve defining the following:

1. *The Terminal Set*
2. *The Functions Set*
3. *The Fitness Function*
4. *The Run Parameters*
5. *The Termination Criteria*

Koza [KDBK99] describes the first two steps as defining the search space for the problem, and the fitness measure as a means of determining the outcome of the search.

2.3.2 The Function and Terminal Sets

The function and terminal sets together form a limited language with which GP can generate programs. The function set contains the primitive functions in the language that take one or more arguments. These functions are selected because they a-priori are believed to be useful for solving the problem. The functions can be typical constructs found in most programming languages such as:

- conditionals (if, if-then-else, or switch statements)

- loops (while, for, do-while)
- recursive function calls
- variable assignments
- indexed memory operations (array manipulation)
- math functions
- logical and bitwise operations

Alternatively they can be user defined functions peculiar to the application domain, such as *If-CellOccupied*, or *sendMessage*. It should be noted that the inclusion of loops and recursion create the possibility that a program loops or recurses indefinitely, thereby making fitness evaluation difficult. Many solutions to the iteration [Koz94b; KDBK99], and recursion [Bra95; Bra96; WL96; YC98; KDBK99] problems have been provided.

The terminal set contains the variables, constants, zero-argument functions, and *random ephemeral constants* for the application. Random ephemeral constants are real numbered or integer constants that are initialised with random values at the start of the run and maintain their values throughout the run.

2.3.3 The Fitness Function

The fitness function measures the success (fitness) of an evolved program. The value is typically obtained by executing the program for a set of test cases, and measuring how closely the programs generates the desired output. The fitness of a program can also contain other measurements such as efficiency and size of code. The measured fitness of a program is used to determine whether it will be selected to take part in the production of the next generation. Good fitness functions are those that are accurate and highly sensitive to differences in performance of the programs.

The fitness measure can be thought of as a means of specifying the high-level requirement specifications for the solution. It describes “what needs to be done but not how to do it” [KDBK99].

2.3.4 The Run Parameters

Two of the most important parameters are the *population size* and the *number of generations*. These parameters determine the amount of exploration and exploitation performed by the GA. A small population that is evolved for a large number of generations is biased for exploitation. Similarly, a large population evolved over a small number of generations is biased towards greater

exploration. By carefully choosing the value of these parameters, one can balance exploration with exploitation.

The reproduction, crossover and mutation probabilities are used to select the genetic operator to be used to create offsprings. They specify the probability with which each of the three genetic operators are chosen. Reproduction and mutation probabilities are usually set to be very small, mutation rates of less than 0.05 are quite common.

The shape and depth of the trees created for the initial population can be specified by the creation method, and the maximum depth parameters respectively. There are two creation methods commonly in use called the *full* and *grow* methods. The full method creates symmetric trees that are of maximum depth along any branch from the root. This is achieved by selecting the root node randomly from the function set, and then continuing recursively to select randomly from the function set for sub-nodes until the maximum allowable depth is reached (at which point a node is selected from the terminal set).

Selecting the grow method creates asymmetric trees. As with full trees, the root is always chosen from the function set, this time however we continue recursively by selecting children from the function and terminal set, until either a terminal is selected or the maximum depth is reached (at which point a terminal is always selected).

A third method called *ramped half-and-half* is used to ensure population diversity in the initial population. The size of trees created is ramped up from 2 to the maximum allowable depth. Half of these trees are created using the full method, and the other half are created using the grow method.

2.3.5 The Termination Criteria

The termination criteria determines when the algorithm should stop. Typical termination criteria include:

- the number of evolved generations exceeds some maximum
- the fitness of the best individual reaches desired value
- the genetic information known to be needed for the solution has been lost

The last first and last of these effectively abort a less than successful run.

2.4 GP Extensions

The original GP technique has been advanced with many extensions. Those that have been used or referenced later in this thesis are described in this section. A more complete survey of

techniques can be found in [LQ95] and in [BNKF98].

2.4.1 Automatically Defined Functions

If we chart the development of programming languages, we see a steady trend of higher and higher levels of abstraction being applied. The earliest languages were machine code and were quickly replaced by assembly level languages. These in turn lead to the development of high level languages such as FORTRAN. The latter facilitated code reuse first in the form of subroutines and later in the more easily parameterised forms of functions and procedures. Useful functions were made available in the form of libraries which are themselves a higher form of functional abstraction. The introduction of object oriented languages which support data abstraction or abstract data types (ADTs) further increased the amount of code reuse.

The advantages of abstraction are that it allows the control of complexity and facilitates code reuse. Code reuse is particularly useful for GP because it means that a block of useful code need not be independently rediscovered at different places within the program where it is needed. In this section we look at a key technique that has been used to introduce abstraction into GP.

Automatically Defined Functions(ADFs) [Koz93; Koz94b] are evolvable functions (subroutines) within an evolving genetic program, which the main result producing branch of the program and possible other ADFs can call. Each ADF is a separate tree; consisting of its own terminal and functions sets. The terminal sets for ADFs that take arguments will also contain terminals for each argument. These terminals get instantiated with the values of the parameters that have been passed when the ADF is called during fitness evaluation. The functions sets for the ADFs can contain primitive functions, as well as other ADFs that have been defined. The result producing branch is just a special ADF which is called by the fitness function when evaluating the program. ADFs effectively act as functional units or building blocks that can be combined to create programs.

The crossover and mutation operations are modified to work with ADFs, so that mutation mutates an ADF at random, and crossover exchanges subtrees between the same ADFs of each individual. Depending on the implementation crossover will either be performed for a single ADF, or on all ADFs.

ADFs have been successfully used on problems that proved too difficult for genetic programming without ADFs. The addition of ADFs adds an extra step to the preparatory steps for a GP application. The extra step defines the architecture in terms of number of ADFs and the arguments that they take, and the result producing branch.

2.4.2 Strongly Typed GP

Koza's early GP system was restricted in that, functions and terminal were designed to be all of the same type. This property that Koza calls *closure* allowed functions to take any terminals or function calls as arguments, and return any terminals or function calls as results. The advantage with this approach was that it greatly simplified tree generation and their manipulation by the crossover and mutation operators.

Montana [Mon94] relaxed this restriction in his Strongly Typed Genetic Programming (STGP) system thus allowing functions and terminals to have type specifications. STGP also supports generic types, which help reduce the number of different function definitions that are needed. In the implementation, the GP algorithm was modified to ensure that arguments to functions are always of the correct type, and that the generated program (or ADF) returns the correct type. Tree generation was facilitated by type possibility tables (computed at the start of a run) that list functions and terminals available at a given tree depth. Care is taken so that when a tree is mutated the root of the subtree replacing the node that is mutated is of the same type. Similarly crossover is restricted so that only subtrees with roots of the same type may be exchanged. Montana argues that the increase in complexity in the GP algorithm is more than compensated by the considerable reduction in the size of the search space, which in turn reduces the effort required to find solutions.

A concern with this approach, is the effect that it has on the connectivity of the search space. Highly restrictive grammars may make the search space too sparse, making it difficult to move from one point to another. This in turn is likely to cause premature convergence.

2.4.3 Steady State GP

The standard GP evolutionary engine is described as *generational* because it works in stages called generations. After the initial population has been created, each subsequent generation is created by applying genetic operators to individuals selected from the last generation to form an entire new population which replaces the old. New individuals are not made available for reproduction until the entire new population has been created.

In *Steady State GP (SSGP)* engine [Rey92; Rey94b; Rey94c], there is no notion of new or old generations, instead there is just a continuous change of individuals in the population. When a new individual is created it replaces an existing individual in the population, and hence immediately becomes available for reproduction. The benefits of this approach are that it is memory efficient, and that it is easy to parallelize. It is memory efficient because only one population needs to be in memory, where as with a standard generational engine, two are required.

2.4.4 Indexed Memory and Data Structures

In his earlier work, Koza used named memory to allow state to be maintained. The function Set-Value in conjunction with a terminal representing a named memory location(variable) can be used to set the value of that memory location. Teller [Tel94] added indexed memory capability to GP. He used an array of memory locations which could be manipulated via read and write functions. The latter take the index of the memory location as an argument.

Langdon [Lan95; Lan96a; Lan96b; Lan96c] used GP with indexed memory to successfully evolve queue and stack abstract data types (ADTs) using a chromosome structure with multiple trees. Each tree in the chromosome was used to represent one of the required ADT member functions. The results suggest that it may be possible to extend the level of abstraction in GP from pure functional abstraction as in ADFs to full ADTs. This may help increase the scalability of GP.

2.4.5 Co-Evolution

In nature organisms interact not just with their environment, but also with each other. Sometimes they are in direct competition with each other as happens with predators and prey, or parasites and hosts. The fitness of these organisms becomes relative to each other. A predator is deemed fit if it can catch prey necessary for its survival. Conversely, a fit prey is better able to escape predators. Dawkins [Daw86] describes how such *competitive co-evolution* leads to an arms race, where each organism evolves to counteract any improvement by evolution of its competitor.

This approach has been applied to GP [Koz91; AP93; Jan94; Rey94a; HSSW95b], and involves creating and evolving two or more populations instead of one. The fitness of individuals in one population is measured relative to individuals in the other and vice versa.

We can do this by testing each individual in one population against:

1. each individual in the other
2. a random subset of the other
3. the current best of the other

The first method is likely to be too compute intensive to be practical and hence one of the other two techniques is normally used.

In *cooperative co-evolution* (which is comparable to symbiosis in nature [Sap94]) the programs are tested together, and pairs of programs that work well together are rewarded with higher fitness.

2.5 Theory

2.5.1 Fitness Landscapes

In the traditional GA individuals in a population are represented as fixed length bitstrings. The total number of possible individuals is equal to the total number permutations of the bitstring, or 2^n where n is the of bits in the bitstring. Suppose that we plot these individuals as points on a horizontal plane such that the distant between individuals is equal to the number of bit changes required to get from the one individual to the other (the hamming distance). Suppose then that we give these points a vertical component proportional to the fitness values of the individuals that they represent. What we end up with is called a fitness landscape, where the peaks represent location of high fitness, and the valleys locations of poor fitness. The actual ruggedness of this landscape would depend on the difficulty of the fitness function. A particularly difficult function might result in a fitness landscape which is flat and virtually featureless, with the exception of a few widely distributed spires (figure 2.5). Conversely a very easy fitness function might consist of a "mountain" with a single peak (figure 2.6).

When we generate an initial random population of individuals, we are effectively placing a mesh over the fitness landscape, simultaneously sampling the fitness function at different places. Genetic operators such as mutation and crossover then provide a means for exploring this landscape. Single bit mutation allows us to move one point at a time across the landscape. Crossover and multi-bit mutation allow us to take giant leaps across the landscape. An assumption of the GA is that if we select individuals of higher fitness, then by applying the genetic operators, we should end up on the landscape at a point of high fitness. Thus there should be some correlation between parents and children, This notion is called auto-correlation.

The fitness landscape provides a powerful tool for visualising the difficulty of a problem. However, if we try to use this tool in GP, we run into one major difficulty; how to decide which points are adjacent to each other. In the fixed-length standard GA we use the hamming distance, but this is not possible in the GP representation which is neither bit oriented nor is it of fixed length. The individuals in GP are program trees which must be compared in some way to determine how far apart they are to be plotted on the fitness landscape. The latter is not very obvious. Kinnear [Kin94] examines the structure of the fitness landscape on which GP operates, and analyses the landscapes of a range of problems of known difficulty in order to determine the correlation between landscape measures and problem difficulty. The landscape analysis techniques that he employs include adaptive walks, and the autocorrelation of the fitness values of random walks. His results indicate that former shows better correlation with the problem difficulty than the latter for GP.

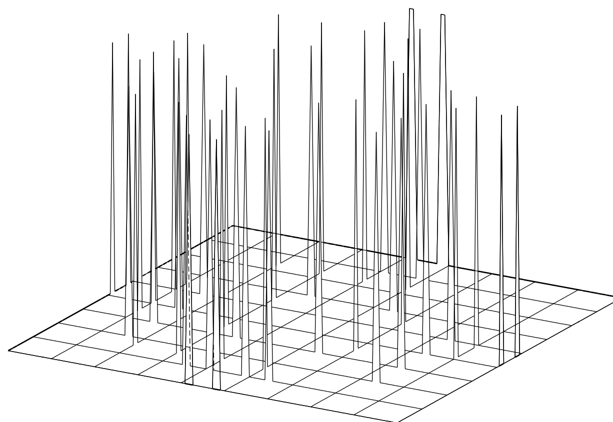


Figure 2.5: A difficult to search fitness landscape.

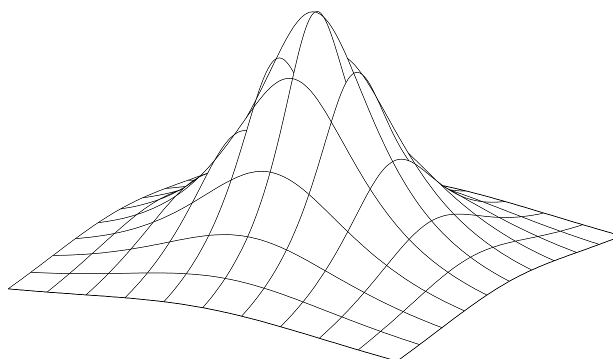


Figure 2.6: An easy to search fitness landscape.

2.5.2 GP Schema Theorem

A number of attempts have been made to provide a schema theorem(ST) for GP. All the early attempts were position-less STs, in which information about the position of the schema components were omitted. Position-less schemas can be instantiated many times in the same program. Koza defined a schema as a set of trees all having one or more subtrees in common [Koz92b]. O'Reilly and Oppacher [OO95] extended this definition to include incompletely defined expressions called *tree fragments*. A tree fragment is a tree with at least one "don't care"(symbol #) leaf. In their definition a schema is multiset of subtrees and tree fragments.

In later attempts positioned schemas were used, in which schemas are represented using rooted trees or tree fragments. Rooted schemas unlike position-less schemas, can be instantiated no more than once in a tree. Rosca [Ros97; RB99] devised a rooted tree schema theorem in which a schema is a tree composed of the same function and terminal sets as used in the run, with the exception that the terminal set includes "#", which can stand for any valid subtree. In Poli and Langdon's schema [PL97], the "don't care" symbol ("=" in their case) is not restricted to appearing in the leaves of schema trees, and furthermore instead of representing any valid sub-

tree, it represents a single node. A schema therefore represents the set of trees that are of the same size and shape, and which have the defined nodes of the schema in the same places. Their schema theorem applies only to one-point crossover, and asymptotically converges to Holland's GA schema theorem. More recently in their hyperschema theorem [Pol00] they allow terminals in a schema to contain either a wild-card that represents any node("="), or any valid sub-tree.

2.6 GP Applications

GP has been successfully applied to a very wide range of problems including:

image processing	*computer graphics*
process control	*autonomous agents*
robotics	*data mining*
electical circuit design	*neural networks*
modeling	*natural language processing*
classification	*pattern recognition*
signal processing	*art*

A survey of some these applications can be found in [LQ95; BNKF98; Koz92b; Koz94b; KDBK99]. William Langdon maintains a bibliography of published material at:

"http://www.cs.bham.ac.uk/ wbl/biblio/README.html".

2.7 GPsys

To help advance research in Genetic Programming we designed and developed an extendable GP system in the Java programming language. The system consists of over 16600 lines of code, at least 50% of which are comments and documentation. This environment was used for all the experiments in this thesis and is also used by many GP researchers worldwide(the current release has over 500 recorded downloads). GPsys can be downloaded from:

"http://cs.ucl.ac.uk/staff/A.Qureshi/gpsys.html".

The design goals of GPsys and how they were realised are shown in table 2.2.

We deemed the most useful GP extensions to be STGP and ADFs, these were consequently in the design. It should be noted that in our ADF implementation, crossover is performed for each ADF. Our original design (GPsys 1) used a SSGP engine to improve memory efficiency, the current release has both a SSGP and a memory efficient generational engine. Many of our design goals were realised just by choosing Java as the implementation language.

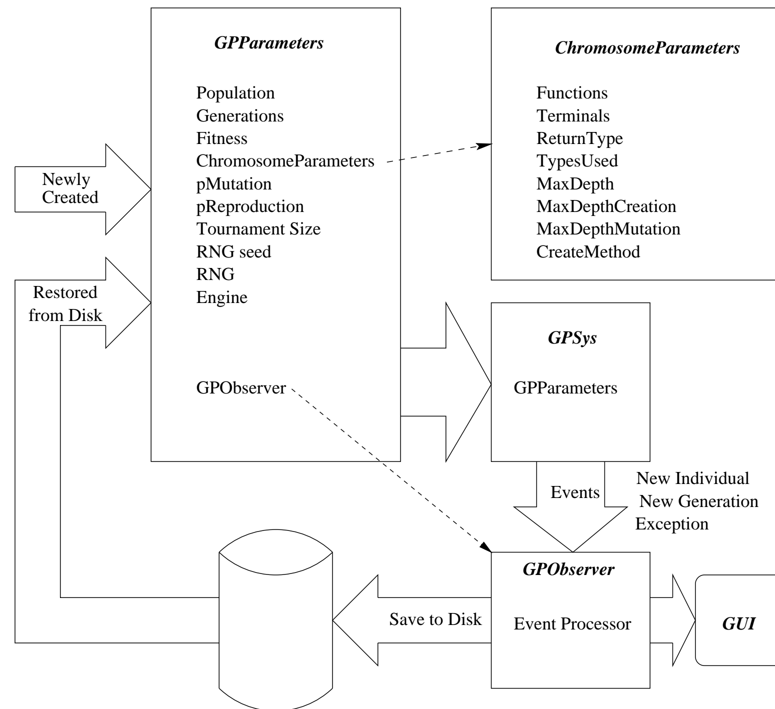


Figure 2.7: GPsys a High Performance GP System

2.7.1 The Java Programming Language

Java is a robust, powerful and efficient, multi-threaded, distributed object oriented programming language. Java compiles into bytecode, which is a machine code for a virtual machine called the Java Virtual Machine (JVM). The JVM interprets the bytecodes to execute the program, and has been ported to many different platforms (operating systems and processors) and provides Java with excellent portability. Most JVMs contain code to speed up the execution of Java programs including just in time compilers and run-time optimisers. The former dynamically compiles bytecode into the machine code for the target processor, the latter monitors program execution and optimises the executing code. Together these techniques overcome any performance arguments against Java.

The Java language has been designed for robustness, which is achieved by minimising programming errors. Programming errors are minimised by replacing the use of pointers by references. References differ from pointers in that only the JVM can generate references, which cannot be used in arithmetic operations to create new references. This feature alone removes many pointer errors that plague languages like C and C++ such as segmentation violations and bus errors. Arrays access is also checked at runtime for out of bound errors. The inclusion of exception handling forces the programmer to consider and deal with run-time exceptions at compile time. This again reduces the probability of errors.

Design Goal	Implementation
easy of use	Plug and Play model Factory/Observer-Observable design patterns
support for most useful GP extensions	ADFs, STGP
extensibility	OO-Design
flexibility	OO-Design
portability	Java
robustness	Java, Exception Handling
high performance	Java
efficiency	SSGP, Memory Efficient Generational Engine
support for persistence	Java Serialization, GZIP Compression
support for parallel/distributed architectures	Java Networking/RMI

Table 2.2: GPsys Design Goals and Implementation

Java has an extremely rich library of pre-built objects which are included in every JVM. These objects include support for graphics, databases, networking, input/output, advanced data structures, string manipulation, compression algorithms and much more. Thus small, very powerful programs can be written with minimal effort.

There is excellent support for distributed computation, in the form of Sockets, or Remote Method Invocation. JVMs are also included in many web browsers, which allows applets (Java code designed to be run in a browser) to be loaded and executed securely within the browser thus opening the possibility for massively distributed computing.

2.7.2 Application Programmers Interface

One of the key goals of GPsys was that it should be easy to use, requiring no code changes to the GP engine by the user. We therefore created a "plug and play" model as depicted in figure 2.7. To use GPsys, one instantiates a GPsys object, and then calls the evolve method, which starts evolution. In order to instantiate GPsys, we must pass to the constructor a *GPParameters* object.

The GPParameters object codes all the parameters that are required by GPsys, and in addition is used by GPsys to store all state information during a run. The GPParameters class definition includes methods that allow a GPParameters object to be saved and restored from disk, hence allowing a GPsys run to be suspended and restarted. The full set of parameters that can be specified include parameters for the run, and parameters for the problem (table 2.3). GPsys supports ADFs, and the parameters for each ADF tree is specified using ChromosomeParameters (table 2.4). The function and terminal set for each ADF is specified as Function and Terminal

arrays in a ChromosomeParameter. Each Function and Terminal in these arrays implements the *factory design pattern* [GHJ+95], so that GPsys can create many instances of these objects by calling factory methods. Many common GP primitives are provided as part of a primitives package in GPsys, including conditionals, arithmetic functions, and indexed memory operators. Some of these primitives support generic types.

The object used to store and measure the fitness of an Individual is specified by the Fitness object in the GPParameters object. Once again, the factory design pattern is used, so that many Fitness objects can be created using this one instance. The Fitness object abstracts the user implementation of a fitness function and fitness representation. Abstract methods define the contract that the user must fulfill to allow the fitness object to be used by GPsys(table 2.5). The termination condition is also specified by the fitness function.

GPsys makes use of the *observer/observable design pattern* [GHJ+95] to provide a clean way of monitoring a run. The users need to create and register an *observer* object with GPsys (the *observable* object) in order to be told when something interesting happens. The observer object must be an instance of a class that implements the *GPObserver* interface, and is registered by passing a reference to it in the GPParameters object. The GPObserver interface specifies the call-back methods that are used to communicate information by GPsys to the user (table 2.6).

2.7.3 Internal Data Structures

The internal data structures used by GPsys are shown in figure 2.9. A Population consists of among other things an array of Individuals. Each Individual has associated with it an array of Chromosomes, one for each of the ADFs and a Fitness. The result producing branch(RPB) is normally the first ADF in the array, although this can be changed. Each Chromosome consists of a tree of Genes.

There are two kind of Genes in the tree, GeneFunctions and GeneTerminals. GeneFunctions represent function calls and hence have an array of Gene arguments associated with them. GeneTerminals are the leaves in the Gene tree and represent Terminals. All Genes hold Primitives which contain the Type and code definition for the primitive which they represent.

There are two kinds of Primitives, Functions and Terminals. Functions represent Primitives which take arguments, and therefore contain Type specifications for each argument that they take. Terminals represent zero-arity primitives, and hence just contain code definitions. ADFunctions and ADTerminals are subtypes of Functions and Terminals which represent automatically defined functions and terminals respectively. They therefore do not contain code definitions, but instead contain an index of the Chromosome that defines them in an Individual's Chromosome array.

Run Parameters	
Parameter Name	Description
populationSize	The size of the population to be used.
generations	The maximum number of generations.
pMutation	The probability that a mutation operation is chosen.
pReproduction	The probability that a reproduction operation is chosen.
tournamentSize	The size of the tournament to be used for tournament selection.
rng	The random number generator to be used.
rngSeed	The seed to be used for the random number generator.
engine	The GP engine to be used for evolution. Possible selections include a generational engine or a steady state engine.
Problem Definition Parameters	
Parameter Name	Description
adf	An array of ChromosomeParameters which define the parameters for each Chromosome (ADF) to be evolved.
fitness	The fitness definition for the problem.
observer	The observer object to which information during evolution is sent.

Table 2.3: GPParameters

Users create their own functions and terminals by extending Function, Terminal, ADFunction or ADTerminal classes. Functions and Terminals are evaluated by GPsys when a GP is executed during fitness evaluation. The return value can be of any type, and depending on the type, the user must override one of the evaluation methods which forms part of the contract of a user defined Function or Terminal. These evaluation methods have been divided for efficiency reasons into those that return Java primitives (byte, short, int, long, char, boolean, float, double) or those that return an object or an array.

The Population object also contains a reference to a CrossoverBookingKeeping data structure. The latter is used to implement memory efficient crossover for the generational evolutionary engine as described by Koza [KDBK99]. Most GP systems maintain two populations during a run, the first is the active population which is used to create the next population. This new population then becomes the active population in the next iteration. The memory efficient crossover process divides the task of generating a new population into two phases. In the first phase we decide which parents from the current population are going reproduce using which genetic operation. This information is stored in a book keeping data structure. In the second phase we execute the

Parameter Name	Description
createMethod	The method used to create the initial population. Possible values include Full, Grow and Ramped Half-and-Half.
maxDepthAtCreation	The maximum creation depth of the Gene Tree for this chromosome.
maxDepthAtMutation	The maximum depth of trees created for mutation.
maxDepthAt	The maximum depth of trees for this chromosome.
functions	The function set for this chromosome.
terminals	The terminal set for this chromosome.
type	The type to be returned by this chromosome.
types	The set of types used by this chromosome.

Table 2.4: ChromosomeParemeters

Function	Description
void add(Fitness f)	Add a fitness to this fitness object.
void divide(int divisor)	Divide the fitness by the specified integer.
boolean equals(Fitness f)	Tests if this fitness = f.
boolean lessThan(Fitness f)	Tests if this fitness < f.
boolean greaterThan(Fitness f)	Tests if this fitness > f.
Fitness instance()	Creates a default Fitness object.
Fitness instance(GPParameters p, Individual i)	Creates a Fitness Object by evaluating the fitness of i.

Table 2.5: Fitness

genetic operations to create the new individuals.

The advantage with this approach is that by carefully scheduling the order in which we execute the genetic operations we can greatly reduce the amount of individuals that we need to hold in memory. A pigeon hole argument can be used to calculate that if the population contains M individuals, then we only need space for at most $M+2$ individuals, instead of the $2M$ individuals normally used. The CrossoverBookKeeping data structure holds four lists. These four lists hold information about parents and the genetic operations in which they are involved. Parents that have 0, 1, 2 and more than two genetic operations outstanding are stored in lists 0, 1, 2 and 3 respectively. When executing the genetic operations, the individual to be replaced is chosen from list 0 (which is always initialised to contain information about the two extra slots mentioned earlier). The parent to be used in the next genetic operation is the parent with the least number

Function	Description
void generationUpdate(GPParameters p, int cm)	Invoked after a new generation has been created. cm specifies how generation was created (via creation, from a stream or evolved)
void individualUpdate(GPParameters p, Individual i, int cm)	Invoked after a new Individual has been created. cm specifies how the Individual was created (randomly, via reproduction, mutation or crossover).
void diagnosticUpdate(java.lang.String s)	Invoked whenever unusual event occurs, s indicates what happened.
void exception(GPException e)	Invoked whenever an exception occurs.

Table 2.6: GPObserver

of outstanding genetic operations.

2.8 Conclusion

We can think of GP from two different perspectives. From a *machine learning* perspective [Mit97], GP is a general technique that allows a machine to learn how to solve a given task from training examples (the test cases). ML researchers would describe GP as a *beam search* in which the population of computer programs is the beam, and the evaluation metric is the fitness function. From an *algorithmic complexity theory* perspective [LV97], we can view GP as compression algorithm which creates a compressed representation (the computer program) of a set of input/output tuples (the test cases).

Primitive
(from gpsys)
+instance
+toString

Terminal
(from gpsys)
+evaluateBoolean
+evaluateByte
+evaluateChar
+evaluateDouble
+evaluateFloat
+evaluateInt
+evaluateLong
+evaluateObject
+evaluateShort

Function
(from gpsys)
+evaluateBoolean
+evaluateByte
+evaluateChar
+evaluateDouble
+evaluateFloat
+evaluateInt
+evaluateLong
+evaluateObject
+evaluateShort

Type
(from gpsys)
+typeID
+typeName
+Type
+toString

0..1
+type *
+argTypes *
+types
0..1
+type

ADTerminal
(from gpsys)
+adfIndex

ADFunction
(from gpsys)
+adfIndex

+terminals *
+functions *

Fitness
(from gpsys)
+add
+divide
+equals
+greaterThan
+instance
+lessThan
+terminationCondition
+toString

ChromosomeParameters
(from gpsys)
+createMethod
+functionsOfTypeAtDepthFull
+functionsOfTypeAtDepthGrow
+maxDepth
+maxDepthAtCreation
+maxDepthMutation
+terminalsOfType
-initialiseFunctionsTypeTable
-initialiseFunctionsTypeTableFull
-initialiseFunctionsTypeTableGrow
-initialiseTerminalsTypeTable
+initialiseTypeTables

0..1
+fitness

«interface»
GPObserver
(from gpsys)
+diagnosticUpdate
+exception
+generationUpdate
+individualUpdate

0..1
+observer

+adf *

GPParameters
(from gpsys)
#creationIndex
+engine
+generations
+pMutation
+population
+populationSize
+pReproduction
+rng
+rngSeed
+tournamentSize
+load
+save
+writeReport

«uses»
«uses»

Figure 2.8: GPsys Parameters UML diagram

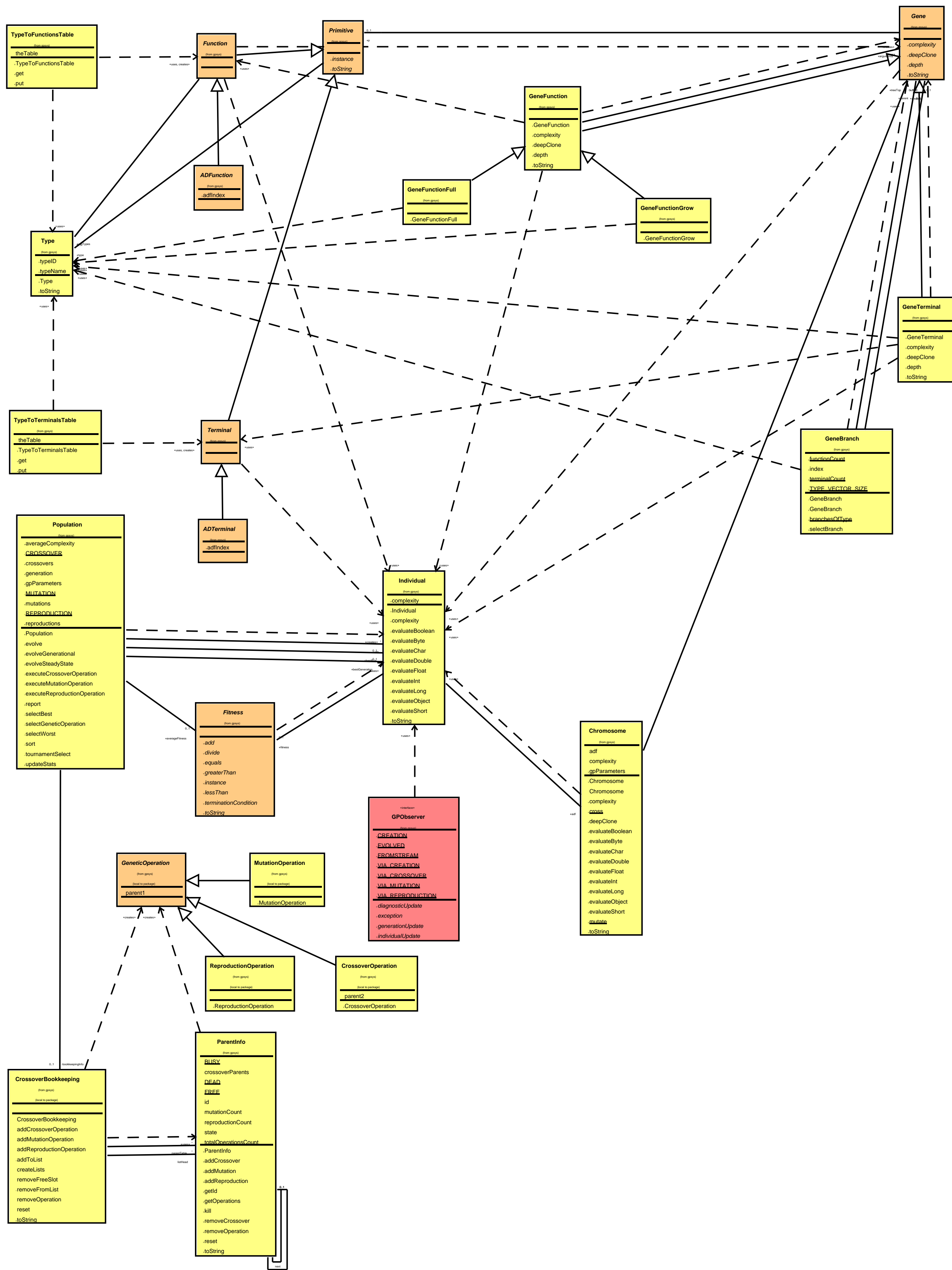


Figure 2.9: GPsys overall UML Diagram

Chapter 3

Intelligent Agents and Multiagent systems

In this chapter we provide a brief overview of the concepts of agents, and multiagents systems and look at the key problems in designing and implementing them. A survey of the application of Genetic Programming to the automatic programming of agents is presented, setting the background for the rest of the thesis.

3.1 Agents

There is no universally agreed definition of the term agent, the definition of agents that we use in this thesis is that used by Jennings and Wooldridge [JSW98; WJ95]:

"an agent is computer system, *situated* in some environment, that is capable of *flexible autonomous* action in order to meet its design objectives."

Their definition is based on the following concepts:

autonomy agents have the ability to act without the direct intervention of humans or other agents, and can control their own actions and internal state

situatedness agents are situated in some environment which they can at least in part perceive and manipulate

flexibility agents possess the properties of *responsiveness*, *pro-activity* and *social ability*

responsiveness the agents perceive their environment and respond in timely fashion to changes that occur in it

pro-activity agents should not simply act in response to their environment, they are able to exhibit goal-directed behaviour by *taking the initiative*

social ability agents should be able to interact with other agents(and possibly humans) to complete their own problem solving and to help others with their activities

They further define an *agent-based* system as one in which the key abstraction used is an agent. Such a system could be comprised of one or many agents. When they are composed of more than one agent they are referred to as *multiagent systems*. A key question in agent research is how to build agents that have the desirable properties stated above. A number of architectures for building agent-based systems have been proposed. These can be classified into deliberative, reactive and hybrid architectures.

3.1.1 Deliberative Architectures

Deliberative architectures are derived from classical approaches to artificial intelligence, and treat agents as knowledge based systems. The emphasis is therefore on knowledge representation techniques and inference methods. Typically a model of the environment, the goals of the agent and its capabilities are represented symbolically, and some form of symbolic reasoning is used to determine which action to perform. A weakness with this approach is that the time taken for such reasoning may invalidate or reduce the utility of the action, so that responsive behaviour may not be possible. Furthermore, symbolic representation of some environmental information may not be easy to accomplish(consider visual information).

BDI agents are a class of deliberative agents that draw inspiration from the theory of human practical reasoning developed by the philosopher Bratman [Bra87]. Such agents are built from components consisting of data structures representing the *beliefs, desires and intentions* of the agent, and functions that represent its *deliberation and means-end reasoning*. The information that the agent has of its environment is represented by its beliefs. The desires represent options available to the agent, which become intentions if the agent decides to commit resource to achieving them. Deliberation is the process of deciding what goals(desires) we want to achieve, and means-end reasoning determines how to achieve them [Wei99; JSW98].

3.1.2 Reactive Architectures

Reactive architectures pioneered by Brooks [Bro86] take a radically different approach to building agents than that of classical AI, and reject symbolic representation and reasoning. Instead they stress the importance of *situatedness and embodiment*, in the creation of intelligent rational behaviour. Intelligent behaviour is seen as *emerging* from the interaction of an agent with its environment and from the interaction of simpler behaviours [Bro91]. To test these ideas, Brooks developed an an agent control architecture called the *subsumption architecture* with these properties. In the subsumption architecture an agent is composed of a collection of *task accomplishing behaviours* arranged in layers The task accomplishing behaviours themselves are extremely simple, consisting of finite state machines that map sensory input to some action. To prevent multiple

behaviours from being simultaneously activated, the layers have priorities, such that lower layers can inhibit behaviours in upper layers.

3.1.3 Hybrid Architectures

Hybrid architectures attempt to combine both deliberative and reactive approaches. These architectures often use vertical or horizontal software layers arranged in a hierarchy, with different levels of abstraction. The distinction between the two layering methods is that with horizontal layering, all layers have access to sensors and effectors, whereas with vertical layering, sensors and effectors are each dealt with by at most one layer. Typically three layers are used, which starting from the lowest layer include the reactive level, the knowledge level and the social knowledge level. The knowledge level has a symbolic representation of the agent's environment, and the social knowledge level has a symbolic representation of other agents [Wei99; JSW98].

3.2 Multiagents Systems

DAI is a subfield of artificial intelligence and is concerned with situations in which several systems, interact in order to solve a common problem [CC96]. These systems may include both humans and machines. Whereas in classical AI, the metaphor of intelligence is based on an *individual human behaviour*, and the emphasis put on knowledge representation and inference methods, the metaphor used in DAI is based on *social behaviour*, and the emphasis is placed on actions and interactions [SDB92].

Research in DAI is traditionally divided into two subfields, *distributed problem solving(DPS)* and *multiagent systems* [CC96; Gas91]. DPS is concerned with dividing the work involved in solving a particular problem amongst a number of nodes that divide and share knowledge about the problem and developing solution. The emphasis in DPS is on task decomposition and solution synthesis. MAS research is concerned with the behaviour of a collection of autonomous agents aiming to solve a given problem, where the problem being solved is often beyond the knowledge and capabilities of any individual agent [Wei99; CC96]. Weiss uses a modern definition of multigents systems which is interchangeable with DAI and covers both DAI subfields [Wei99]. This is the definition that is used in this thesis. Potential benefits of MAS are increased reliability, better performance and lower software complexity.

The characteristics of multiagents systems are as follows [JSW98]:

- Each agent has incomplete information, or capabilities for solving the problem, thus each agent has a limited viewpoint;
- there is no global system control;

- data is decentralised; and
- computation is asynchronous.

Owing to these characteristics the task of designing such agents is difficult. The following list taken from [JSW98] represent the key problems in the design and implementation of MAS:

1. How to formulate, describe, decompose, and allocate problems and synthesise results among a group of intelligent agents?
2. How to enable agents to communicate and interact? What communication languages and protocols to use? What and when to communicate?
3. How to ensure that agents act coherently in making decisions or taking action, accommodating the nonlocal effects of local decisions and avoiding harmful interactions?
4. How to enable individual agents to represent and reason about the actions, plans and knowledge of other agents in order to coordinate with them: how to reason about the state of their coordinated process (e.g. initiation and completion)?
5. How to recognize and reconcile disparate viewpoints and conflicting intentions among a collection of agents trying to coordinate their actions?
6. How to effectively balance local computation and communication? More generally how to manage allocation of limited resources?
7. How to avoid or mitigate harmful overall system behaviour, such as chaotic or oscillatory behaviour?
8. How to engineer and constrain practical DAI systems? How to design technology platforms and development methodologies for DAI?

Our general answer to these questions is that by using GP to evolve agents, we can simultaneously solve many of these problems. Furthermore, we are suggesting that GP could be used as a general methodology for developing multiagent systems.

3.3 Evolving Agents with GP

We can apply GP to agent-based computing in the following ways:

- use gp as a learning or planning technique for agents
- evolve complete agents

- combine both(use GP to evolve agents that themselve use GP as a learning technique)

All of the research applying GP to agents to date, has focussed on the first approaches. Much of the research in GP suggests that GP can be used as very generic learning technique, and therefore its use as a learning technique for agents is highly attractive.

3.3.1 GP as a Learning Technique for Agents

Clack et al [CFLY97; Cla97] for example use GP to evolve document classifying agents. GP is used to search the space of document classification expressions to find expressions that correctly classify the users information needs. The evolutionary process need not ever end, allowing the classifications to co-evolve with changes in the users information needs.

Since plans are effectively just computer programs, the use of GP for plan generation is tempting. The Genetic Planner devised by Handley [Han93; Han91; Han94b] achieves planning by searching the space of plans. The search space is defined in the usual GP way as the set of functions(operators in planning terminology) and terminals. Plans are evaluated for fitness in a simulation of the world in which they are to be executed. Handley showed that he could generate plans for navigation of a robot on 2d world without any reasoning mechanism, and thus avoid traditional planning issues such as the Sussman anomaly, and the dynamic and temporal world problems. Furthermore since we can stop the GP process at any point and ask for the best evolved solution, Handley argues that the genetic planner is an *anytime algorithm*. A key problem with this approach is that it requires a model of the environment, so that we can evaluate the relative success of a plan without executing it. Furthermore measuring the relative success of a plan requires generation of a fitness function, consider for example a plan that needs to switch off a light in a room. Basing a fitness function on the desired outcome, which is that the state of environment has changed so that the light is switched on, would not work. Partial plans that get close to the switch for example would not be rewarded in this scheme. A second problem is the time and space problems associated with evolving a large population of plans.

3.3.2 Evolving Complete Agents

Work done on applying GP to evolve agents has been focussed on mainly reactive agent architectures. There are three key techniques that have been developed. The first is for creating homogeneous agents, and involves evolving a single GP that is instantiated many times, once for each agent [Koz94a; Koz91]. Fitness evaluation is performed by executing the same program in the context of each agent repeatedly for a fixed number of iterations, or until the task that the agents are trying to solve is completed. The fitness value assigned to the GP is measured in terms of how close the agents were to solving the problem at hand. The second technique is used to create heterogeneous co-operating agents, and involves evolving a GP that consists of

multiple trees (similar to ADFs), one for each of the heterogeneous agents [HSSW95a; HS96a; HS97]. During fitness evaluation each agent is instantiated using one of the trees in the GP. The fitness is assigned in the same way as with homogeneous agents, so that the heterogeneous agents are treated as unit or team. The advantage with these architectures is that they avoid the *credit assignment problem* [Min63], i.e. how to assign credit to each of the participating agents. The third technique is to co-evolve competing agents by using separate populations for each of the competing agents [HSSW95b; HS95a]. Fitness evaluation pits agents in one population against agents in the other, so that the fitness of agents in one population are relative to their performance when tested with agents from the other population. Competing agents therefore do not share fitness unlike the team approach mentioned above. Although this technique could also be used to co-evolve co-operating agents [Iba96], the problem of credit assignment makes this approach more difficult.

Koza used GP to evolve wall following behaviour for a simulated autonomous mobile agent using the subsumption architecture [Koz92a]. The wall following problem was originally devised by Mataric [Mat90], and consists of a robot situated in a irregular room that has the task of following the walls of the room. To guide it, it is provided with 12 sonar sensors, each covering 30 degree sectors, and an additional sensor which could detect if the robot had stopped. The robot capable of moving forwards and backwards by a constant distance, rotating left and right by 30 degrees, and stopping. Mataric developed a subsumption architecture comprising of 4 programs that control the robot called stroll, avoid, align and correct. This architecture was sufficient to achieve the task without any modeling or planning as is used in the design of non-reactive robotic systems. Koza, showed that GP could be used to evolve a subsumption architecture for this problem automatically. This paved the way for using GP to evolve reactive agents.

Koza evolved homogeneous reactive agents to solve the “painted desert” problem [Koz94a]. In his definition of the problem, there are 10 ants and 30 grains of sand each of three colours. The 10 ants and 30 grains are placed at random positions on two dimensional torroidal grid of size 10 by 10. The ants are driven by a common program (the ants are homogeneous), and can only sense information about its current grid position. No direct communication between the ants is permitted. The goal is to arrange the grains in vertical bands, where grains in any given band are of the same colour. In addition a band of a given colour must always occupy the same predefined column on the grid (black, grey and striped bands occupy columns 1,2 and 3 respectively). Parallel movement of the ants is simulated by executing the evolved program for each ant sequentially. This process is repeated for 300 iterations. Using simple function and terminal sets and suitable fitness function, Koza was able to evolve a complete solution to this problem, and hence demonstrated that homogeneous reactive multiagent systems that cooperate to solve a task can be evolved using GP. A similar approach was used to program

ants(homogeneous reactive agents) that collectively find and transport food in an efficient manner to their nest [Koz91].

Andre [And94] used GP to evolve programs that are capable of storing a representation of their environment (map-making), and then using that representation to control the actions of a simulated robot. He used GP to effectively co-evolve two programs (each making use of ADFs) for each individual. He split the fitness evaluation of each individual into two phases. In the first phase, the first program is run to extract and store features from the environment. Only the first program is provided with the set of functions to probe the environment. In the second phase, the second program is executed to use the stored representation to control the actions of a robot. The fitness value was awarded only for the success of this second phase. He found that using this approach, he was able to successfully evolve programs that solve simple problems by examining their environment, storing a representation of it, and then using the representation to control action. These programs can be viewed as communicating agents.

Reynolds [Rey94a] used competitive co-evolution to evolve vehicle steering control programs for pursuers and evaders in the game of tag. He obtained near optimal solutions without the use of either expert pursuers or expert evaders.

Haynes [HW95] evolved programs to control an autonomous agent that needs to survive in a hostile environment. The simulated environment consisted of a 2 dimensional grid of cells which can contain other agents, mines and energy. The goal of the agents was to sense and mark the location of the mines and energy by moving through the minefield. Sensing was made possible through tallies available at each cell which count the number of these items available in neighbouring cells (like the game minesweeper). The agent also has access to memory containing information about visited cells. Using a fitness function that uses fluctuating environments he was able to evolve a robust implementation that could handle any environment. Haynes and Sandip [HWS94; HW95; HWS95; HWSS95; HS95b; HSSW95a; HS95a; HSSW95b; HS96c; HS96a; HS97] evolved both homogeneous and heterogeneous agents for the pursuit domain. They also competitively co-evolved predators and prey. Their work used the techniques mentioned at the beginning of this section and is detailed in the next chapter.

Luke and Spector [LS96] compared *cloned* (homogeneous), *free* (heterogeneous with crossover allowed between trees representing different agents) and *restricted* (crossover permitted only between trees representing the same agents) breeding policies for a predator/prey domain called the Serengeti. They also compared a range of coordination mechanisms including *no sensing* (in which the predators were unable to sense each others positions), *deictic sensing* (in which the predators could sense each other in a relative manner such as the position of the nearest agent), and *name-based sensing* (the position of other agents can be sensed referenced by their names).

They found that the heterogeneous approaches produced better results than the homogeneous approaches, and furthermore that the restricted breeding method worked best. Name-based sensing was found to have produced better results than deitic sensing, which in turn produced better results than no sensing.

Qureshi [Qur96] was first to show that GP could be used to evolve agents that use *send* and *receive* operators for explicit communication. He evolved homogeneous agents for a co-operative domain that learn both how and what to communicate. This work is detailed in chapter 5.

Iba compared homogeneous, heterogeneous and co-evolutionary (co-operative) breeding of 2 agents for the tile world problem [Iba96]. He used two different tile world problems, and found that for the first, the heterogeneous worked best, and for the second the co-evolutionary approach worked best. In his co-operative co-evolution approach, he used three populations, one population for each agent and a third as common pool for evolving shared building blocks. Individuals from this shared pool were migrated from the common pool to the other populations (10 per generation). The fitness of the individuals from the two agent populations were evaluated in the first generation by picking random partners from the other, and subsequently by teaming with the best of the other population. Iba also performed experiments in which he evolved both homogeneous and heterogeneous agents that use send and receive primitives similar to that used by Qureshi [Qur96] for a robot navigation domain [INU97; Iba98; Iba99]. The *send_i* deictic operators that he used sends the the position vector to the ith nearest agent (there are 4 of these operators, one for each of the four agents), the receive operator returns the first message from a FIFO queue for the receiving agent. In the same experiment he also used specialised *Send_iS, Send_iR, and Send_iY* operators (12 in total one of each of the four nearest agents) to send stop, random and yield commands to the specified agents. These commands cause the recipients to stay in the same place, move randomly, and move to one of the adjacent vacant square respectively. The targeted agents do not execute any explicit receive primitive to achieve these tasks, nor do they evolve any code to perform the intended task, or have in control of whether the code gets executed. Instead the behaviour is precoded in the simulator, and hence these operators are more like direct control operators. He also evolved heterogeneous predatory agents for the pursuit domain that use name-based primitives called *COM1, COM2, COM3, COM4* which request position information from predatory agents 1, 2, 3 and 4 respectively. The effect of this operator is to return the displacement vector from the calling agent to the target if the receiving agent is within some fixed range of the prey, otherwise the only argument to the function is returned. The agent that is the recipient of request however does not evolve any code to respond to the request, instead this is precoded in the simulator. The operator is therefore more like a conditional sensor. More details of this work are provided in the next chapter. Iba also evolved heterogeneous agents that use ACL-like communication commands (*propose,*

Figure 3.1: Soccerserver screenshot

accept and reject) to negotiate cooperation in the tile world domain [Iba99].

The approach used by Haynes to evolve a team, represents each agent as a separate tree, rather like an ADF. The number of agents to be evolved is therefore specified in advance by the number of trees. By using Koza'a architecture altering operations [KDBK99], Bennett [Ben96a; Ben96b] developed a method for evolving both the number of agents and their code.

A number of researchers [LHF+97; Luk98; AT99] have investigated the use of GP to evolve agents for robotic soccer played in a simulator (Soccer Server [Its95] as shown in figure 3.1). In the RoboCup competition (the softbot part), soccer teams consisting of 11 players (softbots) play in real-time tournaments against other teams. Luke et al [LHF+97; Luk98] evolved players for their team using GP. Each player consisted of a move program and a kick program. Luke et al investigated the possibility of evolving both homogeneous and heterogeneous agents for this domain. The evolution of heterogeneous agents would have required that each individual in the GP population consisted of 22 trees (11 move and kick tree pairs). They therefore chose to evolve pure homogeneous agents and hybrid *pseudo-heterogeneous* teams. The latter consist of a smaller number of heterogeneous agents than required, which are instantiated multiple times to form *squads* which together generate a team of 11 (the team therefore consists of sets of homogeneous agents which are heterogeneous with respect to each other). For

both homogeneous and pseudo-heterogeneous approaches they disallowed crossover between kick and move trees. Furthermore they used their restricted crossover approach mentioned earlier to restrict crossover in the pseudo-heterogeneous approach so that it occurs only between the same squads. To allow fitness evaluation, they used a competitive fitness approach (they were hence co-evolving teams), pitting the team represented by each individual against the team of another individual chosen randomly from the population. Their fitness measure was based crudely on the number of goals scored. They found that the number of goals scored by individuals in the initial population were surprisingly high. The best evolved solution which was of homogeneous type, was submitted to the RoboCup-97 competition and won the first two games against human-crafted opponent teams.

Andre and Teller [AT99] used a similar approach to create a robotic soccer team which they call "Darwin United". They created heterogeneous agents (represented as 11 trees per individual) which also share 8 ADFs (8 further trees per individual). A restricted breeding scheme is used so that crossover occurs between trees representing the same player, and between the same numbered ADFs. Unlike the approach used by Luke et al, Andre and Teller use indexed memory, so that the evolved agents are no longer just reactive. They also use a much more complex fitness evaluation approach. A fitness value for a team consist of an ordered list where items ranging from the least important (getting close to the ball and kicking it) to the most important (scoring a goal and winning the games). Fitness evaluation involves pitting the team represented by the individual being evaluated by graduations of progressively skillful players. The individual gets to play successive teams if it succeeds against earlier teams, the score is the average across the games played. The teams that the individual is pitted against range from an empty field (no team) to the winning team from the 1997 Robocup competition . If the team represented by this individual succeeds then it is also pitted against the teams represented by three individuals who also got to this stage.

Chapter 4

Task Allocation and Conflict Resolution

Task allocation and conflict resolution are both key research areas in multiagent systems. In this chapter we show that GP can be used to evolve agents that automatically allocate tasks amongst themselves both *statically* and *dynamically*, and can resolve conflicts arising from their interaction.

4.1 The Pursuit Problem

The pursuit problem (also called the Predator/Prey problem) is a well studied testbed for DAI research. Benda et al [BJD86] formulated the original problem definition as consisting of four *predators* that need to capture a single *prey*. The predators and the prey are situated on a two dimensional world consisting of cells (figure 4.1). Movement in this world, is possible in orthogonal directions only, and takes the agents from one cell into another (figure 4.2a). The goal for the predators is to move so as to *capture* the prey (figure 4.2c). To capture the prey, the predators must occupy all cells immediately adjacent to the prey (which we call *capture positions* - figure 4.2b). The prey moves randomly, and is slower than the predators (usually implemented by ensuring the prey is stationary some percentage of the time). In most the implementations, only one agent may occupy a cell at a time, and therefore conflicts can occur if one or more agents try to occupy the same cell (in figure 4.2d agents 0 and 1 are in conflict). Many variations to the original definition have been devised by changing key domain parameters, these parameters are described in [SV97] and listed in table 4.1.

Gasser et al proposed a solution based on what they call the *Lieb configuration*. In the Lieb configuration the grid is divided into 4 quadrants by diagonal lines that pass through the cell occupied by the prey, and the predators each occupy a different quadrant. They proposed that the predators first try to achieve the lieb configuration, and then follow a set of *Lieb Rules* to capture the prey.

Stephans and Merx [SM90] investigated the pursuit domain using three control strategies gov-

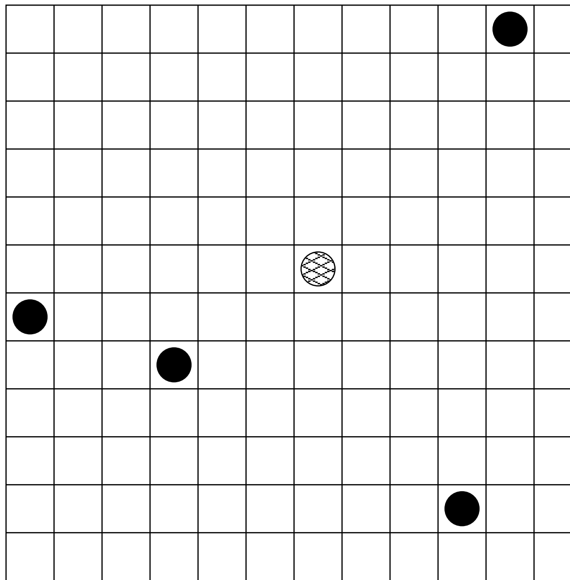


Figure 4.1: The environment of the pursuit domain consists of a two dimensional (usually torroidal) world in which four predators must try to capture the prey by surrounding it.

Table 4.1: Pursuit Domain Parameters

Parameter	Examples
grid size	finite, infinite
grid shape	square, hexagonal, continuous, torroidal, edged
capture definition	prey surrounded, predator on same square as prey
legal moves	orthogonal only, diagonals permitted
movement	agents take turns to move, agents move simultaneously
sensors	what can be sensed, by whom and from how far away
predator communication	permitted, disallowed, range, implicit, explicit
prey movement	random, stationary, deterministic
predator/prey numbers	4x1, 6x2

erning the predator agents. The control strategies included *local control, distributed control and central control*. In their local control strategy, the predator agents pursue their local goals and only communicate when they occupy a capture position by broadcasting that position. This information is picked up by the other predators and update the list of potential local goals. In their distributed control strategy, the predators use both a shared convention, and communication to achieve capture. At the start of each move cycle, each agent computes the straight line distance to each of the capture positions and chooses the closest position as their intention. This intention is then broadcast to the other predators. The shared convention is that the agent furtherest (straight

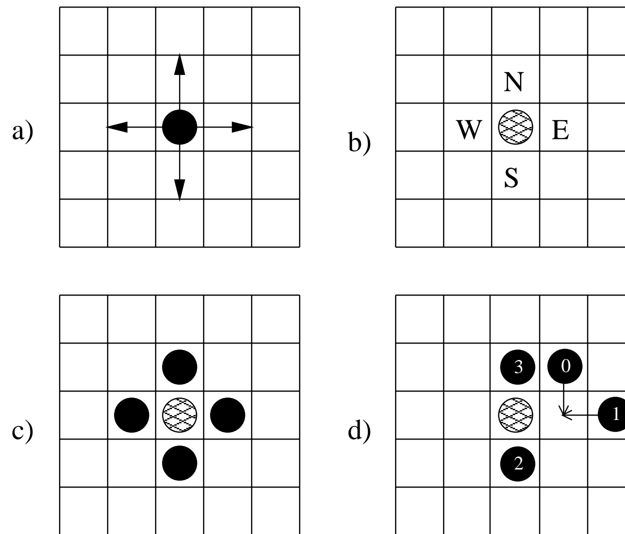


Figure 4.2: In the pursuit domain; a) movement is restricted to orthogonal directions, b) capture positions are cells immediately adjacent to the prey, c) the prey is captured when all capture positions are occupied by predators, d) conflicts occur when one or more agents try to occupy the same cell.

line distance) from their intended capture position is allocated that position. In the central control strategy, one predator (which has complete and accurate information about the position of the other predators) plans and broadcasts commands to other three. In all three control strategies, the predators had accurate information of the position of the prey. They tested each of the control strategies using 30 randomly generated initial placements of the predator agents (the prey was always centered) on 30x30 grid. All agents moved at the same speed, the moves were ordered to avoid collisions, and the prey moved randomly. In their results they report that the central control strategy was the most successful, and the local control strategy least successful. Central control however was less efficient than distributed control, and suffers from having a central point of failure.

Korf [Kor92] proposed that the pursuit problem can be solved without any explicit coordination. He investigated the use of purely greedy strategies for the predators. Korf conducted experiments using 100x100 *orthogonal, diagonal and hexagonal* grid variations, in which the prey actively tries to evade the predators by moving to a neighbouring cell that is the furthest from the nearest predator. In the diagonal grid configuration, the the agents can move diagonally as well as orthogonally, and consequently eight predators instead of four were used. In the hexagonal grid, 6 predators were used to cover each of the exit from a cell. Conflicts were avoided by ordering the movement of the agents, and the prey moved at 90% of the speed of the predators (the prey stays still with a probability of 0.1). The initial positions of the agents were randomly generated. In his simplest solution (which solves problems for all three grid configurations),

the predators try to minimise their distance to the prey. The more efficient solution which he employs in the diagonal and hexagonal versions of the game, uses an attractive force towards the prey and a repulsive force between the predators. Korf concludes that these results lend support to the theory that much coordination and cooperation can be viewed as an emergent property of the interaction of greedy agents in the presence of environmental constraints.

Manela and Campbell [MC93] agreed with Korf that the 4 predators, one prey game could be solved without explicit coordination, but argued that the pursuit game could be made more interesting by having more than one prey. They used a GA to optimise parameters of hand coded predators that communicate. They also introduced the concept of *boredom* (similar to timeouts) to prevent predators from repeatedly executing the same actions without getting any closer to the goal.

Haynes et al [HWS94; HWSS95; HS95b; HSSW95b] used GP to evolve homogeneous predator agents for the pursuit domain. They argued that the ordered execution of moves as used by Korf is a form of communication that helps coordinate the predators. The agents in their version of the pursuit game therefore move concurrently. Agents that try to move to the same cell are bumped back to their original position. A predator may push another predator out of the cell that it occupies if it decides not to move. However this push operator is not available in the function set presented to the GP system. This form of conflict resolution therefore is executed by the environment rather than the agents. They use a 30x30 torroidal, orthogonal game board, in which the agents can move orthogonally or stay still, and randomly generated training test cases consisting of the prey in the center, and the predators placed randomly. The prey moves at 90% of the speed of the predators. They used a STGP system to allow functions and terminals of different types to be combined in one tree, which they argue reduces the search space. They demonstrated that the latter is superior to standard GP applied to this domain through comparison experiments. Their trees return the direction to move, which can be any of North, East, South, West or Here (random instances of which are generated as constants in the terminal set). Here indicates that the agent is to stay put. They provide a function called *CellOf* which takes two parameters, the first being an agent, and the second a compass direction (*tack* in their terminology), and return the cell that is positioned in the specified tack relative to the cell occupied by the specified agent. The terminal set includes a reference to the prey and current predator. Other functions that they use include *IfThenElse,* $<$ and *MD*, where MD returns the manhatten distance between the two cells that are provided as arguments. The fitness function that they employ awards agents for getting close to the prey, with further bonuses for occupying capture positions, and capturing the prey. To ensure *stable captures* as opposed to *shadow captures* they continue the simulation even when the prey has been captured, unlike some of the other approaches mentioned above which stop at this point. They used 100 time steps in their training, however when comparing the evolved

predators with the handcrafted greedy algorithms devised by Korf, they use up to 2000 timesteps. They also varied the type of prey, using randomly moving, move away from the nearest predator (MAFNP), and competitively co-evolved preys. They found that GP evolved solutions that are competitive with the hand coded greedy algorithms presented by Korf. Their results with a co-evolved prey however was less successful. They had anticipated that the performance of the predators and prey would oscillate as one populations learns to outwit the latest strategy used by the other. Instead they found that a simple strategy employed by the prey, in which the prey chooses a random direction and continues to move in that direction (referred to as the *Linear Prey*) successfully evaded all of the predator strategies that they considered. Further investigation showed that these strategies also performed badly against a still prey. Their analysis suggested that the act of the prey moving helps the predators to get out of deadlock situations created by their greedy approach. Later they evolved Heterogeneous predator agents [HSSW95a; HS96a; HS96b; HS97] using the same games rules (including the implicit push operator), function and terminal sets used in their homogeneous approach. The heterogenous agents were evolved as a team by representing each agent as a separate tree in the same individual, as mentioned in the previous chapter. This resulted in very a successful strategy being evolved, and they note that this success is attributed to the ability of heterogeneous agents to avoid potential deadlocks.

Iba [Iba98] also used GP to evolve heterogeneous predators for the pursuit domain. In his game configuration, Iba used a orthogonal grid, and a prey that tries to avoid the predators by moving away from their centre of gravity. 15 training and 10 generalisation test cases were used, in which the agents were placed in randomly generated *Lieb* configurations. Each individual was evaluated in a simulator that runs for a maximum of 30 time steps, so that each agent could move a maximum of 30 times. Simulation was stopped when either the prey was captured or the the maximum simulation time expired. The agents could choose to stay in the current position or to move into one of the four orthogonal cells. The fitness was calculated by summing the total distances of each of the predators from the prey over each time step of the simulation. Lower fitness values therefore correspond to fitter individuals. The fitness function effectively rewards individuals that capture the prey in the shortest simulation time. It is unclear from the description whether the agents move in order or concurrently. The function and terminal set operate over vector types, and the evolved trees return this a value of this type. A mapping functions maps the returned vector to one of the five movements mentioned earlier. The terminal set included primitives that return the displacement vector to the prey, and each of the other agents, and hence global information was available to the predators at all time. The function set included various vector operators, and some conditionals. Thirty runs were executed, each with a population of 500 individuals evolved over 100 generations. The best individual of this experiment resulted in a success rate (ranging 0 to 1) of close to 1. Iba repeated variations of this experiment to explore

the use of communication. Curiously, in these experiment, he employed a randomly moving prey instead of the evading prey used earlier. The range of the primitives used to sense the location of the prey and the other agents were reduced so that they can only sense agents within a circle of radius 5 around their position. Three experiments were conducted, the first of which was identical to the earlier experiment except for the changes mentioned. The second experiment added what Iba calls communication primitives to the function set. As mentioned in the previous chapter, we view these primitives as conditional name-based sensors. The communication primitives called COM0, COM1, COM2 and COM3 request position information from predatory agents 1, 2, 3 and 4 respectively. The effect of this operator is to return the displacement vector from the calling agent to the target if the receiving agent is within some fixed range of the prey, otherwise the only argument to the function is returned. However, a penalty associated with using these operators is that the predator must wait one time unit before moving. In the third experiment in addition to the communication primitives he added a VIEW primitive which evaluates the one and only argument if and only if the prey is visible to the caller. The motivation for the latter was to enable a reduction in the reliance on communication if the prey is already visible. Iba reports that communication with the VIEW primitive gave the best results and reduced the amount of communication necessary.

4.2 Using the Pursuit Domain to Investigate Task Allocation and Conflict Resolution

In the pursuit domain conflicts arise when one or more agents try to occupy the same cell. If such conflicts are not resolved, then they can result in *deadlock*. Just as in concurrent and distributed systems, there are two ways of handling deadlocks in the pursuit domain; *deadlock avoidance*, and *deadlock detection and resolution*. Deadlock avoidance is the simplest and most effective approach that we can employ. Here the predators will need to coordinate their movement to eliminate or minimise conflicts. Deadlock detection and resolution is far more complex. Agents in conflict must first be able to detect the conflict and then coordinate their behaviour so as to resolve it.

We note that in much of the research above, conflict resolution has been avoided by providing environmental mechanisms for resolving conflicts or avoiding them. Examples include the ordered execution of moves used by Stephans [SM90] and Korf [Kor92], and the implicit push operator used by Haynes et al [HSSW95b; HS96b]. One of the important points that Haynes raised for the homogeneous agents that they evolved was that agents driven by deterministic algorithms can get out of conflict situations by the act of the prey moving. Hence, still preys were very effective at causing deadlocks. Furthermore given a large number of simulation cycles the predators are

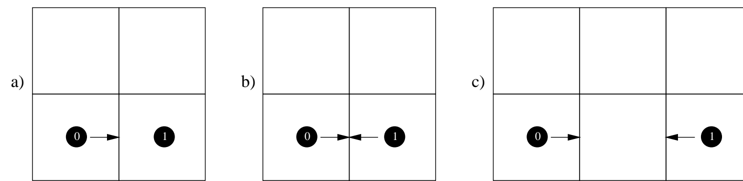


Figure 4.3: Collision conditions: a) agents 0 tries to move a cell occupied by agent 1, b) agents 0 and 1 try to exchange positions, c) agents 0 and 1 try to move to the same cell.

more likely to get out of deadlock situations as a result of the prey moving randomly and hence later capture the prey. We note that in a separate non-GP study Haynes and Sandip [HLS96; HS96d] employed a case-based reasoning approach to learning conflict situations which are subsequently avoided by the predators. The main purpose of this chapter is to directly evolve conflict resolution strategies.

4.2.1 The Pursuit Rules

Our pursuit games were designed to maximise the probability of conflicts, and reduce inadvertent resolution of conflicts (those that occur due to the movement of the prey). The rules of our pursuit games were as follows:

Goal: To capture the prey by occupying each of the four orthogonal positions around the prey and maintaining these positions for the rest of the simulation.

- A simulation lasts for 50 cycles
- A new simulation is run for each test case
- For each simulation cycle each agent is allowed one move
- The moves can be any one of North, East, South, West or Here
- Here is used by an agent to forfeit its move and remain still
- Each of the agents move concurrently so that there is no ordering of moves
- No two agents can occupy the same cell
- If a collision occurs then the agents involved have their moves cancelled
- Collisions occur when:

  one or more agents try to move to a cell that is already occupied (figure 4.3a)

  two adjacent agents try to exchange cell positions (figure 4.3b)

  two or more agent try to move to the same cell (figure 4.3c)

- The randomly moving prey moves 90% of the time.

Our rules differ from those of Haynes and Sandip [HSSW95b; HS96b] in that we only allow 50 cycles for each test case instead of the 100 or more (upto 2000) reported. This reduces the probability of deadlocks being resolved by the movement of the prey. Furthermore our agents cannot exchange positions which would involving crossing each others paths. It is not clear from the papers by Haynes and Sandip whether or not this was possible. This restriction was introduced to maximise the likelihood of conflicts occurring. Our pursuit game *does not* feature any implicit or environmental mechanisms for resolving conflicts.

4.2.2 The Fitness Function

We used the same fitness function as reported by Haynes and Sandip [HSSW95b; HS96b]. After each cycle of a test we measure the manhatten distance between each of the agents and the prey, the sum of which is added to the current fitness value. After each simulation we give further rewards for each agent that is still occupying a capture position, and add yet a further bonus if the prey is captured. This fitness function rewards movement close to the prey, with bonuses for occupying capture positions and a further bonus for capturing the prey.

Table 4.2: Fitness Evaluation Pseudo code for the Pursuit Problem

```
initialise the positions of the agents according to the test case
fitness = 0
for cycle = 0 to MaxCycles
 move the predators according to the evolved code
 if the prey is not stationary move the prey (only 90\% of the time)
 check to see if the moves are valid
 undo invalid moves
 commit valid moves
 foreach predator p, fitness += gridWidth / mahattenDistance(p, prey)
endfor
fitness += MaxCycles * gridWidth * NoOfCapturePositionsOccupied
if the prey is captured
  fitness += MaxCycles * gridWidth * 4
```

4.2.3 Fitness Evaluation

For training purposes, we used 30 randomly generated test cases, with the prey in the center and the predators in random positions. For each test case, we deployed both randomly moving and stationary preys. The latter again helps to maximise conflict situations as discussed earlier. We therefore had 60 test cases. The computational costs of evaluation made it infeasible to run every test case for each individual of each generation. There are four agents whose code need

to be evaluated 50 times for each test case for each individual for each generation. For just one generation the computational costs are 4 * 50 * 60 * M, where M is the population size. Assuming a population size of 3000 as used in our experiments, this figure evaluates to 36 million evaluations. We could reduce the computational cost by reducing the population size, however this parameter is known to be one of the most important for the successful application of GP. We therefore used a sampling technique whereby for each generation, we select 10 test cases at random (without reselection) from the 60 test cases and use them to evaluate each individual. This allows us to fairly compare the fitness of individuals of the same generation whilst minimising the cost of evaluation. The cost of evaluating a generation is now reduced to 1/6th. However, a penalty with this technique is that we need to run the evolutionary process for a longer number generations to give a good chance for each test case to be evaluated many times. Hence we doubled the number of generations that we would normally use, bringing a reduction in computation to 1/3rd of the original number.

A key problem with this approach is that as we move from one generation to the next, the best of the generation may outperform than the current best of the run, not because it is generally better, but because the sampled test cases were easier. Ideally we should weight the fitness value according to the difficulty of the test cases that were passed. Unfortunately this is not easy in the pursuit domain, we discuss issue further chapter 7. To overcome this problem we devised a technique whereby after each generation, we re-evaluate the best individual of the generation using the full 60 test cases and store that individual in the variable bestRunFull if the fitness value is better than the last value of bestRunFull. This mechanism provides an effective way to to track the best individual of the run.

After each run, we took best individual of each run, and re-evaluated the individual using 1000 randomly generated new test cases. For comparison purposes the same one thousand was used for each run. The results of this evaluation were used to measure and compare the generality of the evolved solutions.

4.2.4 The Function and Terminal sets

Table 4.3 lists the functions and terminals that were common to all the pursuit experiments that were performed. Additional primitives for experiment variations are described in the appropriate sections. The functions and terminals operate over three types, *Cell, Direction and Agent*. The Cell type represents cells on the playing field, the Direction type allows the evolved programs to return a suitable direction in which to move, and also allows cells orthogonal to a given cell to be accessed. Only four compass directions are permitted (*North, East, South and West*, an additional direction called *Here* represents the null direction. The latter is used as a return value by evolved programs to indicate the agent is to stay in the current cell, and is also used in conjunction with

Table 4.3: Experiments 1-4 : Common Functions and Terminals

Primitive	Purpose
Direction IfNorth(*Cell c1*,*Cell c2*,*Direction d1*,*Direction d2*)	**if** *c1* is North of *c2* **return** *d1* **else return** *d2*
Direction IfEast(*Cell c1*,*Cell c2*,*Direction d1*,*Direction d2*)	**if** *c1* is East of *c2* **return** *d1* **else return** *d2*
Direction IfCellsEqual(*Cell c1*,*Cell c2*,*Direction d1*,*Direction d2*)	**if** *c1* is equal to *c2* **return** *d1* **else return** *d2*
Cell CellYofX(*Agent x*,*Direction y*)	**return** cell *y* of cell of *x*
Direction North	the *North* direction
Direction East	the *East* direction
Direction South	the *South* direction
Direction West	the *West* direction
Direction Here	no direction
Agent Me	the agent being evaluated
Agent Agent4	the prey
Direction Arg1, Arg2	arguments of current ADF

the *CellYofX* function to access the cell of a given agent. The CellYofX function is the only function that operates on an agent and is equivalent to the CellOf function used by Haynes and Sandip. The functions *IfNorth* and *IfEast* allow the relative position of cells to be compared and depending on the result, allow one of two direction resulting expressions to be evaluated and returned. These functions are aware of the torroidal geometry of the playing field, and have been designed to simplify the task of sensing the relative direction of the prey. The terminal *Me* is instantiated at run-time to be the current agent that is being evaluated.

4.2.5 Behavioural Analysis

To enable behavioural analysis of our evolved code, at the end of each run, we re-evaluated the best Individual of the run using the full set of training test cases. During this evaluation, at the start of each test and after each subsequent simulation cycle, we wrote the state of the game board to a file. This file is effectively a *video* in which each game board state is a *frame*. A Java "video player" application was written which allows these videos to be viewed, as shown figure 4.4. The video player has controls that allow each test for each experiment to be selectively viewed and compared. A pause button together with frame advance and reversal allow each test to be carefully analysed. Each frame shows the board state, with agents represented as numbers. The pursuit agents are numbered 0, 1, 2, 3 and the prey is labelled 4. This numbering is useful

for analysing who does what and when.

4.3 Experiment 1 - Basic Pursuit

Using our version of the pursuit game, and our primitives, we evolved both homogeneous and heterogeneous pursuit agents. To improve scalability we used ADFs, so that the the architecture for each agent consists of one result producing branch (RPB) and one ADF. The homogeneous agents all run the same code, hence the architecture requires only one set of RPB and ADF to be evolved. The heterogeneous agents require 4 sets. The detailed architecture of for each approach is shown in table 4.4.

4.3.1 Results

The results of the experiments are detailed in tables 4.5 and 4.6, and shown as graphs in figures 4.5 and 4.6.

The evolved heterogenous agents show a very high capture rates. The best individual of all the runs (best of run 6) successfully captures the prey in 57 out of the 60 training test cases. This individual also generalises very well, successfully capturing the prey in 1700 out of the 2000 generalisation test cases. Behavioural analysis of these agents show that they use strict task allocation to divide the capture task between them. Each agent assumes a different capture position around the prey. This allows them to easily avoid conflicts and results in a high capture rate. The generalisation of the solution is also very good for the same reason. The specific capture position assignment used by the best individual of run 6 is shown in figure 4.7. Different runs evolved different assignments.

As expected the homogeneous agents performed very badly, with the best individual of all runs succeeding at capture in only 9 out of the 60 test cases. Behavioural analysis of this individual revealed that without the environmental conflict resolution, the small number of permitted moves, and the still prey test cases, the agents frequently got themselves in conflict situations which they were unable to resolve, resulting in deadlock. Homogeneous agents unlike heterogenous agents must allocate tasks dynamically. Without knowing which tasks have already been allocated it is improbable that they choose a task which does not conflict with that of another agent. One way around this problem would be to assign each agent a unique identity and to allow the agents to access their identity and compare their identities with the set of assignable identities. This should allow homogeneous agents to select tasks based on identity. We were also interested in seeing how tasks would be allocated for a mixture of homogeneous and heterogenous agents. Both of these issues are investigated in subsequent experiments.

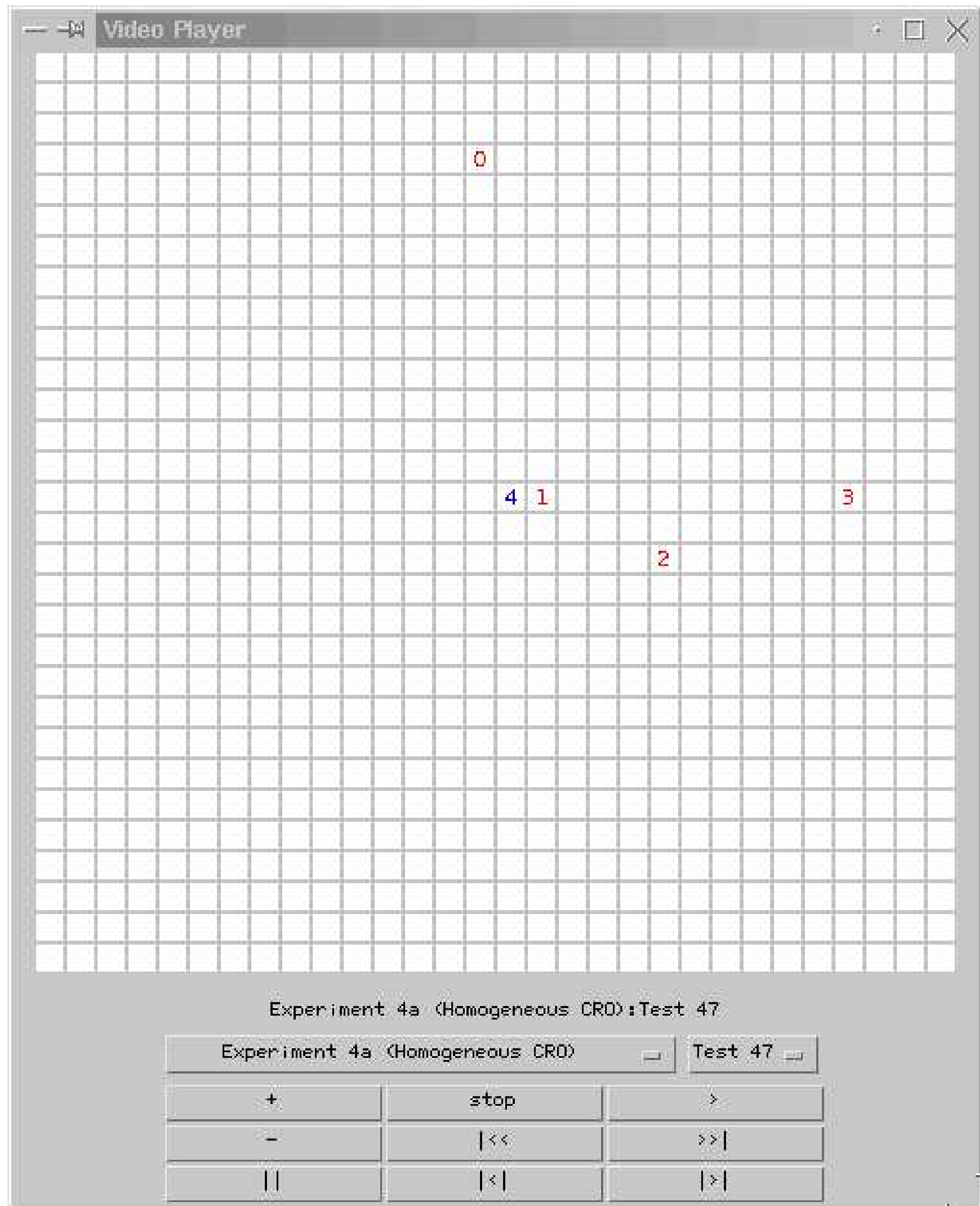


Figure 4.4: Video Player used to Trace Movement of Pursuit Agents

Table 4.4: Experiment 1 : Problem Definition

Objective:	Move the agents in directions that enable capture.	
Homogeneous Agents:	Functions	Terminals
Adf0	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast IfCellsEqual CellYofX, ADF0	Me, Agent4, North, South, East, West, Here
Heterogenous Agents:	Functions	Terminals
Adf0	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast IfCellsEqual CellYofX, ADF0	Me, Agent4, North, South, East, West, Here
Adf2	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF3)	IfNorth, IfEast IfCellsEqual CellYofX, ADF2	Me, Agent4, North, South, East, West, Here
Adf4	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF5)	IfNorth, IfEast IfCellsEqual CellYofX, ADF4	Me, Agent4, North, South, East, West, Here
Adf6	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF7)	IfNorth, IfEast IfCellsEqual CellYofX, ADF6	Me, Agent4, North, South, East, West, Here
Fitness test cases:	10 randomly selected per generation from 60 fixed test cases.	
Fitness:	Average of the raw fitness scores of the 10 test cases.	
Parameters:	Pop = 3000, G = 500.	
Termination Condition:	Best solution captures prey for all 60 test cases.	

Table 4.5: Experiment 1a : Fitness of best evolved homogeneous agents

Homogeneous - Basic		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
0	460	6616.667	50.000	3	6148.785	50.000	38
4	280	7061.233	47.850	5	6649.372	48.386	122
2	202	7395.900	46.650	8	6742.492	48.535	140
6	262	7447.050	48.067	5	7000.521	48.111	149
7	288	7486.783	46.733	7	6922.912	48.186	141
5	216	7598.467	46.883	8	6748.692	48.313	125
1	284	7798.900	46.467	8	6717.876	48.953	86
8	428	7864.367	46.417	9	6895.240	48.185	148
3	174	7881.050	46.933	7	6864.084	48.132	139
9	312	7906.450	46.083	9	7046.939	48.117	145
Mean	290.600	7505.687	47.208	6.900	6773.691	48.492	123.300
1.5m	340	6523.000	50.000	0	5832.845	50.000	0

Table 4.6: Experiment 1b : Fitness of best evolved Heterogenous agents

Heterogeneous - Basic		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
3	582	9396.550	44.450	22	8191.741	45.564	500
2	670	10097.233	42.683	31	8736.327	44.376	750
4	574	11003.933	43.750	34	9575.257	43.896	843
0	430	12668.250	39.950	41	11081.221	42.028	1004
5	668	13215.467	38.517	45	12524.272	39.121	1361
9	652	13634.417	38.867	49	12062.410	41.320	1267
8	700	14253.783	35.933	50	12581.798	38.674	1366
7	582	14678.183	37.933	54	13547.649	37.638	1545
1	680	14704.650	34.167	52	13715.610	36.388	1528
6	724	15750.567	28.700	58	14586.280	32.969	1700
Mean	626.200	12940.303	38.495	43.600	11660.257	40.197	1186.400
1.5m	652	3786.200	50.000	0	3255.050	50.000	0

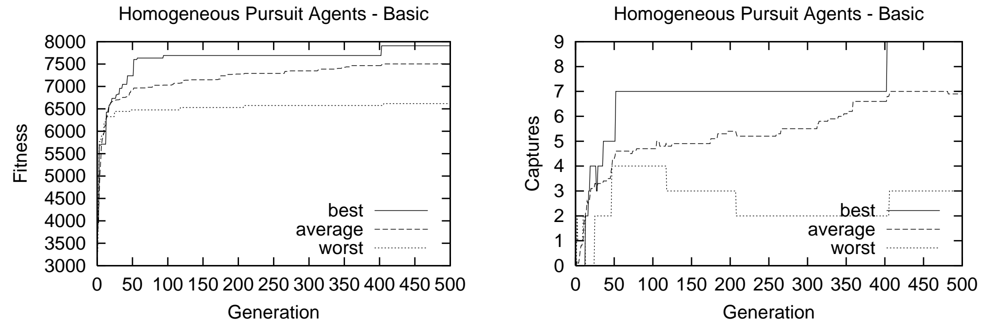


Figure 4.5: Experiment 1a : Performance Graphs of Evolved Homogeneous Agents

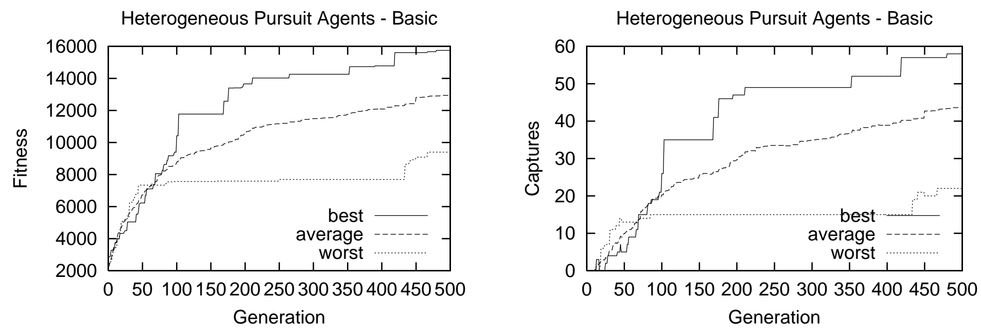


Figure 4.6: Experiment 1b : Performance Graphs of Evolved Heterogenous Agents

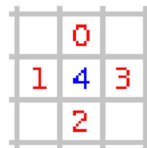


Figure 4.7: Experiment 1b: Capture Posistion Assignment

4.4 Experiment 2 - Identity

In the previous experiments we evolved heterogenous agents that were very successful at capturing the prey and noted that their success was as a result of static allocation of tasks between the team members. In this experiment we assigned identities to each of the agents and provided operators that allow agents to enquire their identity and to compare the identity with the set of all identities allocated (see table 4.7). The purpose of this experiments was to see if we can evolve homogeneous agents that use their identity to allocate tasks between them. The experimental setup is described in table 4.8.

Table 4.7: Experiment 2 : Additional Functions and Terminals

Primitive	Purpose
Direction	**if** *agent.id* == *id2* **return** *d1*
`IfAgentIdEquals`(*Agent agent*,*AgentId id2*,*Direction d1, d2*)	**else return** *d2*
Direction `id0`	the identity *id0*
Direction `id1`	the identity *id1*
Direction `id2`	the identity *id2*
Direction `id3`	the identity *id3*

Table 4.8: Experiment 2 : Problem Definition

Objective:	Move the agents in directions that enable capture.	
Homogeneous Agents:	Functions	Terminals

`Adf0`	`IfNorth, IfEast`
	`IfCellsEqual`
		`North, South`
		`East, West`
		`Here, arg0, arg1`

`RPB (ADF1)`	`IfNorth, IfEast, ADF0`
	`IfCellsEqual, CellYofX`
	`IfAgentIdEquals`
		`Me, Agent4, North, South,`
		`East, West, Here`
		`id0, id1, id2, id3`

Fitness test cases:	10 randomly selected per generation from 60 fi xed test cases.	
Fitness:	Average of the raw fi tness scores of the 10 test cases.	
Parameters:	Pop = 3000, G = 500.	
Termination Condition:	Best solution captures prey for all 60 test cases.	

4.4.1 Results

The results of the experiment are shown in table 4.9 and the graphs in figure 4.8. As can be seen, the evolved homogeneous agents were significantly more successful at capturing the prey

Table 4.9: Experiment 2 : Fitness of best evolved homogeneous agents (with identity)

Homogeneous - Identity		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
0	380	8306.683	45.867	12	7212.681	48.199	226
4	164	8518.450	44.950	12	7354.320	47.915	192
2	334	8676.600	44.233	14	7175.482	48.038	163
7	370	9273.217	44.000	15	7708.118	47.059	223
8	322	9664.500	44.217	16	9285.950	44.345	465
3	358	10261.467	42.200	18	8390.556	46.540	251
6	296	10757.017	40.450	23	9054.132	44.421	427
9	430	11961.617	37.983	39	9541.213	42.336	795
5	186	12462.533	35.667	34	11309.306	39.231	838
1	374	12723.683	38.133	38	10983.227	42.174	884
Mean	321.400	10260.577	41.770	22.100	8801.499	45.026	446.400
1.5m	142	5938.200	50.000	0	5117.752	50.000	0

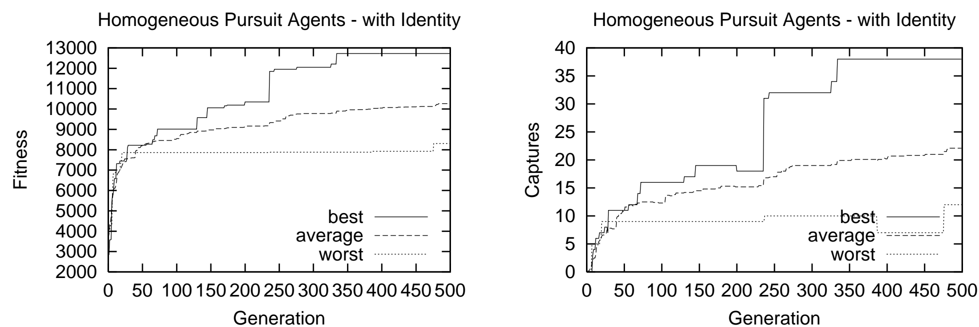


Figure 4.8: Experiment 2 : Performance Graphs of Evolved Homogeneous Agents (with identity)

(percentile comparison) than the homogeneous agents of experiment 1a. Behavioural analysis confirms the allocation of tasks between the agents based on their identities. In fact the best individual of all the runs(best of run 1), employed the same assignment of capture positions as the best evolved GP for the heterogeneous agents of experiment 1b (best of run 6). This is probably the result of a bias created by the initial placement of the agents in the training set. Just as with experiment 1a there were variations on the capture position assignments for the best GPs from the other runs. Whilst the evolved agents were much better than in experiment 1a, they still fall short of the heterogenous agents of experiment 1b. A possible explanation is that the homogeneous agents have far fewer opportunities for crossover in the single set of ADFs than the heterogenous agents where crossover occurs over multiple sets of ADFs, one for each agent.

Furthermore the separate set of ADFs also allow better functional decomposition. If we were to repeat the experiments using multiple ADFs and the ability to invoke any of these ADFs from the RPB based on the identity of the agent, we should be able to substantially improve the results.

4.5 Experiment 3 - Mixed Agents

The purpose of these next set of experiments was to investigate how tasks are allocated to an evolved mixture of homogeneous and heterogenous agents. There are three possible experiments for our instantiation of the pursuit domain:

1. *experiment 3a(31)* one set of homogeneous agents and one agent that is heterogeneous with respect to them

2. *experiment 3b(22)* two sets of homogeneous agents, where agents in one set are heterogeneous with respect to the other.

3. *experiment 3c(211)* one set of homogeneous agents and two agents that heterogeneous with respect to each other and the homogeneous set

In experiment 3a we evolved two sets of agent code, the first set is used by the predatory agents labelled 0 to 2, and the second by agent 3. This effective makes agents 0 to 2 homogeneous and agent 3 heterogenous with respect to them. A similar setup is used for experiment 3b, where two sets of agent code are also evolved, but this time the first set is used by agents 0 to 1, and the second set by agents 2 to 3. This effectively creates two sets of homogeneous agents, but with the two sets being heterogenous with respect to each other. Experiment 3b required 3 sets of agents code to be developed, the first set is for homogeneous agents 0 and 1, and the second and third are for heterogenous agents 2 and 3 respectively.

The problem definitions are similar to that of experiment 1 (see table 4.10, except that the architecture is is modified appropriately. Experiments 3a(31) and 3b(22) have the same architecture, only two sets of agents code are evolved, so we need only Adf0, Adf1, Adf2 and Adf3. In experiment 3a(31) Adf0 and Adf1(RPB) are used by all three homogeneous agents, the agent that is heterogeneous with respect to them uses Adf2 and Adf3(RPB). In experiment 3b(22) Adf0 and Adf1(RPB) is used by the homogeneous agents of the first set, Adf2 and Adf3(RPB) are used by the second set of homogeneous agents. In experiment 3c(211) three sets of agent code is evolved, the architecture therefore requires 3 Adf pairs in total. The 2 homogeneous agents share Adf0 and Adf1(RPB), the remaining agents use Adf2, Adf3(RPB) and Adf4, Adf5(RPB) respectively.

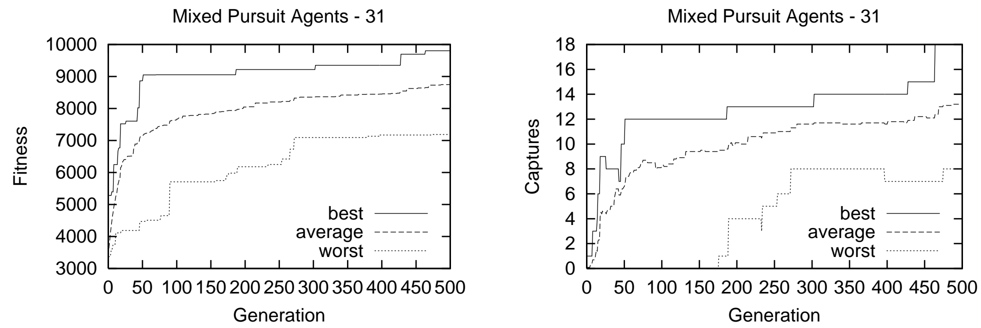


Figure 4.9: Experiment 3a(31) : Performance Graphs of Evolved Mixed Agents

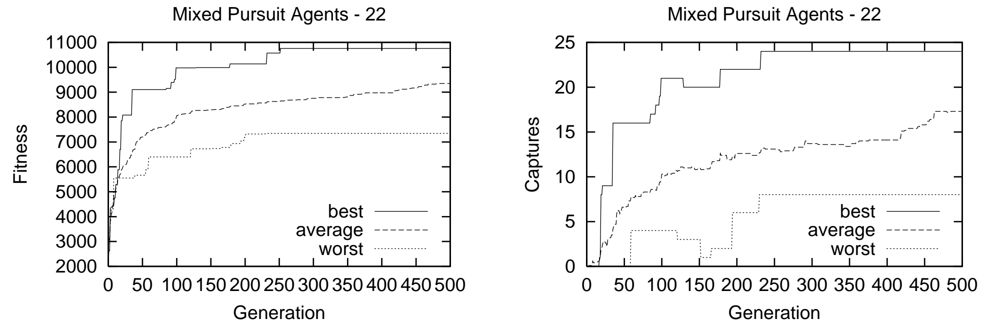


Figure 4.10: Experiment 3b(22) : Performance Graphs of Evolved Mixed Agents

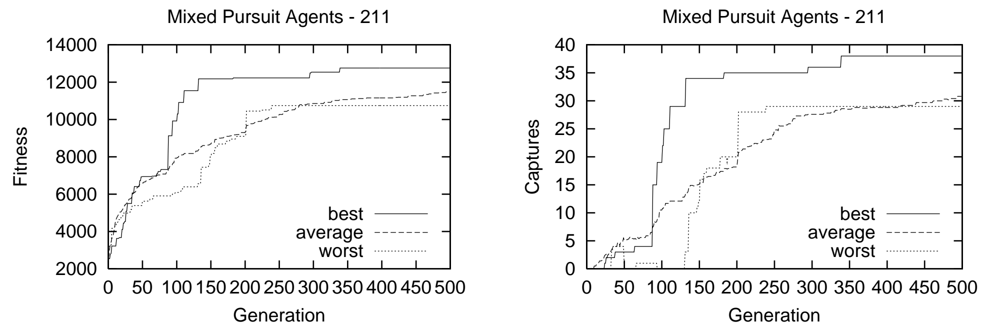


Figure 4.11: Experiment 3c(211) : Performance Graphs of Evolved Mixed Agents

4.5.1 Results

The results of the experiment are listed in tables 4.11, 4.12, and 4.12, and shown by the graphs in figures 4.9, 4.10 and 4.10.

Behavioural analysis of the best solutions for all three experiments, shows that they made use of as much static task allocation as possible. In experiment 3a(31), 3 captures positions were allocated to homogeneous agents 0 to 2 which shared these positions, and the remaining capture position was allocated to agent 3. Most of the conflicts occurred between the 3 homogeneous

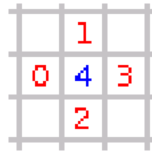


Figure 4.12: Experiment 3a(31): Capture Position Assignment

agents which need to dynamically choose from one of the three capture positions allocated to them. The best individual of all the runs for this experiment (best of run 3) assigned the capture positions North, West and South of the prey to the 3 homogeneous agents, and the capture position East of the prey to agent 3 as shown in Figure 4.12.

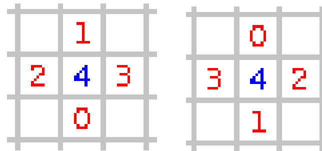


Figure 4.13: Experiment 3b(22) : Capture Position Assignment

The best solution for experiment 3b(22) allocated two sets of capture positions for each of the two sets of agents. Again conflicts were avoided between the two sets of agents, but there were conflicts within each set. The best individual of all the runs (best of run 9) assigned the capture positions North and South of the prey to homogeneous agents 0 and 1, and assigned capture positions East and West of the prey to homogeneous agents 2 and 3 as shown by figure 4.13.

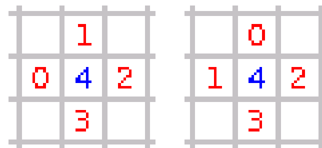


Figure 4.14: Experiment 3c(22) : Capture Position Assignment

The best solution for experiment 3c(211), was also the best result for all the experiments as we might expect. Two capture positions were allocated to the homogeneous agents, and of the remaining captures positions, one was allocated to agent 2, and the other to agent 3. Again conflicts were mainly between the homogeneous agents, but were minimal compared to the other experiments as there was more static task allocation. The best individual (best of run 8) assigned the capture positions North and West of the prey to homogeneous agents 0 and 1. The remaining capture positions East and South of the prey were assigned to agents 2 and 3 respectively as shown on figure 4.14.

These results suggest that GP tries to optimise the allocation of tasks to the agents.

Table 4.10: Experiment 3 : Problem Definition

Objective:	Move the agents in directions that enable capture.	
Mixed Agents(31 and 22):	Functions	Terminals
Adf0	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast IfCellsEqual CellYofX, ADF0	Me, Agent4, North, South, East, West, Here
Adf2	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF3)	IfNorth, IfEast IfCellsEqual CellYofX, ADF2	Me, Agent4, North, South, East, West, Here
Mixed Agents(211):	Functions	Terminals
Adf0	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast IfCellsEqual CellYofX, ADF0	Me, Agent4, North, South, East, West, Here
Adf2	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF3)	IfNorth, IfEast IfCellsEqual CellYofX, ADF2	Me, Agent4, North, South, East, West, Here
Adf4	IfNorth, IfEast IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF5)	IfNorth, IfEast IfCellsEqual CellYofX, ADF4	Me, Agent4, North, South, East, West, Here
Fitness test cases:	10 randomly selected per generation from 60 fixed test cases.	
Fitness:	Average of the raw fitness scores of the 10 test cases.	
Parameters:	Pop = 3000, G = 500.	
Termination Condition:	Best solution captures prey for all 60 test cases.	

Table 4.11: Experiment 3a(31) : Fitness of best evolved mixed agents

Mixed - 31		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
9	496	7187.717	47.617	8	6582.446	48.367	175
8	404	7794.700	47.200	8	6979.263	48.194	215
4	422	8206.400	45.333	12	6531.860	48.444	163
0	566	8717.350	45.467	11	8291.702	46.494	264
6	566	8861.500	44.700	13	7614.285	47.069	215
7	224	8943.217	45.417	15	7801.929	46.564	280
1	290	9108.100	43.850	13	7850.674	46.885	249
2	308	9416.250	43.000	16	8342.736	45.478	354
5	452	9429.150	44.300	18	7810.500	46.904	284
3	432	9802.317	42.550	18	8094.508	46.878	289
Mean	416.000	8746.670	44.943	13.200	7589.990	47.128	248.800
1.5m	364	5397.900	50.000	0	5146.313	50.000	1

Table 4.12: Experiment 3b(22) : Fitness of best evolved mixed agents

Mixed - 22		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
4	362	7346.567	47.617	8	6371.912	48.766	118
3	374	7628.767	46.983	11	6740.513	47.939	217
1	390	9202.883	43.383	16	7806.854	47.132	227
5	392	9358.717	45.200	16	8053.090	46.926	315
0	286	9376.183	43.017	16	8443.133	45.120	363
8	454	9472.033	42.183	17	8422.005	45.156	381
2	596	9798.183	43.016	19	8485.865	45.280	397
7	444	10218.000	41.717	22	8279.560	45.466	381
6	190	10355.533	39.867	24	8601.506	44.723	412
9	304	10760.950	40.183	24	8438.601	45.214	388
Mean	379.200	9351.782	43.317	17.300	7964.304	46.172	319.900
1.5m	318	4862.600	50.000	0	4131.602	50.000	0

Table 4.13: Experiment 3c(211) : Fitness of best evolved mixed agents

Mixed - 211		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
7	422	10739.567	40.700	29	8411.547	45.066	525
1	424	10763.500	44.967	29	8803.552	45.783	578
6	648	10799.917	43.100	28	10041.422	44.066	778
5	628	10961.967	40.667	27	9986.056	42.937	726
3	476	11034.333	41.500	26	9381.893	44.870	531
4	606	11577.667	41.817	32	10388.950	42.896	808
9	606	11741.733	40.300	32	10114.998	42.828	723
0	504	12105.700	38.500	34	10537.085	41.878	836
2	332	12291.117	38.083	33	10162.118	43.302	708
8	642	12757.817	36.917	38	10839.444	41.383	840
Mean	528.800	11477.332	40.655	30.800	9866.707	43.501	705.300
1.5m	436	5461.900	50.000	0	4381.487	50.000	0

4.6 Experiment 4 - Conflict Resolution

The purpose of the next set of experiments (which is also the main focus of this chapter) is to see if we can evolve agents that make use of local information to avoid and/or resolve conflicts. The local information is provided by an operator that allows the agents to detect whether or not a cell neighbouring the cell that they or the prey occupy is also occupied (see table 4.14). We are particularly interested in how the addition of this operator will help homogeneous agents to resolve conflicts, as it is likely that heterogenous agents will still use static task allocation to avoid conflicts. The experimental setup is detailed in table 4.15.

Table 4.14: Experiment 4 : Additional Terminals and Functions

Primitive	Purpose
Direction `IfCellOccupied`(*Cell cell*,*Direction d1*,*Direction d2*)	**if** *cell* is occupied **return** *d1* **else return** *d2*

4.6.1 Results

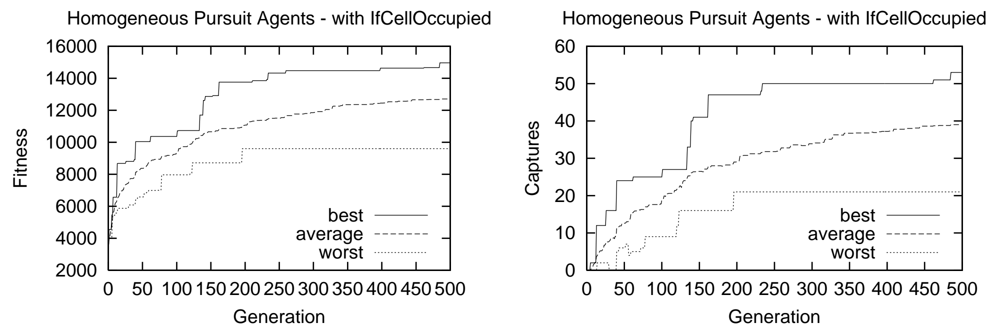


Figure 4.15: Experiment 4a : Performance Graphs of Evolved Homogeneous Agents (with CRO)

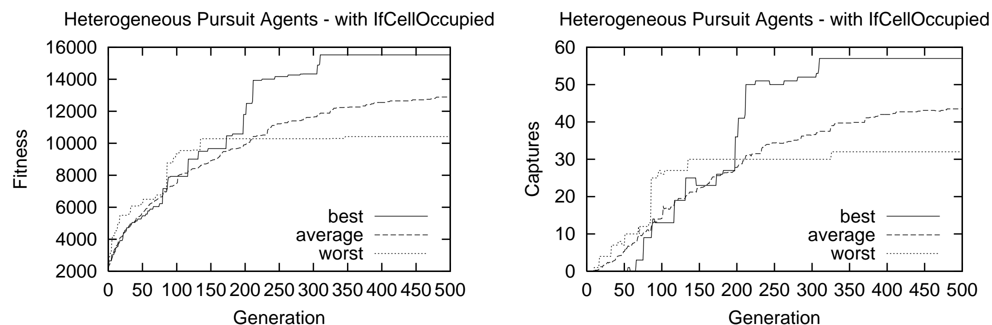
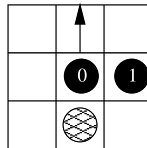


Figure 4.16: Experiment 4b : Performance Graphs of Evolved Heterogenous Agents (with CRO)

The results of the experiment are listed in tables 4.16 and 4.17 and shown graphically in fig-

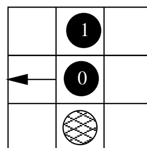
ures 4.15 and 4.16. As expected there was little difference in performance for the heterogenous agents, which continue to use static task allocation to avoid conflicts. The results for the homogeneous agents however showed a substantial improvement in capture rate when compared to the homogeneous agents of experiment 1. Behavioural analysis of the best individuals revealed that the mechanism used by the agents to improve their performance was far more sophisticated than expected expected. We had intended the *IfCellOccupied* operator to be used by the homogeneous agents to avoid movement towards a cell that is already occupied, thus resolving the conflict. This was evident in some test cases, however in many test cases, the agents used what could be viewed as either polite *social conventions* or *stigmergic communication* to resolve conflicts. Careful behavioural analysis of all 60 training test cases, revealed a total of 6 distinct behaviour patterns which when combined yield very sophisticated conflict resolution strategies. These 6 behaviour patterns are described below. In the descriptions N_{prey} is read the cell north of the prey, and $E(N_{prey})$ is read the cell that is east of the cell that is north of the prey (north-east of the prey). Agents named agent 0 and agent 1 were used for illustration, but since the agents are homogeneous the behaviour is not peculiar to these agents.

Behaviour EN



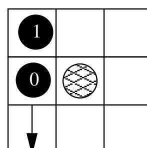
Agent 0 occupies cell N_{prey}, and agent 1 occupies the cell $E(N_{prey})$. Agent 0 responds by moving upwards freeing N_{prey}.

Behaviour NN



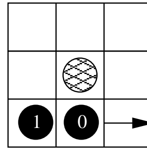
Agent 0 occupies N_{prey}, and agent 1 occupies $N(N_{prey})$. Agent 0 reacts by moving to position $W(N_{prey})$ freeing N_{prey}.

Behaviour NW



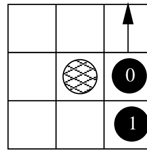
Agent 0 occupies W_{prey}, and agent 1 occupies $N(W_{prey})$. Agent 0 reacts by moving to position $S(W_{prey})$ freeing W_{prey}.

Behaviour WS



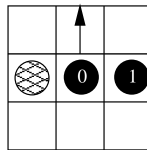
Agent 0 occupies S_{prey}, and agent 1 occupies $W(S_{prey})$. Agent 0 reacts by moving to position $S(E_{prey})$ freeing S_{prey}.

Behaviour SE



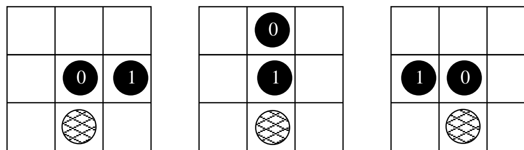
Agent 0 occupies S_{prey}, and agent 1 occupies $S(S_{prey})$. Agent 0 reacts by moving to position $E(N_{prey})$ freeing S_{prey}.

Behaviour EE



Agent 0 occupies E_{prey}, and agent 1 occupies $E(E_{prey})$. Agent 0 reacts by moving to position $E(N_{prey})$ freeing E_{prey}.

The above simple behaviours can be combined to create more complex patterns of behaviour. A good example is the pass through behaviour shown below.



The pass through behaviour starts with agent 0 occupying N_{prey}, and agent 1 occupying $E(N_{prey})$, and hence agent 0 invokes behaviour EN and moves up. Agent 1 simultaneously occupies the vacated position N_{prey}. Next agent 1 invokes behaviour NN, moving to cell $W(N_{prey})$, the freed cell N_{prey} is simultaneously re-occupied by agent 0. The effect is that agent 0 allowed agent 1 to pass through position N_{prey}. This example works because the two agents both play their parts, one agent invokes a conflict resolution behaviour, the other cooperates in the

resolution.

Further examples of such behaviours are shown by frames captured from the video player application used for behavioural analysis. To maximise illustration, the examples are chosen from test cases in which the prey was still, although the same behaviours are used by the agents when capturing moving prey. To minimise space, the frames were cropped around the prey and frames were omitted when there was no change in the cropped region. In the examples shown by figures 4.17 and 4.18, the frames are to be read left to right, top to bottom.

In the example shown in figure 4.17, at the fifth frame, agent 3 is setup to follow behaviour EE and move north. This happens in the next frame, and agent 2 takes occupies the now vacant position. This in turn sets up agent 0 to invoke behaviour EN. In the next frame agent 0 therefore moves north, and agent 3 occupies the vacated position. Now agent 3 is setup for behaviour NN, and hence in the next frame moves west, and agent 0 re-occupies its previous position. In the same frame agent 1 moved to the position E_{prey}, and is therefore setup for behaviour NW. Agent 0 moves south in the next frame and agent 3 occupies E_{prey}. Finally the displaced agent 1 occupies the only vacant capture position by moving east. This example uses a total of 4 different behaviours EE, EN, NN and NW to resolve conflicts and capture the prey. The example in figure 4.18 similarly shows the use of behaviours NN, NW and WS. Note how the movement of the predators(driven by the best individual for this experiment) around the prey in both examples is anti-clockwise. The best individuals of some of the other runs used similar behaviours, but moved clockwise instead.

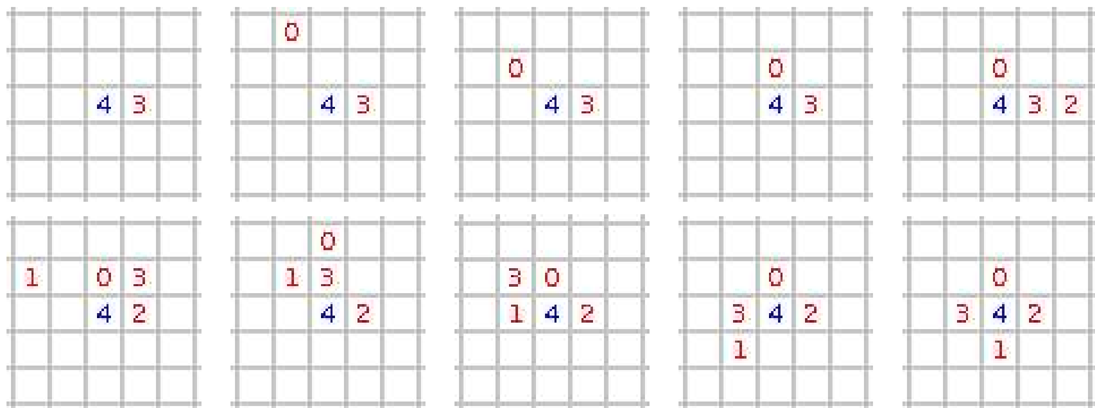


Figure 4.17: Experiment 4a(test 13) : End Game Trace of Agent Movement

4.7 Comparison with Random Search

For comparison, the equivalent number of individuals that are created during a run (1.5 million) were randomly created using a technique similar to that used to generate the initial population. The last row in the result tables for each experiment details the performance of the best of these

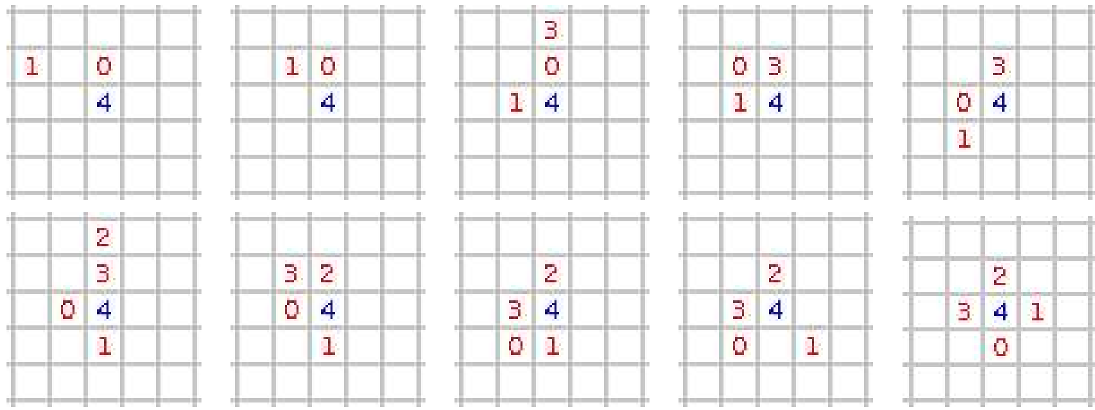


Figure 4.18: Experiment 4a(test 31) : End Game Trace of Agent Movement

individuals. As can be seen from tables 4.5,4.6,4.9, 4.11,4.12,4.13, 4.16 and 4.17, in nearly all of the experiments, the best individual created via random search did not manage to capture a prey for any of the training or generalisation test cases. In the one exception (experiment 3a), a single capture was achieved for 1 of the 2000 generalisation test cases. This shows the the solutions could not have evolved through chance alone. GP for this application is clearly superior to random search.

4.8 Conclusion

We have shown that GP can be used to evolve agents that:

- allocate tasks amongst themselves both statically and dynamically
- use their identities for task allocation
- try to optimise the allocation of tasks
- detect and resolve conflicts

Table 4.15: Experiment 4 : Problem Definition

Objective:	Move the agents in directions that enable capture.	
Homogeneous Agents:	Functions	Terminals
Adf0	IfNorth, IfEast, IfCellsEqual	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast, IfCellsEqual IfCellOccupied, CellYofX, ADF0	Me, Agent4, North South, East, West Here
Heterogenous Agents:	Functions	Terminals
Adf0	IfNorth, IfEast, IfCellsEqual IfCellOccupied	North, South East, West Here, arg0, arg1
RPB (ADF1)	IfNorth, IfEast, IfCellsEqual IfCellOccupied, CellYofX, ADF0	Me, Agent4, North South, East, West Here
Adf2	IfNorth, IfEast, IfCellsEqual IfCellOccupied	North, South East, West Here, arg0, arg1
RPB (ADF3)	IfNorth, IfEast, IfCellsEqual IfCellOccupied, CellYofX, ADF2	Me, Agent4, North South, East, West East, West, Here
Adf4	IfNorth, IfEast, IfCellsEqual IfCellOccupied	North, South East, West Here, arg0, arg1
RPB (ADF5)	IfNorth, IfEast, IfCellsEqual IfCellOccupied, CellYofX, ADF4	Me, Agent4, North South, East, West Here
Adf6	IfNorth, IfEast, IfCellsEqual IfCellOccupied	North, South East, West Here, arg0, arg1
RPB (ADF7)	IfNorth, IfEast, IfCellsEqual IfCellOccupied, CellYofX, ADF6	Me, Agent4, North South, East, West Here
Fitness test cases:	10 randomly selected per generation from 60 fi xed test cases.	
Fitness:	Average of the raw fi tness scores of the 10 test cases.	
Parameters:	Pop = 3000, G = 500.	
Termination Condition:	Best solution captures prey for all 60 test cases.	

Table 4.16: Experiment 4a : Fitness of best evolved homogeneous agents (with CRO)

Homogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
7	331	9603.850	43.033	21	9401.273	44.210	624
2	297	11745.433	39.017	34	10355.528	42.105	798
4	323	11940.217	38.050	33	11498.195	39.086	1013
0	316	12542.667	38.750	38	10536.450	41.184	880
3	204	12632.417	34.617	39	11901.102	37.699	1114
6	295	12639.467	35.783	38	10860.907	39.818	890
5	299	12940.100	35.600	40	11382.413	38.935	1029
8	235	13730.600	32.700	44	11692.839	37.294	1072
1	412	14344.967	32.217	50	13030.682	35.325	1381
9	489	14969.417	32.400	53	14596.335	32.008	1695
Mean	320.100	12708.913	36.217	39.000	11525.573	38.767	1049.600
1.5m	115	5894.800	50.000	0	5665.396	50.000	0

Table 4.17: Experiment 4b : Fitness of best evolved Heterogenous agents (with CRO)

Heterogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
4	615	10415.467	43.200	32	8000.069	46.290	607
3	706	10851.017	44.617	30	8355.619	47.090	509
8	556	12122.917	39.767	37	10137.040	42.784	892
5	744	12334.017	41.283	42	9802.103	43.930	933
9	757	12545.467	43.050	44	9377.983	45.388	772
0	532	13069.000	40.300	43	11934.951	41.045	1201
6	617	13235.800	37.500	47	11140.286	41.012	1154
1	846	14016.517	36.617	50	13270.028	37.664	1447
7	574	14850.400	33.800	53	13725.092	37.403	1573
2	730	15528.817	30.217	57	14763.829	32.877	1735
Mean	667.700	12896.942	39.035	43.500	11050.700	41.549	1082.300
1.5m	553	3975.100	50.000	0	3338.355	50.000	0

Chapter 5

Communication

Sophisticated multi agent systems will require agents to be able to communicate not just with their environment, but with other agents to achieve their goals. Such communication may be achieved implicitly through changes in the environment or explicitly via direct message passing. In this chapter we show that GP can be used to evolve agents that communicate with each other explicitly.

5.1 Experiment 5 - Communicating Agents

The purpose of this experiment is to see if GP can be used to evolve agents that communicate to solve problems. For this purpose we have defined a simple problem for which it is necessary for communication to take place and furthermore, for which it is easy to measure the effectiveness of any communication. The problem involves two homogeneous agents A and B that steer two vehicles placed on a two dimensional grid. The vehicles are constantly moving in the direction that they are currently pointing. All movement is discrete, so that the vehicles always have integer (x,y) coordinates. Initially the vehicles are pointing in random compass directions. The steering mechanism allows the orientation of the vehicles to be changed to any one of the 8 compass directions. The goal is to evolve the code for agents A and B such that they steer the vehicles to enable them to meet. The vehicle model closely follows that used by Reynolds [Rey94a].

Agents A and B know only their own positions, but both have access to two bi-directional unbuffered communication channels which can be used to send and receive messages to each other. The communication channels are "wired" so that one end is connected to one vehicle and the other to the other vehicle. A message sent from one end can only be received at the other end, and vice versa. The channels can only be used to send and receive messages in the form of integers. When an attempt is made to read from a channel the last message received is delivered (default value 0).

Clearly what we would like is for agents A and B to evolve behaviours that use the communi-

cation channels to exchange position information. The vehicles are given an equal amount of energy at the start of a simulation, and a single energy unit is consumed each time a vehicle moves one step in any direction. The game ends when both vehicles run out of energy or when they meet.

The fitness of the evolved code is measured using a function composed of two parts. The first part measures the degree of *success* and the second measures the *efficiency*. When comparing two fitness values, the success measure is given priority so that the efficiency is considered only when the success values are identical. The success is measured as follows:

$$Success(A) = Success(B) = \frac{D_{before}(A, B)}{D_{before}(A, B) + D_{after}(A, B)}$$

Where D_{before} and D_{after} are the Euclidean distances between A and B before and after execution respectively. This function produces a range of values varying between 0 and 1, where a value of 0 indicates that the agents have moved an infinite distance apart, and a value of 1 means that they have met.

The efficiency is defined as the percentage of energy remaining at the end of a simulation:

$$Efficieny(A) = Efficieny(B) = \frac{totalEnergy_{Remaining}(A, B)}{totalEnergy_{Allocated}(A, B)}$$

Where $totalEnergy_{Allocated}$ is the total amount of energy allocated to the agents before the simulation, and $totalEnergy_{Remaining}$ is the total energy remaining after simulation.

We had started off by providing each vehicle with a constant surplus amount of energy for each test case. The result was that the code evolved made use of *social conventions* to obviate the need for communication. The agents would always evolve to meet at an “agreed” location (often location 0,0). We therefore rationed energy for each test case to a total value of 1.5 times the initial Euclidean distance between the agents. This provides sufficient energy to allow solutions that do not use diagonals to evolve, but at the same time inhibits solutions that merely use social conventions.

An important aspect of our fitness function is that it takes into account the difficulty of the test case in the fitness evaluation. Instead of merely measuring the distance between the agents after the simulation, we also take into account how far apart they were at the start of the test case. This ensures that test cases in which the agents are located at closer starting positions are not advantaged over test cases where the distances are greater. This concept of weighting test cases according to their difficulty is discussed in greater detail in Chapter 6.

5.2 Architecture

Tables 5.1 and 5.2 provide details of the architecture used to evolve programs to steer the agents. There is a single result producing branch (ADF0) which returns a compass direction used to steer the vehicle.

Table 5.1: Experiment 5 : Actions Performed by Terminals and Functions

Primitive	Purpose
`Direction If`(*boolean test*,*Direction d1*,*Direction d2*)	**if** *test* **return** *d1* **else return** *d2*
`boolean GE`(*int v1*,*int v2*)	**if** $v1 >= v2$ **return** *true* **else return** *false*
`boolean LT`(*int v1*,*int v2*)	**if** $v1 < v2$ **return** *true* **else return** *false*
`int Add`(*int v1*,*int v2*) `int Sub`(*int v1*,*int v2*) `int SendA`(*int m*) `int SendB`(*int m*)	**return** $v1 + v2$ **return** $v1 - v2$ send message *m* via channel A; **return** *m* send message *m* via channel B; **return** *m*
`int GetX` `int GetY`	get the current *X* coordinate; **return** *X* get the current *Y* coordinate; **return** *Y*
`int RecvA`	get the message received at channel A; **if** no message received yet, **return** 0; **else return** last message received
`int RecvB`	get the message received at channel B; **if** no message received yet, **return** 0; **else return** last message received

`Direction North`
`Direction NorthEast`
`Direction East`
`Direction SouthEast`
`Direction South`
`Direction SouthWest`
`Direction West`
`Direction NorthWest`
	the *North* direction
	the *NorthEast* direction
	the *East* direction
	the *SouthEast* direction
	the *South* direction
	the *SouthWest* direction
	the *West* direction
	the *NorthWest* direction

For fitness evaluation, we used the same sampling technique mentioned in the previous chapters. One hundred test cases were randomly generated. Each test case consists of random initial positions and orientations for each agent located within the region (-50,-50) to (50,50). For each

Table 5.2: Experiment 5 :

Primitives:	Functions	Terminals
`Adf0 (RPB)`	`Add, Sub, If, GE, LT SendA, SendB`	`North, NorthEast, East SouthEast, South, SouthWest West, NorthWest RecvA, RecvB`
Test cases:	100 randomly generated fixed test cases.	
Energy:	$1.5 * Distance_{before}(A, B)$	
Fitness:	Average of the raw fitness scores of the 100 test cases.	
Parameters:	Pop = 1000, G = 100, grid range (-50,-50) to (50,50).	

generation we select a sample of 5 unique test cases from the 100 previously generated ones and use these five to evaluate each individual in the population. The fitness value awarded is the average of the raw fitness for each of the five test cases. The best individual of the generation is then re-evaluated using the complete set of 100 test cases, and becomes the best of the run if it has the highest fitness recorded so far. At the end of a run, the best individual is further re-evaluated to test for the generality of the solution by using 1000 randomly generated test cases.

Each individual in the population was evaluated by using it to steer both A and B. This is achieved by alternately executing the GP once in the context of A, and then in the context of B. The result of each execution is used to steer the respective vehicle, which is then moved forward one unit. This process is repeated until either the two agents both run out of energy, or they meet. The pseudo code in table 5.3 illustrates this approach.

Table 5.3: Evaluation Pseudo Code

```
agent = A
while ((A.hasEnergy() || B.hasEnergy()) && (!met(A,B))) {
  if (agent.hasEnergy()) {
    Direction d = eval GP in context of agent
    agent.turnto(d)
    agent.move()
    agent.energy--
  }
  if (agent == A) agent = B else agent = A
}
```

The ordered execution of the agents has the important consequence of *delayed communication* between the agents. If the agents were to use communication to exchange position information,

the sender will always be at a different position by the time the recipient receives the message. In our problem definition, for the agents to meet, they must be at exactly the same coordinates at the same time. The agents will therefore also need to compensate for the communication delays.

5.3 Results

The results over ten runs of the experiment are listed in table 5.4, and summarised by the graphs in figure 5.1. For comparison purposes the same number of individuals that are generated during a run were created using a technique similar to that used for generating the initial population. The fitness of the best individual found using this method is also listed at the end of the table, and as can be seen performed significantly worse than the weakest evolved individual (with 17 meetings versus 47), and used significantly more energy (3% of energy saved versus 12%).

Table 5.4: Experiment 5 : Results for Evolved Communicating Agents

Evolved Code		Training Tests (100)			Generality Tests (1000)		
Run	Size	Success	Effi ciency	Meetings	Success	Effi ciency	Meetings
0	84	0.92711	0.11856	47	0.92452	0.12282	475
5	69	0.93485	0.14233	61	0.92100	0.13778	546
3	51	0.93925	0.19065	62	0.94333	0.21326	634
9	122	0.99048	0.05868	66	0.99403	0.07720	672
7	79	0.99359	0.26075	86	0.99608	0.26222	899
1	62	0.99523	0.18096	91	0.99513	0.18649	828
6	65	1.00000	0.28359	100	0.99925	0.29343	994
2	131	1.00000	0.37188	100	0.99984	0.38698	998
8	88	1.00000	0.37268	100	0.99968	0.38796	998
4	80	1.00000	0.39126	100	0.99969	0.39508	999
Mean	83.1	0.97805	0.23713	81.3	0.97726	0.24632	804.3
100k	72	0.771502	0.026155	17	0.782516	0.026597	156

Furthermore in four of the ten runs, code was evolved which successfully passed all 100 test cases. Each of these solutions used the two channels to exchange X and Y coordinates, and adapted their movement to achieve the goal. In addition, the evolved solutions found a means for overcoming delays in communication.

5.3.1 Run 4

Run 4 evolved the best solution for both the training tests and the generality tests. The GP tree representing this individual is shown in figure 5.2, the nodes labelled *pruned* represent branches in the GP which contain redundant code and are hence not shown. The complete code can be

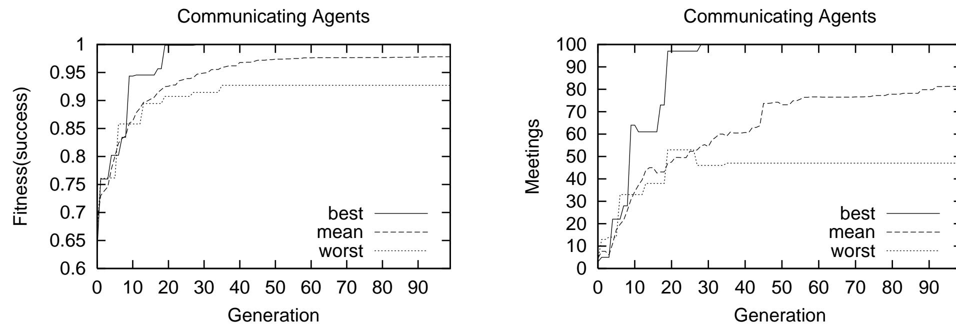


Figure 5.1: Experiment 5 : Performance Graphs of Evolved Communicating Agents

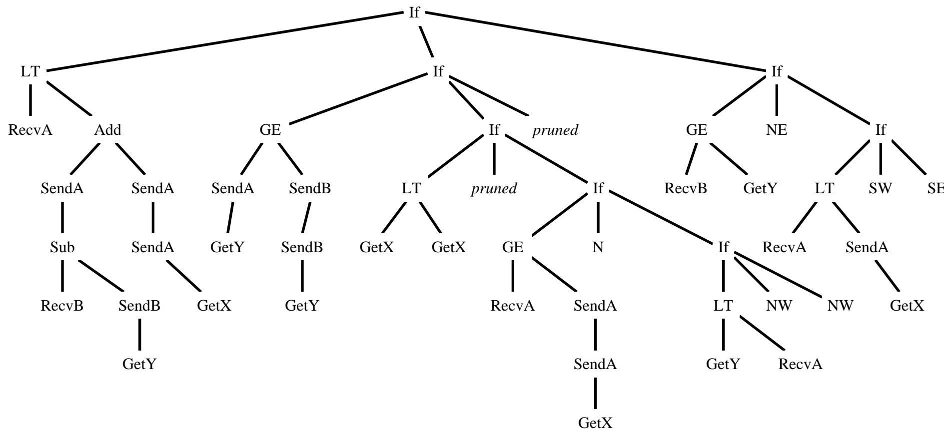


Figure 5.2: Experiment 5 : Evolved Parse Tree for best individual of Run 4

:

Table 5.5: Experiment 5 : Simplified code for Run 4

```
SendA X
SendB Y
If (RecvA - X < RecvB - Y)
    If (RecvA >= X) N else NW
else If (RecvB >= Y) NE
else If (RecvA < X) SW else SE
```

found in Appendix A. We can simplify the code as shown in 5.5. Clearly Channels A and B are used for exchanging X and Y coordinates respectively. The information received via these channels is compared with local X and Y coordinates to determine movement. It should be noted that the code used mainly diagonal movement instead of orthogonal movement. This is the result of the restricted amount of energy allocated, and the efficiency pressure of the fitness function.

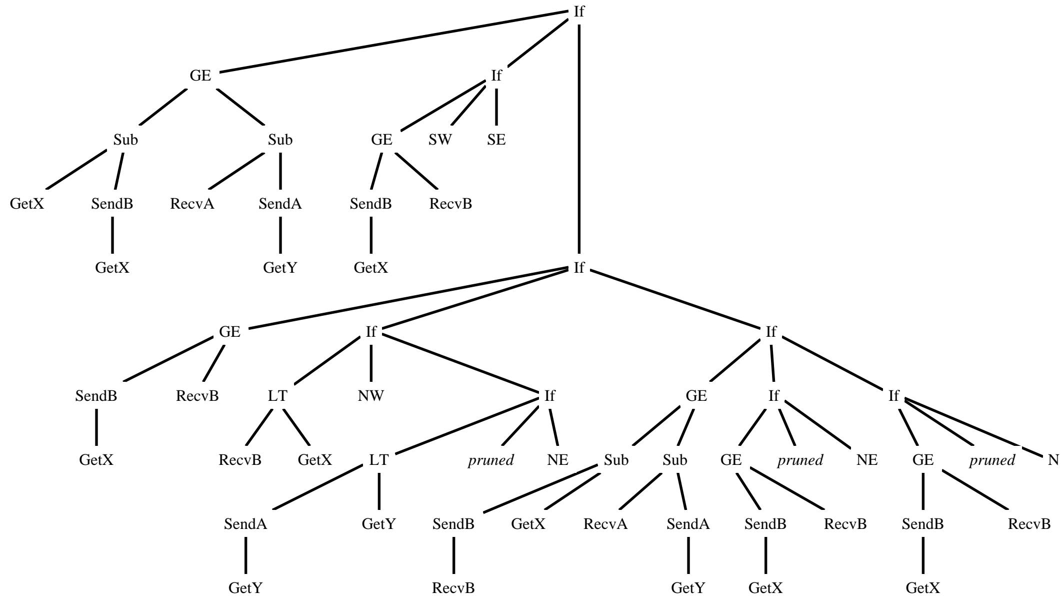


Figure 5.3: Experiment 5 : Evolved Parse Tree for best individual of Run 2

Sample traces of agents movement are shown in figure 5.6. To illustrate the effects of evolution we have compared the best evolved individual for this run for test case 45 at generation 41 with generation 99 (the final generation). One of the key differences is that at generation 41 whilst the evolved code has discovered how to communicate, the code does not take into account the staleness of the information. The solution therefore often misses meetings by one pixel distances. The code for generation 99 not only minimises the energy used, it also uses strategies to overcome communication delay.

Note that the agents are only capable of moving in the directions N, NE, NW, SE, SW. The terminals representing the other directions have been lost during evolution. However despite the absence of the S direction, the agents have learned to achieve movement in this direction by alternately moving SW and SE as shown by test case 21 in figure 5.6.

5.3.2 Run 2

The GP tree representing the best evolved solution for Run 2 is shown in figure 5.3, and the simplified code that it represents is shown in 5.7. As is shown, this time Channel B is used to communicate X coordinates, and Channel A for Y coordinates.

Figures 5.8, show snapshots of the output trace for this individual for some sample test cases.

The snapshots were taken at the point when the agents met or when they ran out of energy. Note how the solution involves diagonal movement at angles other than the 45 degree compass directions. This is made possible by alternately moving in different directions. For example by repeatedly moving in sequences of SE, SE, NE, we end up moving in the SE direction at far less steep angle than by just moving in the SE direction.

5.4 Conclusion

We have shown that GP can be used to evolve agents that learn:

- what information is to be transmitted and received
- how to transmit/receive the information
- which channel to use to transmit/receive which piece of information
- how to adapt behaviour according to the communicated information to solve a problem
- how to deal with the staleness of data associated with communication delay

We hence conclude that it is possible to use GP to evolve agents that explicitly communicate with each other to solve a global problem. The key limitations of our work is that we have evolved only simple homogeneous agents which carry no state, working on a symmetric problem. An earlier version of this work was presented at the GP 96 conference and published in the proceedings. It is the first paper showing that GP can be used to evolve communicating agents.

Table 5.6: Experiment 5 : Trace of Agent movement for Run 4

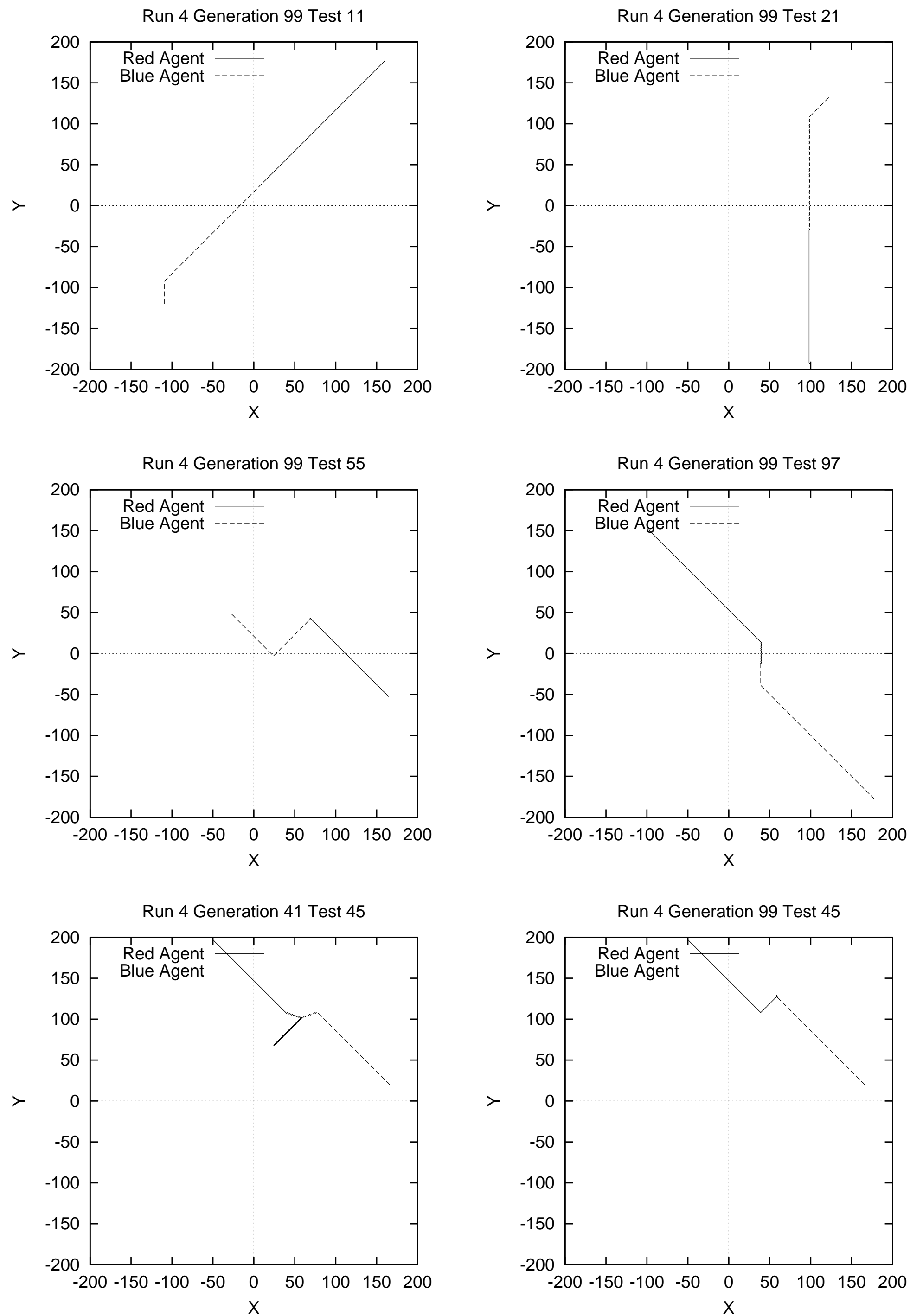
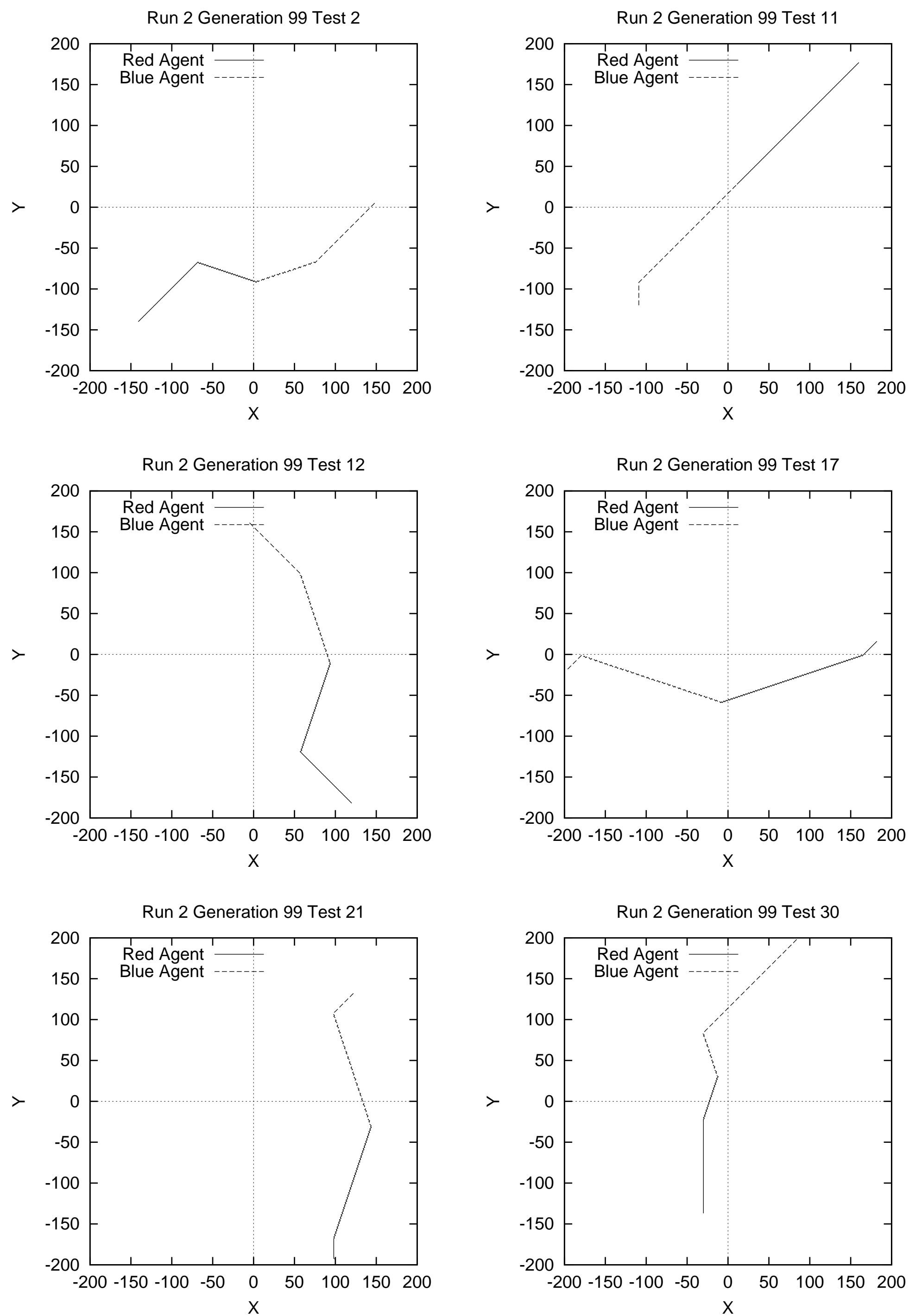


Table 5.7: Experiment 5 : Simplified code for Run 2

```
sendA Y
SendB X
if (Y >= RecvA)
  if (X >= RecvB) SW else SE
else if (X >= RecvB)
      if (RecvB < X) NW else NE
else if ((RecvB - X) >= (RecvA - Y)) NE else N
```

Table 5.8: Experiment 5 : Trace of Agent movement for Run 2



Chapter 6

Socialisation

We would like to be able to design agents that work within an existing society of other agents. The new agent must be capable of interacting both with the environment and with other agents to help achieve individual or group goals. In human programmed MAS, this ability is programmed into the new agents. The focus of this chapter is to show that GP can be used to evolve such agents automatically. In particular we show that the agent learns through evolution to assume a purpose in the team, communicate with team members, resolve conflicts and interact with the environment to achieve goals. We name this process socialisation, as it is similar to the sociological term which describes how humans are trained to become integrated members of a society.

In previous chapters we have shown that GP can be used to evolve homogeneous and heterogeneous agents to solve problems. We evolved agents that learn to decompose a task, communicate with each other and resolve conflicts. To build open an MAS, we must also be able to evolve agents that work within a society of previously created agents. We therefore have devised three experiments in this chapter to determine if GP can achieve this task. All three build on experiments performed in the previous chapters.

6.1 Communicating Agents

In the first experiment we make use of the best evolved communicating agent of chapter 5 (the best agent from run 2) and instead of using two instances of the same agent, we evolve a new agent that replaces the counterpart. The new agent must learn how to communicate with the other agent to allow them to meet. Unlike the original experiment the new agent cannot arbitrarily exchange information using the channels allocated, but must learn the previously assigned purposes of the channels. Success in these experiments would suggest that GP can be used to create agents that learn how to communicate with a society of existing agents to solve a problem.

The architecture for the experiment remains identical, just the fitness function is changed so

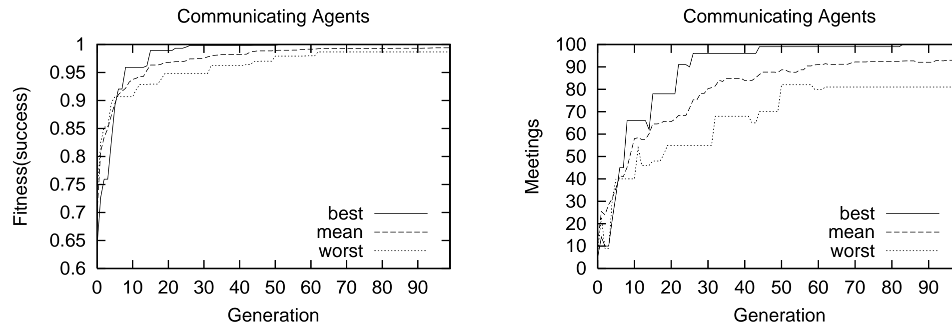


Figure 6.1: Experiment 6 : Performance Graphs of Socialised Communicating Agents

that the first agent is always instantiated using the best evolved agent from run 2 of the original experiment.

6.1.1 Results

Table 6.1: Experiment 6 : Results for Socialised Communicating Agent

Evolved Code		Training Tests (100)			Generality Tests (1000)		
Run	Size	Success	Efficiency	Meetings	Success	Efficiency	Meetings
7	71	0.986758	0.190388	81	0.979776	0.218998	839
5	131	0.986959	0.267658	90	0.987769	0.263201	893
0	220	0.987354	0.264737	84	0.992426	0.271754	861
3	122	0.991897	0.250398	93	0.990781	0.272061	933
2	115	0.995644	0.271641	92	0.998126	0.282659	921
9	80	0.996498	0.227828	97	0.998936	0.244127	991
8	133	0.997794	0.228757	95	0.998234	0.252121	954
6	129	0.998048	0.357939	98	0.999793	0.371336	998
4	94	1.000000	0.293016	100	0.999067	0.308986	993
1	115	1.000000	0.373473	100	0.999760	0.383389	999
Mean	121	0.994095	0.272584	93	0.994467	0.286863	938.2
100K	34	0.876944	0.034519	18	0.883205	0.051225	238

The results over ten runs of the experiment are listed in table 6.1, and summarised by the graphs in figure 6.1. As in previous experiments the same number of individuals that are generated during a run were created using a technique similar to that used for generating the initial population. The fitness of the best individual found using this method is listed at the end of the table, and as can be seen performed significantly worse than the weakest evolved individual both in terms of fitness, number of meetings and efficiency.

Two of the 10 runs generated complete solutions, whilst the remaining 8 were very close. Structural analysis of the two solutions shows that both have learned how to correctly communicate with the previously evolved agent. Furthermore, the agents also learned how make use of the communicated information taking into account the effects of communication delay in order to achieve meetings. Note that this task is actually quite complex. The new agent needs to predict how information sent to the other agent is used by the other agent to alter its behaviour. At the same time the new agent needs to learn how to make use of the information that is received to govern its own behaviour to ensure successful meetings. A simplified version of the best evolved solution (run 1) is shown in table 6.2 (the GP tree is too large to be displayed). As can be seen the solution correctly communicates X and Y coordinates using channels A and B respectively. There are however some differences in how the information is used, so that the to agents are no longer homogeneous.

Table 6.2: Experiment 6 : Simplified code for Run 2

```
sendA X
sendB Y
If (RecvB + RecvA >= X + Y)
  If (RecvB - Y >= RecvA - X)
      If (X < RecvA) NE
      else If (Y < RecvB) NW
      else E
  else If (Y < 2RecvB - X) E
  else If (Y >= X/2)
      If (Y < RecvB) NW
      else E
  else E
else If (RecvB < Y)
  If (RecvA >= X) S else SW
  else W
```

6.2 Pursuit Agents

The second and third experiments make use of the best evolved solutions of the homogeneous and heterogeneous conflict resolution experiments for the pursuit domain of chapter 4.

In the homogeneous agents case, we instantiate two predator agents using the previously evolved solution and grow a new program which is instantiated twice to serve the purpose of the remaining two predators. The new agents must learn to work within the homogeneous society of existing agents which have been previously evolved to dynamically allocate tasks amongst

themselves and to resolve any resulting conflicts. We know from our analysis of the best evolved agent of the homogeneous conflict resolution experiment that the agents deal with conflicts by avoidance, and detection/resolution. Conflict resolution was achieved mainly by means of social rules or conventions. The new evolved code is instantiated twice so that it actually forms a homogeneous subgroup of agents. These agents must learn the social rules used to resolve conflicts and are also free to determine new rules amongst themselves to help resolve conflicts.

The purpose of the third experiment is to see if a two new heterogenous agents can be evolved to work within a heterogenous society of agents. We know that the best heterogeneous evolved solutions for pursuit problem make use of static task allocation to minimise conflicts. We are therefore interested to see if we can evolve new agents that determine their tasks in the team without overlapping with tasks chosen by the previously evolved agents.

Once again the architecture for the two experiments is identical to that used in the conflict resolution experiments of Chapter 4, except that for the heterogeneous predators, instead of evolving trees for four different agents, we need only evolve trees for 2 different agents. The fitness function was modified appropriately so that predators 0 and 1 use instances of the previously evolved code, and predators 3 and 4 make use of the individual being evaluated. For the homogeneous society, agents 3 and 4 would be two instances of the individual (consisting of Adf0 and Adf1). For the heterogeneous agents, Adf0 and Adf1 are used by the predator 2, and Adf2 and adf3 are used by predator 3. Both experiments were executed with a population size of three thousand as before, but the number of generations was restricted to 250. The same test case sampling scheme was used for fitness evaluation as described in the previous experiments.

6.2.1 Results

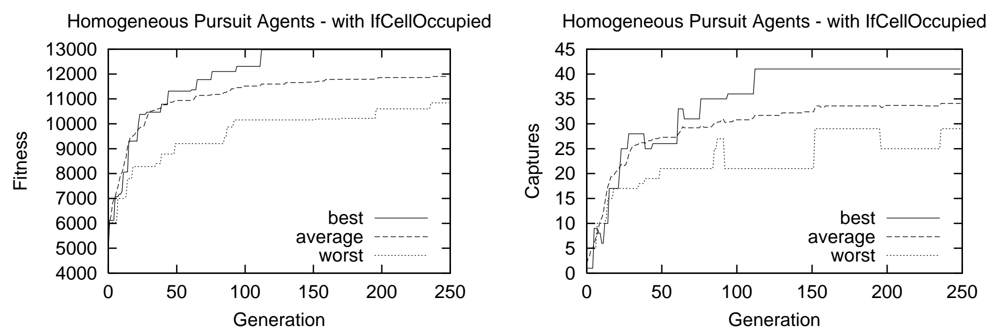


Figure 6.2: Experiment 7a : Performance Graphs of Socialised Homogeneous Pursuit Agents

The results of 10 runs of the homogeneous predator experiments are shown in table 6.3 and the graphs of figure 6.2. The evolved homogeneous subgroup were not as successful as the original agents, scoring a maximum of 41 captures out of the 60 test cases. Behavioural analysis of the

Table 6.3: Experiment 7a : Fitness of best socialised homogeneous pursuit agents

Homogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
2	297	10843.400	41.133	29	11081.581	41.333	1049
0	373	11275.700	39.617	28	11041.713	40.887	890
8	280	11714.733	38.350	32	10830.088	41.455	843
4	104	11745.183	40.767	32	10468.151	42.623	785
6	243	11803.683	38.717	33	10747.575	41.314	851
7	220	12107.250	39.750	37	10740.483	41.825	975
3	172	12132.550	39.650	38	10360.154	42.987	911
9	130	12195.583	40.500	37	10918.627	42.340	935
1	345	12227.517	39.517	34	11340.375	41.290	944
5	261	12971.517	35.917	41	10910.338	41.247	877
Mean	242.500	11901.712	39.392	34.100	10843.909	41.730	906
750K	151	11135.600	40.300	6	9933.580	43.589	420

best individual revealed that it had learned two of the behaviours described in experiment 4a (EE and NN). The remaining four behaviour were not learned resulting in deadlocks when there were conflicts. The evolved subgroup however did learn to make use of all the conflict resolution capabilities of the previously evolved agents, hence the relatively high capture rate.

This result is still very promising, it suggest in principle that using GP we can evolve agents that learn how to make use of social conventions programmed into an existing society of agents, and furthermore we can actually train the new agents to behave according to the same social conventions. We believe by using some of the scalability methods mentioned in the next chapter we could greatly improve the performance of these agents.

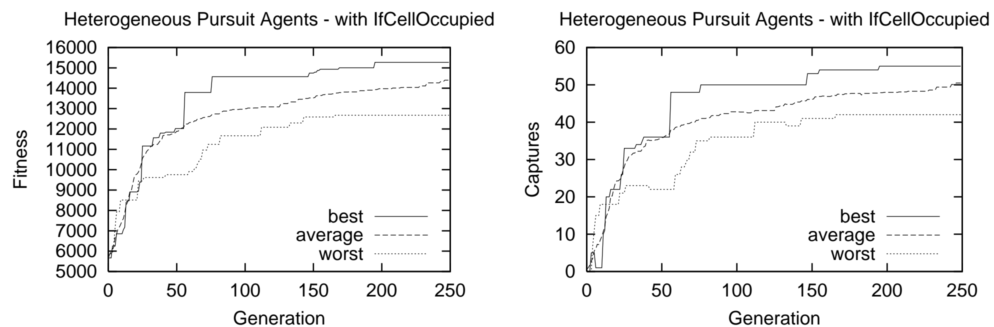


Figure 6.3: Experiment 7b : Performance Graphs of Socialised Heterogenous Pursuit Agents

The results of 10 runs of the heterogenous predator experiments are shown in table 6.4 and the

Table 6.4: Experiment 7b : Fitness of best socialised Heterogenous pursuit agents

Heterogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
4	310	12674.417	36.050	42	11779.794	39.085	1244
1	303	12768.533	40.783	42	11537.858	41.185	1134
7	442	13177.233	37.150	43	12149.423	39.099	1265
6	351	14491.217	35.683	51	12948.046	38.845	1415
8	431	15024.067	36.100	55	12889.357	38.387	1418
3	409	15076.650	30.083	53	13753.141	34.912	1521
9	398	15111.967	30.850	54	13485.342	35.467	1476
0	442	15164.967	31.217	55	14230.562	35.466	1657
2	441	15209.433	33.550	55	13593.414	37.841	1553
5	270	15272.000	30.867	55	14390.490	32.522	1640
Mean	379.700	14397.048	34.233	50.500	13075.743	37.281	1432.300
750k	167	9748.600	40.600	4	6449.118	47.500	146

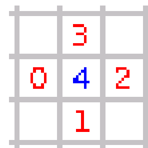


Figure 6.4: Experiment 7b : Capture Position Assignment

graphs of figure 6.3. The evolved heterogeneous agents were spectacularly successful, three runs of which achieved a capture rate of 55 out of 60. The generality was also very good, the best being run 0 which scored a capture rate of 1657 out of 2000. Behavioural analysis of the best evolved code confirmed that the new agents each selected one of the captures positions which were not allocated to the previously evolved agents. The new agents learned through interaction with the society their role in that society. Figure 6.4 shows the capture position assignments used by the best individual.

6.3 Conclusion

We have shown that GP can be used to create socialised agents. These are agents that have been evolved to work within an existing society of agents. We have also shown that the society can be homogeneous, or heterogeneous, communicating or non communicating. We have demonstrated that the new agents can learn to identify their role in the society and can communicate with existing members both implicitly and explicitly. They can also detect and resolve conflicts between

themselves and the existing members. The latter can also involve learning the social conventions of the society.

A important concept in software engineering is code reuse. Code reuse reduces the effort required in large complex systems, whilst increasing the robustness of systems. The ability to use GP to work with previously coded agents provides excellent code reuse and improves the scalability of GP.

Chapter 7

The Scalability of Genetic Programming

Even for small sized problems, the memory and processing requirements of GP can be quite significant. Consider for example the pursuit problem, with homogeneous predators. Let us assume a population size of 10,000 individuals, using a GP system that uses 4 32-bit integers for each node in the program tree.

If we assume that at worst each individual consists of 1000 nodes, then our estimate for space requirements would be more than $10,000 * 1000 * 4 * 4 = 160Mbytes$. If we assume the use of a generational engine, then depending on the implementation we can assume memory requirements that are up to double the calculated value.

If we then assume that we execute ten runs of this problem, where each run evolves the population for 500 generations, and that evaluation of each generation involves running the fitness function for each individual in the population, then the total number of fitness evaluations is $10 * 500 * 10,000 = 5 * 10^7$. For the pursuit problem, if the fitness evaluation involves testing the program for over a total of 60 test cases, where for each test case, the program is executed for 4 times (for each predator), for each of the 50 cycles, then the total number of evaluations of the program is $5 * 10^7 * 60 * 50 * 4 = 6 * 10^{11}$ evaluations.

Clearly this brings into question the scalability of GP for solving medium to large problems. This issue is of particular importance when we use GP for evolving complex multi-agent systems. The key factors are the space required to store the population, and the time taken to execute the fitness function. The impact of both of these factor can be reduced by improving the performance of the GP system so that it requires a smaller population, running for a fewer number of generations to solve problems.

7.1 Solving Space Problems

We can reduce the memory requirements of the genetic programming algorithm by a several ways. These can be categorised into three main groups, improving the implementation of the GP

system, improving its performance and tuning the problem definition to optimise performance.

7.1.1 Implementation

We can implement the GP algorithm in ways that reduce the space requirements. The most efficient way of representing a population in a GP system is to use a directed acyclic graph (DAG). Handley [Han94a] used this method to represent the entire population of parse trees as a single directed acyclic graph (DAG). One advantage with this approach is that eliminates duplication of identical subtrees. Handley reported a 15-28 fold reduction in the number of nodes per generation when using this representation.

An alternative is to use a linear representation, where each tree is represented in polish notation as a byte array [KDBK99]. This is possible since the total number of terminals and functions is unlikely to exceed 256, even when including ephemeral constants. This approach significantly reduces the memory requirements. A program of 1000 nodes can now be represented using only 1KB instead of 16KB as in the above calculation.

There are two widely used GP engines, the Steady State Engine and the Generational Engine. The Generational engine as defined by Koza generates an entire new population from the old, replacing it. This would suggest that the implementation would require space sufficient to hold twice the population of individuals to work. However, Koza [KDBK99] describes a memory efficient crossover algorithm that requires space little more than populations size. This approach has been implemented in a number of GP systems including GPsys. The steady state engine works by placing newly created individuals into the population as they are created, and hence only require a single population to be in memory.

Each of the above techniques will still cause problems as the population size and/or the program size scales up. As the memory requirements increase, the operating system will use more and more virtual memory, causing the process to start paging. Instead of using memory, we could use as database to store the population. This is possible since, at most we need only two Individuals to be in memory at any given time. Fitness evaluation only requires one individual to be in memory, and crossover requires two. Thus we could store the population in a database and retrieve individuals as and when required. An important requirement of this approach is that the time taken to download an individual from the database should be significantly smaller than the evaluation time. Current RDBMS allow very fast storage and retrieval of a very large number of objects (in the order of Terra-bytes) and hence retrieval of individuals from a database should not impede performance significantly. Furthermore, since there are no concurrent access requirements in our application, we would not require database locking and therefore should be able to significantly increase the performance of database access.

Using a RDBMS or an ODBMS, we should be able to apply GP to particularly difficult domains where the population sizes are very large and/or the size of the evolved programs need to be very large. Using the same calculations used at beginnings of the chapter, we can quite easily increase the population size by 2 to 3 orders of magnitude without running into problems with current databases. Alternatively or in addition, we could also increase the size of the individuals by 3 orders of magnitude. This would allow us to evolve Megabyte size code using populations of millions of individuals. Whilst this seems hopeful, in the next section we shall see that the real constraint is the computational load.

The use of databases brings additional benefits:

- results stored in a relational database allow easy post-processing, and there many tools that integrate RDBMS with spreadsheet calculators
- assuming the use of transactions to write each new generation to the database, we can easily restart runs after a system crash or reboot using the last saved generation
- we could store every generation of a run to allow analysis of evolutionary flows. We would also need to store information about the genetic operator used to create the individual together with the trees/subtrees of the parents involved. This may help us better understand the GP algorithm.
- We can have a population that grows by one generation each generation so that no individuals are ever removed. This could be used to reduce loss of population diversity.

7.1.2 Tuning the problem definition

We could choose GP parameters that lower the space requirements. The most obvious ones are to restrict the size of the population and the size of individuals in the population.

Restricting the size of individuals in the population is usually a requirement of nearly all GP systems. However, there are two problems with this approach. The first is that it we cannot predict in advance the size of solution programs. Whilst algorithmic complexity theory suggest a lower bound on the size of such a program, the actual size of the evolved program cannot known. A possible solution is the application of *parsimony* pressure in the fitness function. The second problem is that to make GP more scalable we will need to be able to evolve bigger programs. However, the use of ADFs and automatically evolved data structures may help.

The single most important factor for a successful GP application reported by GP practitioners is the population size. Therefore reduction of the population size is unlikely to help make GP more scalable. However, this possibility has been investigated by Gathercole and Ross [GR97a] who

report that small populations evolved for a large number of generations can be better than large populations over a few generations. It is noted that they use a high mutation rate (60%).

Considerable work has been done to demonstrate that Automatically Defined Functions can be used to evolve code re-use [Koz94b]. Code reuse can be used to reduce the size of an evolved solution as well as to reduce the computational effort required to find it, and should consequently be used wherever possible.

7.1.3 Performance

We can optimise the GP algorithm to minimise the required population size. This would involve making better use of a smaller population, the main obstacles of which are premature convergence and bloat.

When a GP run has prematurely converged, the programs start to increase in size until they reach the maximum allowed sized. The code that is added to the solutions is referred to as bloat or introns and has no effect on the fitness of the evolved program. The cause of bloat has been studied by Langdon and others [LP97a; LP97b; Lan97a; LP97c; Lan97b; Ang98]. Suggestions have been made as to how we can reduce bloat and hence the size of evolved solutions. However, this is likely to make little difference as the appearance of bloat marks premature convergence.

Premature convergence leads to the population converging on a less than optimal solution early in the run. One of the explanations for premature convergence is the lack of genetic diversity in the population. This happens when the proliferation of fit individuals earlier in the run, obviate the need for primitives that are needed later. Small populations are more prone to this problem, as convergence happens more rapidly. If however we detect loss of diversity and employ some technique to re-inject it into the population, we may be able to reduce the effective population size. Ways of maintaining population diversity include changing the fitness test cases, so as to prevent individuals that work on easiest of test cases from dominating the population. This approach was used in the experiments in this thesis. Other alternatives include using competitive co-evolution to co-evolve the difficulty of the test cases together with the programs used to solve them. This creates a continuous arms race and prevents simple solutions from dominating the population. This technique was used recently to evolve a checkers program that can win even expert checkers players without ever having played an expert [CF99].

The use of demes present a simple yet effective means of combating premature convergence and is discussed further later in this chapter.

Another common method used to defend against premature convergence is to use mutation to help re-create diversity and re-introduce lost genetic material. Koza initially avoided the use of mutation in GP and relied entirely on crossover, but has since changed his policy.

7.2 Solving Processing Problems

There are two ways that we can reduce the computational requirements for a GP run. We can reduce the cost of evaluation and/or we can reduce the number of evaluations that are required. The former relies on improving the efficiency of the fitness functions, and the latter on improving or optimising the use of the GP algorithm.

In addition we can use various techniques to speed up the evaluation times, including hardware solutions and parallel and distributed processing. The processing requirements can sometimes be aggravated by paging caused by excessive memory utilisation. The solution to the latter has already been covered in the previous section, and therefore will not be examined here.

7.2.1 Speeding Up Fitness Processing

7.2.1.1 Use Hardware

There is considerable work in progress with using evolutionary techniques to evolve hardware. The technique involves evolving connections for Field Programmable Gate Arrays(FPGAs) which can be reconfigured at high speeds, and allows low level signalling to be exploited without simulation. It is doubtful however that this technique is any faster than simulation using Spice [KDBK99] on the fastest general purpose processors. The FPGAs tend to have bandwidths of 10MHz or less whereas the top end Pentium processor have exceeded 1GHz.

7.2.1.2 Machine code GP

The principle is to get GP to directly evolve machine code. The idea being that the code generated should be extremely fast in execution [NBF99; Nor97]. However, the cost is that we lose the abstraction provided by high level languages resulting in larger code sizes, particularly when we deal with complex problems. This in turn affects the search space; there are fewer permutations and combinations of operators and operands in a 100 node tree than a 10000 node tree.

7.2.1.3 Use Effi cient High Level Languages

It is argued that using efficient high level languages like C and C++ offers performance advantage over languages like LISP, and we note that Koza now uses a C implementation of GP. Languages such has Java however, offer alternatives such as run time optimization and results with GPsys suggest that this may not be necessary.

7.2.1.4 Compiled Code

The code generated by GP is typically interpreted many times during fitness evaluation. We can therefore speed the evaluation time significantly by writing the code to a file and compiling it to generate machine code. This would only make sense if the evaluation time takes many times

longer than the compilation time. Harris [HB96a; HB96b] used this approach to speed up the evaluation time of feature detection code generated by GP.

7.2.1.5 Parallel and Distributed Architectures

The use of parallel and distributed architectures provides one of the most significant ways of making GP scalable. The scalability comes from being able to add processing power incrementally as and when required.

Parallel computers minimise the costs associated with having multiple processors by sharing computer hardware, such as the motherboard, disks, memory, VDU and other peripherals between the processors. Their main disadvantage is that they present a single point of failure. There is always a limit to how many processors can be added to a parallel processing system and hence scalability is limited.

In comparison, distributed architectures are more costly per CPU, but offer greater flexibility in that it is easy to add or remove processing power. There is theoretically no limit to how many computers can be added and hence distributed architectures are extremely scalable. Each networked computer can fail without affecting others and hence a distributed system is more fault tolerant. The main disadvantages are cost and management issues.

The most recent trend is to combine both parallel and distributed architectures to produce super computing clusters, which offer the best of each approach. The Linux Beowulf architecture is one example.

There are a number of techniques we could employ to exploit parallel and distributed processing:

- evaluate individuals from a population in parallel
- evolve demes in parallel, allowing communication between demes
- execute a number of runs in parallel

The first technique works very similar to standard GP, except that we farm out evaluation of individuals to a number of processors. These processors may be on the same parallel processing computer hosted by a multithreaded operating system, or distributed across networked computers. The latter is only effective if the time taken to evaluate an individual is significantly larger than the time take to communicate individuals and evaluation results. A disadvantage with this approach is that it is not fault tolerant. If one or more of the processors fail we need to ensure that the evaluation of the individuals allocated to it are restarted elsewhere. As with all distributed systems, complex failure semantics make the latter difficult.

The last technique is extremely simple, and is used by most GP practitioners. Here we execute

a separate run on each processor or host. There are no communication overheads with this approach.

The standard GP breeding policy is described as panmictic since it potentially allows any individual in the population to breed with any other individual. An alternative is to divide the population into subpopulations called “demes”, or islands. These demes are typically positioned on a two dimensional grid, and breeding is restricted so that any two parents have a much higher probability of being selected from within the same deme or from geographically close demes.

This approach has been investigated both by D’Haeseleer and Bluming in [DB94], and by Tackett and Carmi in [TC94]. Tackett and Carmi use explicitly defined demes of different sizes for a problem with tunable difficulty (the donut problem). Their results suggest that use of demes significantly improves performance. In contrast D’Haeseleer and Bluming instead of using explicit demes, introduce locality in the form of geographic neighbourhoods, and select parents from the same neighbourhood and also place the child into the same neighbourhood. They have detected the spontaneous emergence of deme like structures in their results and report that the introduction of locality significantly improves population diversity which in conjunction with slightly increased generality of individuals, yields a substantial improvement in the generality of the population as a whole. Recalling that loss of population diversity is one of the reasons for premature convergence, demes therefore delay convergence.

One of the advantages of the demic approach is that it is easy to implement using parallel or distributed architectures and has very low communication overheads. A second advantage is that it is extremely fault tolerant, the loss one or more demes does not halt a demetic run. Koza and Andre [KA95; AK95; AK96a; AK96b] developed a GP system using a network of transputers to exploit the parallelism inherent in the demic model. Each processor was allocated a separate deme. The results of his experiments with EVEN-N-PARITY problems indicated that more than linear speed-up in solving the problem using GP was obtained. Figure 7.1 a) shows the architecture used by Koza. The demes were connected to form a torroidal structure which has the advantage of good communication between neighbouring demes as well the ability to maintain high connectivity in the face of node failure. Bennett and Koza have more recently employed the use of a Beowolf architecture [IKSS99] which effectively combines distributed and parallel architectures to provide scalable processing power. This architecture has been used for evolving programs using GPPS which has 3 orders of magnitude greater processing requirements than traditional GP systems.

The Internet with its millions of networked computers offers excellent possibilities for distributing work. This has been exploited by the SETI project which works by means of a specialised screen saver. The latter kicks into operation during long periods of inactivity and connects to a

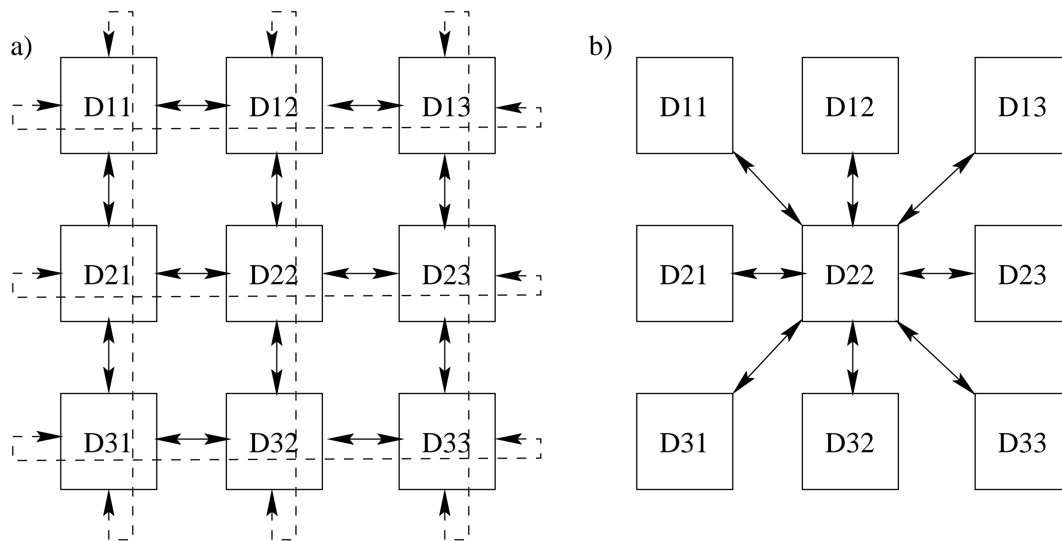


Figure 7.1: Parallel Demetic GP

specific computer to download data to be processed. The results are shipped back to the same computer after completion. It would easy to create a demic GP system which works on the same principle. The architecture would probably be shaped as in part b) of figure 7.1. The computer hosting the GP system would form the hub, allowing communication between demes to occur and collection of results. This approach was used by Chong [Cho99a; Cho98; Cho99b; CL99] via a Java based GP system running as an applet.

Poli [Pol96b; Pol96a; Pol97b] devised a new form of GP called Parallel Distributed Genetic Programming(PDGP). PDGP uses a graph-like representation a provides fine-grain parallelism. Poli has shown in experiments with PDGP on standard GP problems that it performs significantly better than standard GP. An additional advantage with PDGP is that it allows symbolic and neural processing elements to be combined freely [Pol96c; Pol97a].

7.2.2 Reducing Fitness Function Evaluation Costs

Reducing the evaluation cost ultimately relies on making the fitness function as efficient as possible. The most effective way of achieving the latter is by using test case sampling. The remainder are simple rules of thumb.

7.2.2.1 Deadlock Detection

When evolving MAS programs, we normally repeatedly execute the evolved program for a given test case until the maximum number of iterations permitted have been expended. In situations where deadlock occurs we would be executing the programs needlessly until we have reached the maximum number of iterations. Detection of deadlock would allow the test case to be aborted and hence save processing time.

7.2.2.2 Caching

During fitness evaluation, particularly involving ADFs, it is possible that the evaluation of a branch or ADF gives the same results regardless of how many times it is executed, i.e. it is idempotent. For this case we can cache the results in memory and use this value instead of re-evaluating the code repeatedly.

Handley [Han94a] exploited the his DAG representation of the population to implement an extremely efficient method of caching. Duplicated subtrees are represented just once in the DAG representation, and hence by caching the computed value of each subtree he was able to avoid re-evaluation for each instantiation, even when the same sub-tree occurs in a different population. This was made possible by restricting functions so that they have no side effects and by fixing the test cases used by the fitness function. This approach reduced the number of node evaluations by 11-20 fold per run.

7.2.2.3 Reducing Wasted Computation By Aborting Runs

If we can detect premature convergence, we can abort runs which will ultimately fail. However detection of premature convergence is non trivial. One simple check is to ensure that the genetic material known to be part of the solution has not been lost.

Teller and Andre [TA97] describe a method that can be used to reduce wasted computation in fitness evaluation by choosing the optimal number of fitness cases for each individual. Gathercole and Ross [GR97b] use the concept of an error limit to prevent individuals which exceed this limit from being further evaluated. This allows them to reduce the number of fitness test cases applied to an individual.

7.2.2.4 Simplifying the Code

We could copy the individual being evaluated and remove any redundant code (also known as introns) from the copy just before evaluating it. This can be justified as long as the time take to evaluate is significantly larger than the time taken to simplify the code. Care must be taken to prevent code with side effects from being removed. An example is a subtraction resulting in zero, where the operands are calls to functions that are expensive. Koza developed a LISP program that automatically removed redundant code [Koz92b] for evaluation.

7.2.2.5 Sampling Test Cases

Reducing the amount of tests cases against which a program must be tested as part of the fitness function is a simple and highly effective way of reducing the evaluation cost. The mechanism that was used for almost all the experiments in this thesis is variation of test case sampling. In test case sampling instead of applying the full set of test cases for each fitness evaluation, we sample

from a set of test cases. Typically a sample size significantly smaller than the test case set is chosen. In our experiments this sample size ranged from 1/6th for the pursuit problem to 1/20th for communicating agents problem. In our approach, for each generation, we randomly sampled a fixed number of test cases from the set of test cases and used them to evaluate each individual of the generation. The fitness value assigned to each individual is the average over the test cases. This approach allows individuals from the same generation to be compared easily. As mentioned in chapter 3, a key problem with this approach is that as we move from one generation to the next, the best of the generation may outperform than the current best of the run, not because it is generally better, but because the sampled test cases were easier. Ideally we should weight the fitness values according to the difficulty of the test cases that were passed. Unfortunately this is not easy in the pursuit domain, where given a particular arrangement of agents it is difficult to measure how much effort is required to enable capture. To overcome this problem we devised a technique whereby after each generation, we re-evaluate the best individual of the generation using the full training set of test cases. This individual becomes the best of the run if its full fitness value is greater than the current best.

To justify our approach we conducted a comparison experiment (experiment 10). We implemented an experiment which was identical to experiment 8 (homogeneous pursuit with conflict resolution operators) except that the fitness evaluation used all 60 test cases for each individual instead of samples of size 10. The significantly higher computational costs meant that we could only run the experiment for 250 generations instead of 500. The results of the experiment are shown in table 7.1 and the graphs of figure 7.2. The total number of evaluations of each individual using this approach is estimated at $250 * 3000 * 60 * 50 * 4 = 9 * 10^9$. For the original experiment, the total number of evaluations is estimated at $500 * 3000 * 10 * 50 * 4 = 3 * 10^9$. The new experiment therefore required 3 times as many evaluations as the original experiment. The results, for the new experiment when compared with the results of the original experiment do not justify the extra computational effort. furthermore, the results from the original experiment show a greater generality.

It could be argued however that the new experiment was only run for 250 generations and therefore was disadvantaged in that the total number of individuals explored were fewer. To make the comparison fairer, we therefore present results from experiment 8 at generation 249 in table 7.2. Clearly these results show that even after just 250 generations the runs produced on average more general solutions. The best individual of the run was only slightly worse for the 60 test cases (passing 50 instead of 53) but was significantly more general. Furthermore, we effectively halved the number of evaluations and can therefore state that using our approach not only reduces the evaluation cost significantly, it also creates more general solutions. The problem with using a fixed of set test cases seems to be overfitting. These results should not surprise us, using the same

fitness test cases for each generation allows successful individuals in early generations to swamp subsequent generations which in turn causes loss in genetic diversity and leads to premature convergence. By contrast in the sampling approach, those individuals successful in one generation may not be so successful in subsequent generations. We believe that sampling effectively creates niches in the population, where each niche contain individuals that are successful for a particular set of test cases. Like demes these niches help preserve population diversity and hence delay convergence. Furthermore as we move from one generation to the next, the individuals which propagate the most are going to be those that can score high fitness regardless of the test cases presented to them, meaning those that are more general. The graphs in figure 7.3 show how the fitness and capture count of the best of the generation varies over generations. The graphs show the averages for the runs, the best run and the worst.

We have one last point that needs testing; the re-evaluation of the best individual of a generation using the full training set of test cases before comparing with the current best of the run. As an alternative we could compare without a full evaluation. The results we would obtain using the latter for experiment 8 are shown in table 7.3. Clearly these results are significantly worse than the original results hence, justifying this approach.

The technique described by Gathercole and Ross [GR94] as Random Subset Selection is similar to the technique described above.

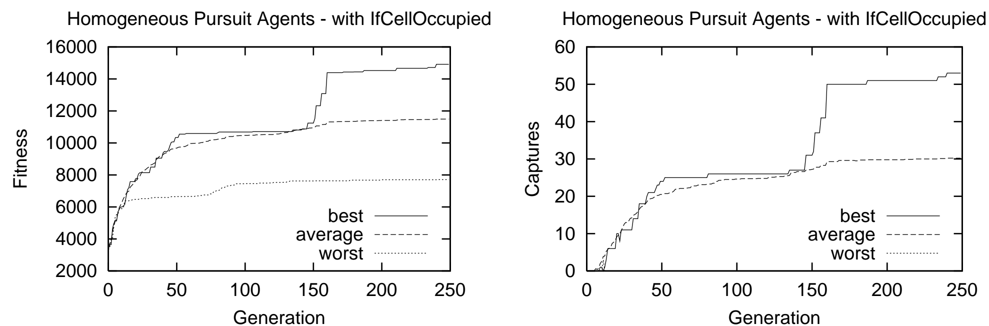


Figure 7.2: Experiment 8 : Performance Graphs of Evolved Homogeneous Agents (Full Evaluation)

7.2.2.6 Redundancy in Test Cases

We note that as we approach the end of a run, the number of test cases that are failed become smaller and smaller. It is likely that population would have converged on solutions to the easiest of the test cases early in the run. It seems wasteful therefore that these same test cases are still used later in the run. One solution might be to assign weights to the test cases dynamically, such that test cases most frequently passed are deemed easier and hence have smaller weights than those that are infrequently or never passed. In this scheme, the fitness sampling approach

Table 7.1: Experiment 8 :Fitness of best evolved Homogeneous agents (Full Evaluation)

Homogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
7	295	7704.550	50.000	0	6641.823	50.000	0
6	249	9781.967	42.733	21	8008.843	46.219	368
9	370	9977.433	43.550	23	7651.716	46.477	352
2	382	10138.950	43.283	22	7854.247	46.382	362
3	226	10595.300	41.817	22	9358.700	43.583	539
0	300	11248.617	40.883	29	10211.205	41.986	768
1	257	12107.433	39.750	35	9364.629	44.225	606
5	245	14179.183	38.067	50	11830.044	39.349	1235
8	266	14296.600	32.333	47	13166.297	34.562	1376
4	237	14918.783	32.117	53	12517.947	36.180	1275
Mean	282.700	11494.882	40.453	30.200	9660.545	42.896	688.100
750k	223	5654.633	49.950	1	5638.654	49.607	32

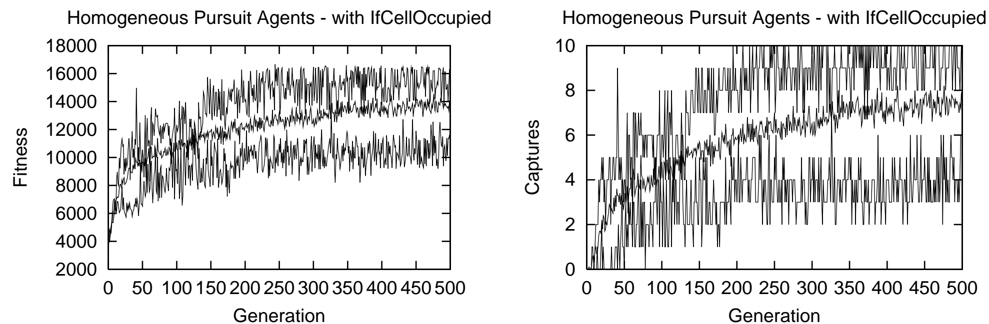


Figure 7.3: Experiment 4a : Performance Graphs of Evolved Homogeneous Agents (Fittest of Generation)

discussed earlier would be augmented to select test cases with probabilities in proportion to their weights. The danger with this approach is that the population may ”forget” how to solve the earlier test cases. However, the dynamic way in which the weights are defined should compensate. This approach has been investigated by Gathercole and Ross and implemented as dynamic subset selection [GR94; Gat98].

7.2.3 Improving the efficiency of the GP algorithm

We could reduce the cost of evaluation, by reducing the number of evaluations that are required. The number of generations required to find a solution, or one that is closest, is highly dependent on the problem domain. To minimise this number, we could:

Table 7.2: Experiment 4a : Fitness of best evolved homogeneous agents at generation 249

Homogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
4	258	8121.167	47.217	11	7343.922	48.300	205
7	331	9603.850	43.033	21	9401.273	44.210	624
2	229	10930.250	39.017	30	9478.439	42.790	697
0	281	11275.600	39.583	29	9255.731	44.354	541
5	228	11673.933	38.683	32	10188.894	41.212	768
3	186	11907.067	36.333	34	10473.023	40.379	812
1	338	12140.450	38.467	36	10992.288	39.824	953
6	227	12307.500	36.383	36	10805.262	40.014	878
8	238	12801.167	36.550	39	11967.796	37.739	1077
9	350	14320.900	32.383	50	13877.423	32.915	1565
Mean	266.600	11508.188	38.765	31.800	10378.405	41.174	812

- reduce the size of the search space
- improve the quality of the search space (make it less sparse)
- improve the genetic operators used to navigate the search space
- improve the starting population for a run

Important configuration parameters are the population size, the choice and number of functions/terminals and the maximum allowed size of trees. Increasing the population size increases the probability of obtaining a solution in earlier generations, but a significant cost is added for evaluating each generation which is likely to cancel out any useful gains. The choice, and number of functions and terminals directly affects the size of the search space; few high level functions and terminals are likely to be better than many simpler ones. We can estimate an upper bound for the number of possible program trees that we can generate. If we assume that the trees are represented in polish notation, then we can treat them as simple strings of code. Let us assume then that maximum size of the strings is restricted to N, and that the symbol set is of size Z (the total number of functions and terminals). The total number of strings is $\sum_{k=1}^{N} Z^k$ and shows the importance these two parameters. Note that this is likely to be a significant overestimate, as we have assumed that all symbols are equal and may be combined arbitrarily. In practice some of those symbols will have been functions whilst others will have been terminals. Terminals cannot have subtrees, and hence there are restriction on how we can generate these strings. Furthermore the use of a strongly typed GP system [Mon93] will further reduce the number of possible strings

Table 7.3: Experiment 4a : Fitness of best homogeneous agents of run (no re-evaluation)

Homogeneous - CRO		Training Tests (60)			Generality Tests (2000)		
Run	Size	Fitness	Av. Cycles	Captures	Fitness	Av. Cycles	Captures
7	362	8641.983	46.050	14	8802.604	44.564	506
6	320	10070.983	40.500	24	10420.545	40.244	803
2	251	10354.500	38.800	28	9487.855	43.035	700
3	262	10766.650	38.217	29	10322.856	40.523	824
0	274	11037.233	39.683	29	10079.818	42.301	730
4	241	11309.000	40.150	29	10424.150	41.860	784
5	281	11807.717	38.533	36	10488.759	40.588	885
8	192	12887.350	34.967	38	12137.954	37.553	1097
1	386	12982.317	34.550	42	12899.421	34.894	1342
9	352	13122.000	35.483	43	13157.326	34.996	1398
Mean	292.100	11297.973	38.693	31.200	10822.129	40.056	906.900

that we can generate. The latter however may impact the quality of the search space in a negative way, by making it more sparse for example and thus making it more difficult to find solutions and therefore cancelling any gains. Through use of ADFs we can restrict the maximum size of the evolved solution and hence reduce the size of the search space. This mechanism is not known to negatively impact the quality of the search space [Koz94b].

Improving the quality of the search space is very difficult. The major factors being the difficulty of the problem itself, the choice of functions and terminals and the fitness function. An ideal fitness function should be sensitive to extremely subtle changes in performance and be more continuous in its measurement than discrete. The challenge is how to describe a problem in a fitness function which has those characteristics.

The standard crossover and mutation genetic operators generate new individuals almost blindly, from their parent(s). The result being that the new individual has a good probability that fit portions of code are disrupted. Creating more intelligent genetic operators that avoid disrupting part of the code marked as useful may help evolution. Teller [TV95; Tel96] developed an intelligent crossover operator that learns to select good crossover points using information such as the execution path of the evolved programs. He reported significant improvements in the rate of production of fitter children using this operator.

Langdon recently analysed how the shape of trees generated for the initial population affect the performance of the a run [Lan99]. He suggested that problems can be divided into two types, those in which the solutions trees are deep and those where they are wide. The creation

method parameter is used in GP systems to specify the shape of the initial population, with values including Grow, Full or Ramped half and half [Koz92b]. The Grow method tends to create many short trees, the full method creates deep trees. If we choose the wrong type for a given problem, we may disadvantage the run. An analysis of the fittest individuals of randomly shaped trees may provide insight into the value to be used for this parameter.

7.3 Conclusion

We have provided a solution to the space complexity issues of GP (the use of a database) and have shown that the key issue is one of fitness evaluation. We have reviewed the techniques that are currently in use to alleviate the demands of fitness evaluation and shown through experimentation that fitness sampling is extremely effective.

Chapter 8

Genetic Programming and Software Testing

If we compare the human software development cycle with software generated by GP we note that whereas test cases are used in software engineering to *verify* that the program conforms to the requirement specifications, in GP the test cases along with the fitness function *are* the requirement specifications. The analog of the traditional software testing phase in GP is the testing that we perform at the end of a run for generalization of the evolved code. Once again we note that instead of comparing the results of tests with the specifications, the test cases are themselves most of the specification. This strong dependency of GP on software testing suggests that formal software testing techniques are extremely important for GP. It is surprising therefore that little work has been done on the use of formal software testing in GP. In the this chapter we consider formal software testing techniques and comment on their suitability for genetic programming. In particular we also look at techniques suitable for testing distributed/multiagent systems.

8.1 Fitness evaluation

When evolving programs using GP we measure a programs fitness by testing the program and measuring success. We test programs by initialising the inputs to some predefined parameters, executing the program and then measuring in some way the difference between the desired output and the output obtained. The tuple consisting of input values and desired outcome define a test case. In GP the desired outcome is effectively encoded in the fitness function and hence only the input values for each test case is required. Many test cases are needed to test a program. Ideally we would like to apply all the possible test cases for a given application so that we get an accurate measure of the program correctness or fitness. In practice however this is seldom feasible due to large number of test cases that are possible. Consider for example, our pursuit problem.

There are 4 predators and one prey that can be placed in any of the cells in a 30 by 30 board.

If we place the prey first, there are 900 positions to choose from, and for each such positions, we need to choose 4 positions for the predators. If the predators are homogeneous then we can choose from any of the 899 cells in an order independent manner. For heterogeneous predators, the order matters and we need to consider permutations of the different orderings. The number of possible combinations for selecting cells for 4 homogeneous agents and from 899 positions is given by the binomial co-efficient

$$\binom{899}{4}$$

and evaluates to about 27 billion positions. Multiplying by 900 for the prey positions gives approximately 24 trillion test cases. For heterogenous agents there is an additional factor $K!$ that we must consider, since the ordering of the predators is important. This gives us approximately 583 trillion test cases. To make things worse, we can multiply the above values by the different number of programs that we will be used to drive the prey, since each is effectively a different test case. In a typical GP run consisting of a population of 1000 individuals, evolved for 500 generations, allowing 50 cycles for each test case will require $1000 * 500 * TestCases * 50 * 4$ evaluations of our program.

Clearly exhaustive testing of our program is not going to be possible. We are therefore forced to take a very small subset of the test cases and use them for training purposes. In our experiments for the pursuit problem, we chose 60 such test cases, there were actually 30 different placements for two different types of prey. Furthermore, out of the 60 test cases, to reduce the time taken to evaluate individuals, we exposed each generation to a sample of only ten test cases, which are chosen at random for each generation. Note that for our implementation of the pursuit problem, since the agents are stateless, a new test case is effectively created by each simulation cycle. Therefore the effective number of test cases seen by each individual during an evaluation can be as many as 50 times the number of scheduled test cases. This is still a small number considering the full space of test cases.

Even when testing the generality of any evolved solution at the end of a run, we still can only sample from the full space of possible test cases because of the high cost of evaluations. For our experiments, we generated 1000 unique random positions for the agents and created 2000 test cases out of them using still and randomly moving preys.

Software testing is central to GP, and we therefore conclude that the GP research community could benefit greatly from the experience gained in the formal software testing of human coded programs.

8.2 Software Testing and Quality Control

The testing of human coded computers programs is now an established part of software engineering. There are number of well developed techniques that are used to test software and we hope to show that these techniques can also be used to test programs evolved via GP.

8.2.1 The Definition of Software Testing

It is intuitive to think of testing as a means for checking whether a program achieves its intended purpose. Using this definition it seems quite reasonable to use a random set of inputs and compare the output with the desired output. However, in his classic book "The Art of Software Testing" [Mye79] Myers argues that a better definition is that:

"testing is the process of executing a program with the intent of finding errors"

A successful test case would therefore be one that detects an error. If we use this definition, it becomes apparent that random-input testing is less effective. Myers describes random-input testing as probably the poorest testing methodology. If we think of the tests in terms of probability of finding errors, it becomes clear that this approach is unlikely to be optimal or even close to optimal.

One weakness with Myers' definition of testing is that it relies entirely on program execution for testing, there are other techniques that can be used which do not require the program to be executed. This leads Hetzel [Het85] to use a more general definition

"Testing is the measurement of software quality. (where quality means meets the customers requirements)"

This definition has the advantage that it allows for metrics other than program success that we may use to judge the quality of the software such a size of code, economy in use of resources and execution time. Furthermore it doesn't just rely on program execution. A weakness with this definition is that requirement specifications whilst being clear on what a program should do, seldom specify undesirable features. This is not surprising as the list of undesirable behaviour is often much larger than the list of desirable behaviour.

Beizer [Bei95] lists the following as part of the reasons for testing software:

- to *falsify* the object with respect to stated or unstated requirements
- to *validate* the object; that is, show that it works

Both of these help us to evaluate the quality of software. Tests used to falsify software are called *dirty tests* or *negative tests*. Those used for validation are called *clean tests* or *positive tests*.

By using both types of tests, we can get a good measure of the correctness of the program. Falsification and validation should therefore important aims of a fitness function.

We know from our discussion earlier in the chapter that exhaustive testing is often impossible. Any testing that we perform is hence incomplete. Our goal is therefore to cover as many errors as possible with the minimum number of test cases. The key issue in software testing is one of *test case design*:

”What subset of all possible test cases has the highest probability of detecting the most errors?”

Many of the techniques developed for software testing help in answering the above question. These techniques can be divided into two sets, *black box testing* and *white box testing*. There are also hybrid techniques that combine the two to maximise test coverage.

8.3 Black Box Testing

Black box, behavioural, functional and dynamic testing are all names for testing techniques that test programs by executing them. They treat a program as a black box which has inputs and outputs and are not concerned with the structure or internal behaviour of the underlying code. Tests are performed by applying different inputs and checking the output for conformance with the requirements. Clearly complete testing would therefore require tests for every possible set of inputs, which is is not possible. Consider for example testing an optimizing “C” compiler! *Equivalence partitioning, boundary-value analysis, cause effect graphing and error guessing* are methods that can be used to design black-box test cases that maximise error coverage using the minimum number of test cases. Beizer [Mye79] lists other techniques, but many of these require building a graph based model of the program being tested and rely on assumptions about the likely structure of the code being tested.

8.3.1 Equivalence Partitioning

Equivalence partitioning is a powerful means of reducing the number of test cases that are needed. The approach involves partitioning the input domain of a program into a finite number of *equivalence classes*, such that testing one value from the class is equivalent to a test of any other value. If one test in the class detects an error than the same error would be detected by any of the other tests in the class. Since equivalence classes can overlap, it is also possible that a a test case may simultaneously be representative of tests in more than one class. This type of test cases is therefore even more useful than one that fits into a single equivalence class.

If we consider the communicating agents of chapter 5, it easy to see the kind of equivalence partitions that we could create. The X-Y axis creates four quadrants, and the agents can be located within the same quadrant or in different quadrants, each possibility could be seen as an

equivalence class. In any case it is easy enumerate the total number of relative positions in terms of quadrants. Alternatively we could partition the test case space by just the relative positions of X-Y coordinates, so that $x1 > x2 \& y1 > y2$ is an equivalence partition. We could even use both schemes, which would invariably result in overlaps, which could satisfied with fewer test cases than the total number of equivalence classes.

8.3.2 Boundary-value Analysis

The general principle behind boundary-value analysis is that test cases that explore situations that are on or near the edges between equivalence classes have a higher probability of finding errors. So instead of just picking any value from an equivalence class, we select values that put each edge of the equivalence class to test. Furthermore this approach is used with input equivalence partitions as well as output equivalence partitions. Typically values are selected on each boundary, and very near to the boundary, or on each boundary segment as well as near to the boundary segments. This technique is also called extreme value testing. When the partitions are defined by inequalities on the set of inputs they are referred to as domains and we use the term *domain testing* instead of boundary analysis.

8.3.3 Cause-effect Graphing

Cause effect graphing is useful for finding errors associated with combinations of input values. These type of errors are not exposed by the previous two methods. The general principle is to translate by hand a requirement specification into a boolean graph (the cause-effect graph) or set of graphs, where a cause is a distinct input condition or equivalence class, and an effect is output condition or system change. The graph is then annotated with constraints and eventually converted into a limited-entry decision table, where each column in the table represents a test case.

8.3.4 Error Guessing

Error guessing involves predicting the type of errors we might have in a system from knowledge of the application domain, or from statistical information about the relative frequency of occurrence of different types of errors.

8.4 White Box Testing

White box, structural logic-driven and static testing are all names for test techniques that work on the source code itself. Tests are created by examining the program logic and selecting inputs that exercise as much of the logic as possible. The analogue of exhaustive input testing for white-box testing is exhaustive path testing. Exhaustive path testing requires that we execute all possible

paths of control flow through a program. The latter is again infeasible due to the very large number of flows created via conditionals, loops and function calls. A number of methods are available for white-box testing including *Statement, Decision, Condition, Decision/Condition and Multiple-Condition Coverage*. All of these methods can be automated to enable white-box test cases generated by a program. Many test generation tools use white-box testing.

8.4.1 Statement Coverage

Statement coverage requires that we design test cases such that every statement in the program gets executed at least once. The weakness with this approach is that it does not exercise conditional statements very well. As a result many paths containing errors may not be exercised.

8.4.2 Decision Coverage

Decision coverage requires that we write test cases such that each decision statement in the program has a true and false value at least once. Assuming that a program contains at least one decision statement (examples include `If, While, For, Switch`) then decision coverage will also provide statement coverage.

8.4.3 Condition Coverage

Where a decision statement consists of more than one condition, such as `If (e1 & e2)` to ensure that each condition is tested, we use condition coverage. Condition coverage requires us to write test cases such that each condition in a decision assumes all possible values at least once.

8.4.4 Decision/Condition Coverage

Condition coverage does not subsume decision coverage. If consider the same example `If (e1 & e2)` we note that condition coverage requires us to create two test cases represented by the tuples (e1 = true, e2 = false) and (e1 = false, e2 = true). This clearly does not cause the statement guarded by the `If` clause to be executed. We therefore still needs Decision coverage. Decision/Condition coverage hence requires that we write test cases such that each condition in a decision as well the decision itself takes on all possible outcomes at least once.

8.4.5 Multiple-condition Coverage

Many programming languages use *lazy evaluation* to evaluate and/or condition operations, so that for example, if the first condition in an `and` expression is false, the second condition is never evaluated. Similarly in an `or` expression, if the first condition is true, the second is never evaluated. To ensure that errors in logical expression are detected, multiple-condition coverage is used. This requires that test case are written such that all possible combinations of conditions in each decision are covered. Multiple-condition coverage subsumes decision/condition, condition

and decision coverage.

8.5 Applying Software Testing to GP

The use of test cases are important for two phases of GP. The first phase is for training purposes and the second is for generalisation.

8.5.1 The Training Phase

During the training phase each program in each generation is tested via the function. The fitness function executes the program using a set of test cases and measures success. The test cases could be designed using software testing techniques such as white-box or black-box testing. However, the programs that we are testing in GP are constantly changing, and therefore tests generated via white-box testing techniques cannot be reused, and need to be re-generated for each program during the evolutionary. Clearly this is likely to be too costly. Black box testing techniques are ideal, as they do not rely on the structure of the code being tested. The implication is that we can design test cases once using black-box techniques and use them throughout for testing each generated program.

Black box-techniques such as equivalence partitioning and boundary-value analysis should help us design the minimum number of test cases to cover a large areas of the test space. This should in theory be much better than the random testing techniques that are commonly used, and hence make our fitness functions more accurate as well as efficient. This in turn should help scale GP. It should be noted that even using black-box techniques, the number of test cases is still likely to be too high for evaluating each individual, and hence the sampling technique detailed in the previous chapter and used in the experiments is still applicable. The main difference being that the test cases that are sampled have been designed for maximum coverage.

As mentioned earlier, clean tests are used for validation, whereas dirty tests are used for falsification. A key issue that needs to be researched is how the ratio of clean versus dirty tests affects evolution.

8.5.2 Generalisation

At the end of a run GP practitioners usually retest the program using unseen test cases as check of for the generality of the solution. This is typically done by using a larger sample of randomly generated tests. As a quality control mechanism this is far from optimal. A better strategy would be to use a combination of white-box and black-box test techniques which together would provide a much more accurate measure of the quality of the program. White-box test techniques can be used at the end of run since we only have a single program for which we need to generate test cases.

8.5.3 The Pesticide Paradox

In human coded software, information about errors that have been detected via a set of tests is used to edit the programs and remove all occurrences of such errors. The usefulness of the set of test cases that were used therefore decreases. A similar process happens in GP. As evolution progresses more and more test cases are passed by individuals in the population. The effectiveness of such test cases is therefore decreased. Software testing techniques have therefore co-evolved with the program errors that they try to detect. This suggest that maybe research into how we can co-evolve test cases for any given application might be interesting.

8.6 Testing Distributed and Multi-agent Systems

There is has been a surprising shortage of research in the testing of distributed systems. One of the reasons is that testing of distributed, multi-threaded applications is many times more difficult than testing non-distributed systems. Research has therefore focussed on formalised design techniques which try to reduce errors. The following is a non-exhaustive list of issues associated with testing distributed systems which apply equally well to multiagent systems:

- *Transient Failures*: distributed programs can suffer failures caused by temporary conditions such as network delays due to high volumes of traffic, or router failures causing network partitions. Communication delays can result in timing errors which can cause a process to fail, which in turn will causes tests to fail. If the same tests were to be repeated the errors may not occur.

- *Reproducibility*: Execution of a distributed application may depend on the order in which component processes are executed as well as the order in which messages are received. Thus execution of the same application using the same inputs hence may not yield the same results.

- *Observability*: An attempt to gain information about the behaviour of a distributed application may change its behaviour, so that for example errors disappear or are introduced by the observer. This is known as the *probe effect*.

- *Timing*: A distributed system does not have a global clock. Hence analysis of system logging from test execution does not accurately provide information about the sequence of events. The notion of logical time based on causal ordering must hence be used.

Krawczyk and Wiszniewski [HK98] provide a detailed analysis of the issues in testing distributed software applications.

Low, Chen and Ronnquist [LCR99] propose a methodology for automatically generating test cases to test BDI agents. Their methods are based on white-box testing techniques. They divide test coverage into node based criteria and plan based criteria. Nodes are similar to statements and hence node based criteria is similar to statement coverage. The key differences is that nodes can fail. A plan consists of set nodes connected by arcs to create a node path. A plan is successfully executed by executing each of the nodes in the path. Failure of plan occurs if any of the nodes in the path fail. Communicative interaction between agents is modeled by the notion of connected plans.

8.7 Conclusion

We propose that GP practitioners can benefit significantly through a better understanding of software testing techniques developed for software engineering. We have shown how black-box techniques could be used to design more efficient and accurate fitness functions, and have also shown that by combining black-box and white-box test techniques we can more accurately measure the quality of programs evolved using GP. We have noted that the lack of documented systematic methods for the testing of distributed and multiagent systems may make it more difficult to apply GP to these domains.

We hope that the strong dependency of GP on software testing, and intensive manner in which testing is used, may help advance research in software testing.

Chapter 9

Conclusion

The elegance of using GP for programming multiagent system comes from being able to search for desired behaviour at both the individual and group level simultaneously. Using GP we can avoid having to deal with the *micro/macro problem*. We generate agents at the micro level, and evaluate them at the macro level. The agents with the best evaluations(fitness) are used to create new agents at the micro level. This process is iterated until desired micro and macro level behaviours emerge. The fitness function does not need to detail how each agent is to behave, or interact with each other or the environment. It just needs to provide a relative measure of how successfully the agents solve the global problem. The *potential* ways in which agents can interact with the environment and other agents is specified by the function and terminals sets. Further advantages are that the GP approach does not rely on extensive domain knowledge, and that it is focussed on performance, which in the end is what matters. Note that performance can be measured in a number of ways, ranging from how effectively a systems meets its goals to how efficiently this is achieved.

9.1 Review of the Work

In chapters 4,5 and 6 we used GP to evolve agents that show well coordinated, coherent behaviour. In chapter 4 we evolved both homogeneous and heterogeneous pursuit agents using game rules and test cases that are biased towards creating more conflicts. We showed that it was easier to evolve successful heterogeneous agents than homogeneous agents. The explanation was that the heterogeneous agents avoided conflicts by statically allocating tasks between them. The homogeneous agents must allocate task dynamically which is difficult. We showed that through the use of identities that homogeneous agents could perform as well as the heterogeneous agents. We noted that the problem with dynamic task allocation is that the agents are more prone to task overlap. This task overlap in turn results in greater conflicts. We therefore added an operator that we thought would be helpful for resolving conflicts. We then tried to evolve agents for the pursuit problem using this additional operator and showed that homogeneous agents had developed

social conventions that help detect and resolve potential conflicts.

In Chapter 5 we showed how GP can be used to create multiagent systems in which the agents communicate explicitly. We showed the evolutionary process can create programs that learn to send and receive information across two channels. The programs decided what information was sent across which of the channels, and how to interpret the information received from a channel to achieve the task in hand.

In chapter 6 we evolved agents that learn to work within a society of existing agents. First an agent was evolved which learns to communicate with the best agent evolved in chapter 5. The evolved program through interaction with the environment and the other agent learned to correctly interpret information received through the communication channels and what information it needs to transmit through the communication channels. Next we showed that we can evolve a pair of heterogeneous agents which learn to work with two other heterogeneous agents(the best evolved heterogeneous agents of experiment 4b). The heterogeneous agents of chapter 4 used strict task allocation, to avoid conflict. The new agents learned to divide the remaining tasks between them. We then showed that we could also evolve a program that drives a pair of homogeneous agents, which must interact with an existing society of homogeneous agents (the best evolved homogeneous agents of experiment 4a) that have predefined social conventions used to resolve conflicts. We showed that the newly program can at least in part learn to behave according to the social convention used by the existing society, and can certainly exploit these conventions.

In chapter 7 we considered the scalability issues in genetic programming, and reviewed possible solutions. We concluded that fitness evaluation is the key scalability issue. Chapter 8 proposed that software testing techniques that are used in software engineering may be useful for genetic programming. We believe that the use of GP to evolve agents shifts the emphasis in the effort required to building a MAS, into designing ways of testing them. This we argue is an area that has had less research emphasis and may limit the use of GP.

9.2 Solving Key Issues in MAS Research

The following is a very rough attempt to use GP to tackle many of the key issues in MAS described in chapter 2. An important point made by Jennings et al [JSW98] was that these issues are intertwined. The answers that we propose all rely on a single coherent methodology that that tries to address these issues simultaneously. We are therefore less likely to shift emphasis on particular problems by solving others. We accept that the view proposed below is an extreme view which is likely to create many more questions than it answers:

We can formulate a MAS problem by defining a fitness function. The fitness function needs to

encode a way of measuring how successful the agents are at solving the problem. The principle way of achieving this is to generate test cases and use them as specifications of the desired behaviour. We can decompose the problem automatically by specifying how many agents are to be evolved and what capabilities the agents possess(specified by the function and terminal sets). Furthermore, we can evolve agents that allocate tasks between themselves both statically and dynamically. Since the quality of result synthesis is the measure that we use when evaluating programs for agents, we already constrain the search to programs that drive agents to synthesising results.

We have shown in chapters 4, 5 and 6 that GP can be used to evolve agents that communicate both implicitly and explicitly. The agents decided what and when to communicate and how, and can develop their own interpretation of what is communicated. In chapter 6 we have even shown that the agents can learn to communicate with existing agents, which use a predefined interpretation of communicated information. If we model communication as function calls, then the act of sending a message is the same as calling a function. The act of responding to a message, is executing the function being called and returning the result. It has been demonstrated that GP can evolve functions (ADFs) [Koz94b] and can make use of them, hence we should be able to evolve sophisticated communicative behaviour quite easily.

By providing a fitness function that measures the success of the evolved agents at the group level, we believe we should be able to eliminate programs which cause the agents to work in an incoherent manner. Similarly harmful interactions may also be removed. We can avoid or mitigate harmful overall system behaviour, such as chaotic or oscillatory behaviour by the design of a fitness function that penalises such behaviour. The communication delays between the agents of chapter 5, caused agents early in their evolution to exhibit such behaviour, however this was eventually removed by natural selection.

We showed in chapter 4 how agents can be evolved that recognize and reconcile conflicting intentions when trying to coordinate their actions. The agents of experiment 4a learned to avoid, detect and resolve conflicts.

GP can create programs for agents that may or may not need to interact with other agents. If the application requires that agents need to interact, then this would have been implicitly encoded in the fitness function, and should have been made possible through the primitives provided. The evolutionary process would therefore favour those programs which cause the agents to interact in desirable ways. As shown in chapter 6 knowledge of other agents can be gained purely by interacting with them. Tasks initiation and completion can be made implicit in the fitness function (by rewarding agents that correctly identify these conditions). In the pursuit domain of chapter 4, we evolved predator agents that moved when the prey was not completely surrounded, and

maintained their position around the prey once it was surrounded. They therefore recognised the state of their coordinated process.

In chapter 4 we evolved agents that as well as having a primary goal, also had a secondary goal, which was to reduce the expenditure of a resource(energy). We therefore showed that we could evolve agents that solve a problem and are simultaneously economical in their use of resources. We could just as easily attach penalties in the fitness function to reduce consumption of any resource whose utilisation needs control, such as communication. We therefore believe that this technique can be used to balance local computation and communication.

We propose that GP can be used as a general methodology for developing DAI systems. By careful design of function sets, terminal sets and fitness functions, we can engineer and constrain practical DAI systems.

9.3 A Methodology for Automatically Programming Multiagent Systems Using GP

Our methodology for automatically programming a MAS requires the following computing environment:

- A Strongly Typed Genetic Programming System Supporting ADFs such as GPsys.
- An environment that is identical or closely simulates the environment in which the agents are to work, and which can safely be used to test the agents in a controlled manner.
- A scalable computer architecture that can support the above.

There are then 8 preparatory steps required to code a GP MAS application. These involve defining the following:

1. The Environment
2. The Agent Classes
3. The Architecture
4. The Function Sets
5. The Terminal Sets
6. The Fitness Function
7. The Run Parameters
8. The Termination Criteria

9.3.1 The Environment

We need to define the environment in which the agents will operate. The environment can be isolated in that there is no predefined agent society operating within this environment, or populated with other agents.

9.3.2 The Agent Classes

We need to choose whether our agents are to be:

- Cooperative or Competitive
- Homogeneous, Heterogeneous or Mixed Agents
- Reactive or Deliberative (with memory)
- Communicative or non-Communicative

The first choice is to decide whether the agents that we wish to generate are to cooperate or compete. Competitive agents will generally require separate populations to be co-evolved, even when they must sometimes cooperate with each other such as trading agents. Cooperative agents can be evolved as heterogenous teams or as homogeneous agents. To create purely reactive agents we can omit memory operators, otherwise we must provide a means of storing state information. Communicative agents will require operators that allow communication to take place, either explicitly like the agents of chapter 5 or implicitly like the agents in chapter 4 experiment 4a. It is helpful to know how many agents we require, although it may be possible to discover this number automatically through evolution [Ben96a; Ben96b].

9.3.3 The Architecture

For each agent class that needs to be evolved, we need to define a set of ADFs including the result producing branch. Alternatively we could parameterise the GP system to evolve this architecture using the architecture altering operations reported by Koza [KDBK99].

9.3.4 Function and Terminal Sets

For each class of agent that we need to evolve, we need to define the function and terminal sets. The latter should contain functions and terminals that can be used to compute decisions, communicate with other agents and sense/manipulate the environment.

9.3.5 The Fitness Function

The key to evolving a MAS is to correctly define the fitness function. The fitness function should measure how closely the agents achieve their intended purpose, awarding higher fitness to those

individuals that are more successful. We can also add further criteria to the fitness function such as a measure of how efficiently the agents achieve their purpose. We can penalise unwanted behaviour by defining laws which have cost repercussions if they are broken, much like penalties are used in human legal systems. Fitness is evaluated by testing the evolved code against test cases representative of the real problems that the agents must solve. This evaluation involves instantiating the desired number of agents of each class from the evolved code, and iteratively executing them for the given test case until the problem is solved or the maximum number of iterations have expired.

9.3.6 The Termination Criteria and Run Parameters

These are the same as described in chapter 2.

9.4 Critical Assessment

The following are key limitations of our work:

- We used only symmetric problem domains of limited complexity.
- We did not apply our technique on any real-world problems.
- We focussed our attention on evolving and co-evolving cooperative agents (both homogeneous and heterogeneous) and did not investigate competitive co-evolution of heterogeneous agents.
- We evolved only purely reactive agents.
- We proposed that software testing techniques may be useful for GP, but did not perform any experiments to support our proposal.

9.5 Future Work

The following lists further research directions stemming from our work:

- Using GP for Real World/Large Scale MAS Applications
- Using GP for Assymetrical MAS Problems
- Competitive Co-Evolution of Heterogeneous Agents
- Evolving Agents With Memory
- Socialisation in More Diverse Agent Societies

- ADFs and Communicating Agents (modeling communication as function calls)
- Design Patterns for MAS
- Agent Learning and the Baldwin Effect
- Evolving Distributed Algorithms
- Scalability of GP
- Software Testing and GP
- Automated Software Testing of MAS
- Investigating Evolution of Human Languages by Evolving Communicating Agents

In addition to the above we propose that Kolmogorov/Chaitin complexity theory may prove useful for GP as discussed below.

9.5.1 Algorithmic Complexity, Computer Programming and Evolution

Algorithmic complexity theory which has its roots in information theory has already proved useful in many diverse domains including probability theory, cryptography, communication and compression algorithms [LV97]. It may also help us to formulate a theoretical framework for Genetic Programming.

The process of programming a computer can be thought of as creating a sequence of bits that code an algorithm that can generate the right output sequences for a given set of input sequences. It is likely that there will be many such algorithms, and therefore many such codes.

The simplest being one that uses a table lookup to map the input to the output, and therefore would have at least the same number of bits as the input/out pairs. We note also that such a program does not try to make some generalisation between the inputs and the outputs and therefore will fail if new inputs not present in its tables are presented. This compares with the concept of overfitting in GP.

Any program code that is shorter than the number of bits required for the mapping table will be of higher complexity than the simplest program. The most complex program will be the shortest such program and will describe an algorithm that abstracts the relationship between the input output pairs and will therefore be more general. Furthermore, we would expect the binary sequence representing this program to be random, i.e. the program should be incompressible and irreducible. Note that if there is no pattern to the mapping between the inputs and the outputs, then the program will be equivalent to a table lookup.

Assuming we have a fitness function which rewards programs that generate the correct output for a given input and also rewards *parsimony*, we would expect the evolutionary process to lead to fit programs that are also small. Indeed this has been demonstrated in a few GP applications.

Such a fitness function could be used to try to find the minimal program ie. one that is incompressible. One way of describing evolution could therefore be as a means of generating a random sequence of bits (the program) which together with a an input sequence of bits encodes a computation required to generate a required sequence of bits (the output).

We can then think of the fitness function as a kind of test of randomness for a finite sequence of bits. Fitness proportionate selection then selects bit sequences in proportions related to their randomness. We can then describe crossover as a process where we take two sequences of bits, each of high complexity (fitness) and exchange a random subsequence of bits from each sequence to yield two new sequences. The point of this operation would be to generate more random sequences of bits (of higher fitness). Mutation too can be thought of as replacing a subsequence of bits from a sequence of bits by randomly generated bits.

From the above we might conclude that the generation of random bit strings is a side effect of the evolutionary process. An obstacle to this theory is the inclusion of introns in both real-life DNA and in programs evolved through GP. Introns are redundant segments of code that have no effect in the program other than increasing its size. However, since they have no effect on the execution and are hence not really part of the algorithm description, we can exclude them from our tests of compressibility.

We could carry out experiments to test this theory by measuring the compression ratios of each individual as well as their fitness, to see if there is a correlation between the two. General compression programs such a GNU gzip could be used to compress the text representation of each individual, and we could calculate the compression ratio as the size before compression divided by the size after compression. This is should be possible for large enough programs, provided we remove introns when evaluating compressibility.

If the above were true as the average fitness of the population increases we should expect the population to converge on bit sequences that are more difficult to compress. A simple test is to use a general compression program like GNU gzip and use it to compress the populations of each generation in a run and see what the relationship is to average fitness and compressibility. The compressibility would be the uncompressed size divided by the compressed size. We should find that the compressibility tends to one as the population converges on the solution.

9.6 Summary

We have achieved our objectives of showing that GP can be used to evolve MAS automatically. We have in particular shown that GP can be used to evolve agents that:

- exhibit coordinated, coherent behaviour (chapters 4, 5 and 6)
- can resolve conflicts (chapter 4)
- communicate explicitly, and in doing so decide what to communicate and how (chapter 5)
- can be integrated into an existing society of agents (chapter 6)

We have examined the scalability issues involved in the use of GP, both generally and in particular as a technique for automatically programming agents, and proposed solutions to these problems. Related to scalability we have examined the importance of of formal software testing techniques in the development GP technology, and looked at the issues involved in the testing of MAS. Lastly we have proposed a methodology for automatically programming multiagent systems using GP.

Appendix A

Best Evolved Pursuit Agents

A.1 Experiment 1a - Basic Homogeneous (Best of Run 9)

ADF[0] : (IfEast ADF0Arg1 ADF0Arg1 South (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 East (IfNorth ADF0Arg0 ADF0Arg0 North (IfEast ADF0Arg0 ADF0Arg1 East North))) East) North) (IfNorth ADF0Arg0 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 East East) (IfEast ADF0Arg0 ADF0Arg1 East (IfNorth ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg1 West (IfNorth ADF0Arg1 ADF0Arg1 North South)) West)))))
ADF[1] : (IfEast (CellYofX Me (IfNorth (CellYofX Me South) (CellYofX Agent4 (IfEast (CellYofX Agent4 North) (CellYofX Agent4 South) North (ADF0 (CellYofX Me West) (CellYofX Me (IfEast (CellYofX Me East) (CellYofX Me West) North East))))) (ADF0 (CellYofX Agent4 North) (CellYofX Agent4 (IfEast (CellYofX Me East) (CellYofX Agent4 Here) (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me Here) West (ADF0 (CellYofX Me West) (CellYofX Agent4 South))) South))) (IfEast (CellYofX Me (ADF0 (CellYofX Me North) (CellYofX Me West))) (CellYofX Agent4 (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 South) West (IfCellsEqual (CellYofX Agent4 South) (CellYofX Agent4 Here) East East))) (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me East) West South) (IfEast (CellYofX Agent4 (ADF0 (CellYofX Agent4 West) (CellYofX Agent4 South))) (CellYofX Me (IfNorth (CellYofX Me South) (CellYofX Me East) Here East)) East (IfEast (CellYofX Agent4 South) (CellYofX Me West) (ADF0 (CellYofX Me West) (CellYofX Me South)) (IfEast (CellYofX Me East) (CellYofX Me South) East North)))))) (CellYofX Me South) (ADF0 (CellYofX Me West) (CellYofX Me (IfNorth (CellYofX Me West) (CellYofX Agent4 (IfEast (CellYofX Agent4 South) (CellYofX Agent4 South) (ADF0 (CellYofX Me East) (CellYofX Me North)) (ADF0 (CellYofX Agent4 West) (CellYofX Me South)))) (ADF0 (CellYofX Me East) (CellYofX Agent4 (IfEast (CellYofX Agent4 East) (CellYofX Agent4 North) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me West) West North) Here))) North))) (ADF0 (CellYofX Agent4 West) (CellYofX Me (IfNorth (CellYofX Agent4 (ADF0 (CellYofX Agent4 South) (CellYofX Me North))) (CellYofX Me (IfEast (CellYofX Me West) (CellYofX Agent4 (ADF0 (CellYofX Me West) (CellYofX Agent4 North))) East (IfEast (CellYofX Agent4 South) (CellYofX Me West) North West))) (ADF0 (CellYofX Me Here) (CellYofX Agent4 North)) (ADF0 (CellYofX Agent4 North) (CellYofX Me South))))))
Fitness : Fitness(7906.45,46.083333333333336,9)
Complexity : 312

A.2 Experiment 1b - Basic Heterogeneous (Best of Run 6)

ADF[0] : (IfNorth ADF0Arg0 ADF0Arg0 East (IfNorth ADF0Arg0 ADF0Arg1 North (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 North (IfNorth ADF0Arg0 ADF0Arg0 North (IfNorth ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg0 South East) East))) West) West)))
ADF[1] : (ADF0 (CellYofX Agent4 (ADF0 (CellYofX Agent4 (ADF0 (CellYofX Me Here) (CellYofX Me West))) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me Here) West South)))) (CellYofX Me West))
ADF[2] : (IfNorth ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 South West) (IfNorth ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 North West) (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg0 South South) (IfNorth ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg1 North (IfNorth ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg1 North South) West)) North)) (IfEast ADF0Arg1 ADF0Arg1 North West)) (IfNorth ADF0Arg1 ADF0Arg0 South East))))
ADF[3] : (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Me (IfNorth (CellYofX Me Here) (CellYofX Agent4 (IfNorth (CellYofX Me Here) (CellYofX Me Here) West (IfNorth (CellYofX Me East) (CellYofX Me East) Here Here))) West (IfNorth (CellYofX Agent4 East) (CellYofX Me East) Here Here))) (CellYofX Me (IfNorth (CellYofX Me West) (CellYofX Me West) (IfEast (CellYofX Me South) (CellYofX Agent4 West) West East) South)) (IfNorth (CellYofX Me East) (CellYofX Me South) West West) (IfNorth (CellYofX Me West) (CellYofX Me West) East South))) (CellYofX Me (IfEast (CellYofX Me North) (CellYofX Agent4 (IfCellsEqual (CellYofX Me Here) (CellYofX Me Here) West Here)) North Here)))
ADF[4] : (IfEast ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 North East) East) North) (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 East North) East)) (IfNorth ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 West (IfEast ADF0Arg0 ADF0Arg0 West West)) (IfEast ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg1 Here (IfNorth ADF0Arg0 ADF0Arg1 North Here)) South))) North) (IfEast ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 North North) (IfCellsEqual ADF0Arg1 ADF0Arg1 North East))) (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 North South) North) North) (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg1 East North) East)) (IfNorth ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 West (IfEast ADF0Arg0 ADF0Arg0 West North)) (IfEast ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg1 South (IfNorth ADF0Arg0 ADF0Arg1 North Here)) South))) North) (IfCellsEqual ADF0Arg1 ADF0Arg0 North North)))
ADF[5] : (IfNorth (CellYofX Me (ADF0 (CellYofX Agent4 East) (CellYofX Me (IfEast (CellYofX Me South) (CellYofX Me Here) (IfNorth (CellYofX Me East) (CellYofX Me West) North Here) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 North) South East))))) (CellYofX Me (IfEast (CellYofX Me South) (CellYofX Agent4 (ADF0 (CellYofX Agent4 South) (CellYofX Me (IfNorth (CellYofX Me East) (CellYofX Me West) East Here)))) West North)) (IfNorth (CellYofX Agent4 South) (CellYofX Me West) North West) (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 East) (ADF0 (CellYofX Agent4 (ADF0 (CellYofX Me East) (CellYofX Me (IfEast (CellYofX Me South) (CellYofX Me Here) (IfNorth (CellYofX Me East) (CellYofX Agent4 West) East Here) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 North) West East))))) (CellYofX Me North)) South))
ADF[6] : (IfNorth ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg0

ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 East Here) South) West) (IfNorth ADF0Arg0 ADF0Arg0 Here South)) South) (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 East West) (IfEast ADF0Arg1 ADF0Arg0 West Here))) (IfEast ADF0Arg1 ADF0Arg1 West (IfNorth ADF0Arg1 ADF0Arg0 Here South))) West) (IfNorth ADF0Arg0 ADF0Arg0 West East))
ADF[7] : (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 (IfNorth (CellYofX Me (IfEast (CellYofX Me West) (CellYofX Agent4 South) West North)) (CellYofX Agent4 (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 Here) West (IfEast (CellYofX Me North) (CellYofX Me South) North Here))) South North)) (CellYofX Agent4 Here) East (IfCellsEqual (CellYofX Me (ADF0 (CellYofX Agent4 West) (CellYofX Me Here))) (CellYofX Me West) Here South))) South (IfEast (CellYofX Agent4 (IfEast (CellYofX Me North) (CellYofX Me Here) Here Here)) (CellYofX Agent4 (IfEast (CellYofX Agent4 (IfEast (CellYofX Agent4 West) (CellYofX Me North) East North)) (CellYofX Me Here) (IfCellsEqual (CellYofX Me (IfNorth (CellYofX Agent4 West) (CellYofX Me South) Here South)) (CellYofX Agent4 (ADF0 (CellYofX Agent4 Here) (CellYofX Me North))) West (IfNorth (CellYofX Agent4 North) (CellYofX Me West) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Agent4 West) Here Here) South)) (ADF0 (CellYofX Me (IfEast (CellYofX Agent4 North) (CellYofX Me South) North Here)) (CellYofX Agent4 (ADF0 (CellYofX Me Here) (CellYofX Me North)))))) (IfNorth (CellYofX Me (IfEast (CellYofX Me West) (CellYofX Agent4 South) South North)) (CellYofX Agent4 (IfEast (CellYofX Agent4 (IfEast (CellYofX Agent4 West) (CellYofX Agent4 South) South South)) (CellYofX Agent4 Here) West (IfEast (CellYofX Me North) (CellYofX Me Here) North Here))) South North) West))
Fitness : Fitness(15750.566666666668,28.7,58)
Complexity : 724

A.3 Experiment 2 - Identity (Best of Run 1)

ADF[0] : (IfEast ADF0Arg0 ADF0Arg1 West (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg0 Here (IfEast ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 East Here) (IfNorth ADF0Arg1 ADF0Arg0 North (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg0 East West) North) Here) East)))) (IfNorth ADF0Arg1 ADF0Arg0 North (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 East South) West) East))))
ADF[1] : (ADF0 (CellYofX Me West) (CellYofX Agent4 (IfEast (CellYofX Agent4 (IfEast (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me Here) (IfAgentIdEquals Me id0 West South) South)) (CellYofX Me (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 (IfAgentIdEquals Me id2 North East)) (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 Here) Here (IfAgentIdEquals Me id1 North West)) (ADF0 (CellYofX Agent4 South) (CellYofX Agent4 South)))) (IfEast (CellYofX Agent4 (IfAgentIdEquals Me id0 (IfAgentIdEquals Agent4 id3 South Here) Here)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 North) (IfAgentIdEquals Agent4 id0 West East) South)) (IfCellsEqual (CellYofX Me Here) (CellYofX Me (IfAgentIdEquals Me id2 Here Here)) South North) (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 Here) Here (IfEast (CellYofX Me North) (CellYofX Me North) Here North))) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 Here) (IfAgentIdEquals Agent4 id0 (IfAgentIdEquals Agent4 id0 North Here) (IfAgentIdEquals Me id0 South Here)) South))) (CellYofX Me (IfNorth (CellYofX Me (IfAgentIdEquals Me id2 North East)) (CellYofX Agent4 (IfAgentIdEquals Me id0 North West)) West (ADF0 (CellYofX Me Here) (CellYofX Me West)))) (IfEast (CellYofX Agent4 (IfAgentIdEquals Me id1 (IfAgen-

tIdEquals Agent4 id0 South Here) East)) (CellYofX Agent4 Here) (IfEast (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 Here) (IfAgentIdEquals Agent4 id0 North South) South)) (CellYofX Me (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 (IfAgentIdEquals Me id2 North East)) (IfCellsEqual (CellYofX Agent4 North) (CellYofX Agent4 Here) West (IfAgentIdEquals Me id0 North West)) (ADF0 (CellYofX Me Here) (CellYofX Agent4 South)))) (IfEast (CellYofX Agent4 (IfAgentIdEquals Me id0 (IfAgentIdEquals Me id3 South Here) Here)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 North) (IfAgentIdEquals Me id0 West Here) South)) (IfCellsEqual (CellYofX Me Here) (CellYofX Me (IfAgentIdEquals Me id2 Here South)) South North) (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 Here) Here (IfEast (CellYofX Me North) (CellYofX Me North) South North))) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 Here) (IfAgentIdEquals Agent4 id0 North West) South)) (IfNorth (CellYofX Agent4 (IfAgentIdEquals Agent4 id0 Here Here)) (CellYofX Me Here) West (IfEast (CellYofX Agent4 Here) (CellYofX Me Here) West North))) Here)))
Fitness : Fitness(12723.683333333332,38.13333333333333,38)
Complexity : 374

A.4 Experiment 3a - Basic31 (Best of Run 3)

ADF[0] : (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg1 East (IfNorth ADF0Arg1 ADF0Arg0 North East)) East) South) (IfNorth ADF0Arg0 ADF0Arg0 South (IfNorth ADF0Arg0 ADF0Arg1 North (IfEast ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 South East) East) South)))) East)
ADF[1] : (IfEast (CellYofX Agent4 (IfEast (CellYofX Me (IfEast (CellYofX Me (ADF0 (CellYofX Me East) (CellYofX Agent4 South))) (CellYofX Agent4 East) (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me South) East East) Here)) (CellYofX Me South) (IfEast (CellYofX Agent4 (IfNorth (CellYofX Me North) (CellYofX Me West) West North)) (CellYofX Me (ADF0 (CellYofX Agent4 South) (CellYofX Me North))) (IfNorth (CellYofX Agent4 North) (CellYofX Me North) West North) Here) (ADF0 (CellYofX Me (IfCellsEqual (CellYofX Me North) (CellYofX Me Here) East East)) (CellYofX Agent4 West)))) (CellYofX Me (IfEast (CellYofX Me East) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 West) (IfCellsEqual (CellYofX Agent4 (ADF0 (CellYofX Me North) (CellYofX Agent4 South))) (CellYofX Me North) (ADF0 (CellYofX Agent4 West) (CellYofX Me North)) West) North)) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me East) Here (IfEast (CellYofX Me North) (CellYofX Me East) (ADF0 (CellYofX Agent4 North) (CellYofX Agent4 East)) West)) (ADF0 (CellYofX Me East) (CellYofX Me South)))) (ADF0 (CellYofX Agent4 (IfEast (CellYofX Me (IfNorth (CellYofX Agent4 North) (CellYofX Me East) West North)) (CellYofX Me East) North North)) (CellYofX Me (ADF0 (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 North) (CellYofX Agent4 North) North South)) (CellYofX Agent4 Here)))) West)
ADF[2] : (IfNorth ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg0 South North) (IfNorth ADF0Arg0 ADF0Arg1 Here (IfNorth ADF0Arg0 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg0 North (IfNorth ADF0Arg0 ADF0Arg0 South North)) North))) (IfNorth ADF0Arg0 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 North North) East) (IfNorth ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg0 North South) (IfCellsEqual ADF0Arg1 ADF0Arg0 Here South))) (IfCellsEqual ADF0Arg1 ADF0Arg0 East South)))
ADF[3] : (IfNorth (CellYofX Me East) (CellYofX Agent4 (IfEast (CellYofX Agent4 North) (CellYofX Me West) Here Here)) (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 North) North South) (IfNorth (CellYofX Agent4 (ADF0 (CellYofX Agent4 (IfEast (CellYofX Me West)

(CellYofX Agent4 Here) South North)) (CellYofX Me East))) (CellYofX Me (IfEast (CellYofX Agent4 Here) (CellYofX Me North) (IfNorth (CellYofX Me Here) (CellYofX Me West) (IfNorth (CellYofX Me West) (CellYofX Agent4 (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 North) North East)) Here West) North) East)) (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me West) (IfNorth (CellYofX Agent4 (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 East) (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 North) (IfNorth (CellYofX Me East) (CellYofX Agent4 West) Here Here) North) Here)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me North) Here Here)) (IfNorth (CellYofX Me Here) (CellYofX Me West) (IfNorth (CellYofX Me West) (CellYofX Agent4 North) Here North) West) Here) North) West))
Fitness : Fitness(9802.316666666668,42.55,18)
Complexity : 432

A.5 Experiment 3b - Basic22 (Best of Run 9)

ADF[0] : (IfNorth ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 West (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg0 South (IfNorth ADF0Arg1 ADF0Arg0 West Here)) (IfCellsEqual ADF0Arg0 ADF0Arg1 Here (IfNorth ADF0Arg0 ADF0Arg0 West Here))) (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 Here West) West)) South)) (IfEast ADF0Arg1 ADF0Arg1 East North))
ADF[1] : (IfEast (CellYofX Me East) (CellYofX Agent4 (IfEast (CellYofX Me (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 South) South East)) (CellYofX Me (IfEast (CellYofX Agent4 North) (CellYofX Agent4 North) East East)) West (IfCellsEqual (CellYofX Me (IfCellsEqual (CellYofX Me West) (CellYofX Me East) East North)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me Here) East Here)) (ADF0 (CellYofX Me East) (CellYofX Me Here)) (ADF0 (CellYofX Agent4 East) (CellYofX Me North))))) (IfEast (CellYofX Me West) (CellYofX Agent4 West) West (ADF0 (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me Here) East East)) (CellYofX Agent4 East))) East)
ADF[2] : (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 North (IfCellsEqual ADF0Arg0 ADF0Arg1 North (IfEast ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 South (IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 Here North) West)) North) North))) (IfNorth ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 West (IfCellsEqual ADF0Arg1 ADF0Arg0 East (IfEast ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg0 South North) West))) North)) (IfNorth ADF0Arg0 ADF0Arg1 East South))
ADF[3] : (IfEast (CellYofX Agent4 (IfNorth (CellYofX Me (IfNorth (CellYofX Me North) (CellYofX Agent4 North) East South)) (CellYofX Me West) East (ADF0 (CellYofX Me South) (CellYofX Agent4 West)))) (CellYofX Agent4 South) (IfNorth (CellYofX Me North) (CellYofX Agent4 (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 Here) West West)) West (IfEast (CellYofX Agent4 (IfCellsEqual (CellYofX Me Here) (CellYofX Me East) Here (ADF0 (CellYofX Me South) (CellYofX Agent4 West)))) (CellYofX Me West) (ADF0 (CellYofX Me West) (CellYofX Me South)) West)) (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Me (IfNorth (CellYofX Agent4 West) (CellYofX Me South) West South)) (CellYofX Me East) East (ADF0 (CellYofX Me South) (CellYofX Agent4 South)))) (CellYofX Me Here)))
Fitness : Fitness(10760.95,40.18333333333333,24)
Complexity : 304

A.6 Experiment 3c - Basic211 (Best of Run 8)

ADF[0] : (IfNorth ADF0Arg1 ADF0Arg0 North (IfEast ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg1 North (IfEast ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 South (IfCellsEqual ADF0Arg1 ADF0Arg1 North East)) (IfCellsEqual ADF0Arg0 ADF0Arg0 East North)) East)) (IfCellsEqual ADF0Arg1 ADF0Arg1 East (IfCellsEqual ADF0Arg0 ADF0Arg1 East North))) (IfCellsEqual ADF0Arg0 ADF0Arg0 East North)) East))
ADF[1] : (IfEast (CellYofX Me South) (CellYofX Agent4 (IfNorth (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me North) (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me Here) South South) (IfNorth (CellYofX Me Here) (CellYofX Agent4 South) East North))) (CellYofX Me South) (IfEast (CellYofX Me (IfEast (CellYofX Me East) (CellYofX Agent4 East) West East)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 East) (CellYofX Me North) North South)) West North) North)) (IfEast (CellYofX Agent4 Here) (CellYofX Me (ADF0 (CellYofX Agent4 West) (CellYofX Agent4 (IfEast (CellYofX Agent4 West) (CellYofX Me West) East South)))) (ADF0 (CellYofX Me North) (CellYofX Me Here)) (IfEast (CellYofX Agent4 (ADF0 (CellYofX Me South) (CellYofX Me North))) (CellYofX Me East) (IfEast (CellYofX Agent4 (IfNorth (CellYofX Agent4 South) (CellYofX Agent4 South) East East)) (CellYofX Agent4 (IfNorth (CellYofX Me East) (CellYofX Agent4 East) West South)) (IfNorth (CellYofX Agent4 East) (CellYofX Me North) West West) Here) West)) (IfCellsEqual (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 (IfNorth (CellYofX Me North) (CellYofX Agent4 East) South (IfCellsEqual (CellYofX Agent4 North) (CellYofX Agent4 North) North East))) (CellYofX Agent4 Here) (IfEast (CellYofX Me South) (CellYofX Me East) West (IfCellsEqual (CellYofX Me North) (CellYofX Me Here) East (IfCellsEqual (CellYofX Me North) (CellYofX Me East) Here South))) (ADF0 (CellYofX Me (IfEast (CellYofX Me Here) (CellYofX Agent4 West) North South)) (CellYofX Me (IfNorth (CellYofX Me Here) (CellYofX Me West) North Here))))) (CellYofX Agent4 (IfEast (CellYofX Me (IfEast (CellYofX Agent4 West) (CellYofX Me East) Here South)) (CellYofX Me (IfNorth (CellYofX Me Here) (CellYofX Agent4 West) South South)) (ADF0 (CellYofX Agent4 East) (CellYofX Agent4 Here)) North)) (ADF0 (CellYofX Agent4 East) (CellYofX Agent4 (ADF0 (CellYofX Me (IfEast (CellYofX Me East) (CellYofX Agent4 East) West East)) (CellYofX Me (IfCellsEqual (CellYofX Agent4 East) (CellYofX Agent4 East) South South))))) South))
ADF[2] : (IfEast ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg0 West (IfCellsEqual ADF0Arg0 ADF0Arg1 West North)) (IfNorth ADF0Arg0 ADF0Arg1 West (IfNorth ADF0Arg0 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg0 West West) South) (IfCellsEqual ADF0Arg1 ADF0Arg1 West North)))) (IfCellsEqual ADF0Arg0 ADF0Arg1 East (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfCellsEqual ADF0Arg0 ADF0Arg1 North (IfNorth ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg0 West West) South) (IfCellsEqual ADF0Arg1 ADF0Arg1 West West))) East)))
ADF[3] : (ADF0 (CellYofX Me (IfNorth (CellYofX Agent4 (IfCellsEqual (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Me East) (IfNorth (CellYofX Me West) (CellYofX Agent4 Here) West South) Here)) (CellYofX Me Here) (IfNorth (CellYofX Me Here) (CellYofX Me South) Here West) West)) (CellYofX Agent4 Here) West (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Agent4 West) (CellYofX Me South) Here North)) (CellYofX Agent4 (IfNorth (CellYofX Agent4 West) (CellYofX Me South) West North))))) (CellYofX Me (IfNorth (CellYofX Me (IfCellsEqual (CellYofX Me West) (CellYofX Agent4 Here) (IfNorth (CellYofX Agent4 South) (CellYofX Agent4 South) West Here) South)) (CellYofX Agent4 Here) North (ADF0 (CellYofX Agent4 Here) (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Me South) (IfNorth (CellYofX Me West) (CellYofX Agent4 South) Here East) Here))))))
ADF[4] : (IfNorth ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfEast

ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg1 West East) (IfEast ADF0Arg0 ADF0Arg1 West North)) (IfNorth ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 East (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 North (IfNorth ADF0Arg1 ADF0Arg1 South North)) (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg0 Here West) South) (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 South Here) (IfCellsEqual ADF0Arg1 ADF0Arg1 South East))) West))) West)) South)
ADF[5] : (ADF0 (CellYofX Me (IfCellsEqual (CellYofX Agent4 South) (CellYofX Agent4 North) (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 East) North (ADF0 (CellYofX Me North) (CellYofX Agent4 West))) (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 East) (IfEast (CellYofX Agent4 East) (CellYofX Agent4 South) Here (ADF0 (CellYofX Me North) (CellYofX Me East))) (IfNorth (CellYofX Me East) (CellYofX Agent4 South) North (ADF0 (CellYofX Me North) (CellYofX Agent4 East)))))) (CellYofX Agent4 Here))
Fitness : Fitness(12757.816666666668,36.916666666666664,38)
Complexity : 642

A.7 Experiment 4a - CRO Homogeneous (Best of Run 9)

(IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg0 Here (IfCellOccupied ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg0 West West) East) (IfCellsEqual ADF0Arg1 ADF0Arg1 East (IfCellOccupied ADF0Arg1 North East))) East)) (IfCellOccupied ADF0Arg0 West (IfCellsEqual ADF0Arg1 ADF0Arg1 North (IfEast ADF0Arg0 ADF0Arg1 (IfCellOccupied ADF0Arg1 West (IfCellOccupied ADF0Arg1 North North)) East)))) (IfEast ADF0Arg1 ADF0Arg0 (IfCellOccupied ADF0Arg1 (IfCellOccupied ADF0Arg1 North North) (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg0 East (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg1 North East) (IfEast ADF0Arg0 ADF0Arg1 North East))) (IfCellOccupied ADF0Arg1 West (IfCellsEqual ADF0Arg0 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg0 South East) North)))) (IfEast ADF0Arg0 ADF0Arg1 East East))) North)
ADF[1] : (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Me (IfCellOccupied (CellYofX Me South) West (ADF0 (CellYofX Me West) (CellYofX Agent4 North)))) (IfCellOccupied (CellYofX Me South) (IfEast (CellYofX Me Here) (CellYofX Agent4 Here) (ADF0 (CellYofX Me West) (CellYofX Agent4 North)) South) (IfCellOccupied (CellYofX Me Here) (IfNorth (CellYofX Me South) (CellYofX Me East) West Here) Here)) (IfCellOccupied (CellYofX Me East) (IfNorth (CellYofX Agent4 South) (CellYofX Agent4 East) North (ADF0 (CellYofX Agent4 Here) (CellYofX Me South))) (ADF0 (CellYofX Me Here) (CellYofX Agent4 West))))) (CellYofX Me (IfNorth (CellYofX Agent4 West) (CellYofX Agent4 (IfCellOccupied (CellYofX Me (IfCellsEqual (CellYofX Agent4 East) (CellYofX Me North) North (ADF0 (CellYofX Agent4 West) (CellYofX Me North)))) South West)) West (IfCellsEqual (CellYofX Agent4 North) (CellYofX Agent4 West) (IfCellsEqual (CellYofX Me West) (CellYofX Me West) East West) (IfCellsEqual (CellYofX Agent4 East) (CellYofX Agent4 South) West (IfEast (CellYofX Me South) (CellYofX Agent4 South) Here South))))) (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Agent4 West) (CellYofX Me South) (IfCellOccupied (CellYofX Agent4 Here) Here West) Here)) (CellYofX Me (IfNorth (CellYofX Me North) (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) West South)) (IfCellsEqual (CellYofX Me South) (CellYofX Me West) (ADF0 (CellYofX Agent4 (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 West) North East)) (CellYofX Agent4 East)) (IfCellsEqual (CellYofX Me East) (CellYofX Agent4 North) South (IfCellOccupied (CellYofX Me West) (IfNorth (CellYofX Me North) (CellYofX Agent4 South) West South) Here))) (IfEast (CellYofX Agent4 Here) (CellYofX Me (IfCellOccupied

(CellYofX Me South) (IfNorth (CellYofX Me North) (CellYofX Agent4 South) South Here) West)) East (IfCellOccupied (CellYofX Agent4 South) West West))))) (IfEast (CellYofX Me (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 (ADF0 (CellYofX Agent4 East) (CellYofX Agent4 East))) (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me (ADF0 (CellYofX Me East) (CellYofX Me South))) (IfNorth (CellYofX Agent4 South) (CellYofX Me Here) (ADF0 (CellYofX Agent4 West) (CellYofX Agent4 South)) (IfEast (CellYofX Me Here) (CellYofX Me North) Here Here)) (IfCellOccupied (CellYofX Agent4 East) (IfEast (CellYofX Me South) (CellYofX Agent4 South) West West) Here)) (IfCellOccupied (CellYofX Agent4 (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 Here) Here North)) North (ADF0 (CellYofX Me East) (CellYofX Agent4 West))))) (CellYofX Agent4 (IfCellOccupied (CellYofX Me South) (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 Here) (IfCellOccupied (CellYofX Me North) West North) Here) North)) West (IfEast (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Me West) (ADF0 (CellYofX Agent4 North) (CellYofX Me South)) (IfCellsEqual (CellYofX Me North) (CellYofX Agent4 East) South East))) (CellYofX Me (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 North) North South)) South West)))
Fitness : Fitness(14969.416666666666,32.4,53)
Complexity : 489

A.8 Experiment 4a - CRO Homogenous (Best of Run 9 Generation 249)

ADF[0] : (IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 West (IfCellOccupied ADF0Arg0 West (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 North West) (IfEast ADF0Arg0 ADF0Arg0 (IfCellOccupied ADF0Arg1 West (IfCellOccupied ADF0Arg1 North North)) East)))) (IfEast ADF0Arg1 ADF0Arg1 (IfCellOccupied ADF0Arg1 North North) (IfEast ADF0Arg0 ADF0Arg1 East North))) North)
ADF[1] : (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Me (IfCellOccupied (CellYofX Me South) West North)) (IfCellOccupied (CellYofX Me South) (IfEast (CellYofX Me Here) (CellYofX Agent4 Here) (ADF0 (CellYofX Me West) (CellYofX Agent4 North)) Here) (IfCellOccupied (CellYofX Me Here) (IfNorth (CellYofX Me North) (CellYofX Agent4 East) West North) Here)) (IfCellOccupied (CellYofX Me East) (IfNorth (CellYofX Agent4 East) (CellYofX Me East) North (ADF0 (CellYofX Agent4 Here) (CellYofX Me South))) (ADF0 (CellYofX Me East) (CellYofX Me West))))) (CellYofX Me (IfNorth (CellYofX Me West) (CellYofX Me (IfCellOccupied (CellYofX Me (IfCellsEqual (CellYofX Agent4 East) (CellYofX Me South) North (ADF0 (CellYofX Agent4 West) (CellYofX Me Here)))) South East)) West (IfCellsEqual (CellYofX Agent4 (IfEast (CellYofX Me Here) (CellYofX Me South) South North)) (CellYofX Agent4 West) (IfCellsEqual (CellYofX Me West) (CellYofX Me West) East North) (IfCellsEqual (CellYofX Agent4 East) (CellYofX Agent4 West) West (IfEast (CellYofX Me Here) (CellYofX Agent4 Here) Here South))))) (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) (IfCellOccupied (CellYofX Agent4 East) West Here) West)) (CellYofX Me South) (IfCellOccupied (CellYofX Agent4 Here) Here Here) South)) (CellYofX Me (IfNorth (CellYofX Me North) (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) Here South)) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me West) (ADF0 (CellYofX Agent4 (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 Here) North East)) (CellYofX Agent4 South)) (IfCellsEqual (CellYofX Me East) (CellYofX Agent4 East) Here (IfCellOccupied (CellYofX Me West) (IfNorth (CellYofX Me North) (CellYofX Agent4 South) West South) Here))) (IfEast (CellYofX Me East) (CellYofX Me Here) East (IfCellOccupied (CellYofX Agent4 South) West West))))) (IfEast (CellYofX Me South) (CellYofX Agent4 (IfCellOccupied (CellYofX Me Here) (IfCellsEqual (CellYofX Me South) (CellYofX Agent4

Here) (IfCellOccupied (CellYofX Me North) West North) Here) North)) West (IfEast (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 North) (ADF0 (CellYofX Agent4 North) (CellYofX Agent4 West)) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 Here) Here East))) (CellYofX Me (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 North) North North)) South West)))
Fitness : Fitness(14320.9,32.38333333333333,50)
Complexity : 350

A.9 Experiment 4a - CRO Homogenous (Best of Generations Run 9)

ADF[0] : (IfEast ADF0Arg0 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 West (IfCellOccupied ADF0Arg0 North (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg1 North West) (IfEast ADF0Arg0 ADF0Arg0 (IfCellOccupied ADF0Arg0 West (IfCellOccupied ADF0Arg1 North North)) North)))) (IfEast ADF0Arg1 ADF0Arg1 West (IfEast ADF0Arg0 ADF0Arg1 East West))) North)
ADF[1] : (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) (IfCellOccupied (CellYofX Agent4 East) West Here) West)) (IfCellOccupied (CellYofX Me South) (IfEast (CellYofX Me Here) (CellYofX Agent4 Here) (ADF0 (CellYofX Agent4 West) (CellYofX Me North)) Here) (IfCellOccupied (CellYofX Me Here) (IfNorth (CellYofX Me North) (CellYofX Agent4 West) West North) Here)) (IfCellOccupied (CellYofX Me East) (IfNorth (CellYofX Agent4 East) (CellYofX Me East) North (ADF0 (CellYofX Agent4 Here) (CellYofX Me South))) (ADF0 (CellYofX Me Here) (CellYofX Agent4 West))))) (CellYofX Me (IfNorth (CellYofX Me West) (CellYofX Me (IfCellOccupied (CellYofX Me (IfCellsEqual (CellYofX Agent4 East) (CellYofX Me South) North (ADF0 (CellYofX Agent4 West) (CellYofX Agent4 Here)))) South East)) West (IfCellsEqual (CellYofX Agent4 (IfEast (CellYofX Me Here) (CellYofX Me South) South North)) (CellYofX Agent4 West) (IfCellsEqual (CellYofX Me West) (CellYofX Me West) East North) (IfCellsEqual (CellYofX Me East) (CellYofX Agent4 West) West (IfEast (CellYofX Me Here) (CellYofX Agent4 Here) Here South))))) (ADF0 (CellYofX Agent4 (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) (IfCellOccupied (CellYofX Agent4 East) West Here) West)) (CellYofX Me South) (IfCellOccupied (CellYofX Agent4 Here) Here Here) South)) (CellYofX Me (IfNorth (CellYofX Me North) (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) Here South)) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me West) (ADF0 (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 Here) North East)) (CellYofX Agent4 South)) (IfCellsEqual (CellYofX Me East) (CellYofX Agent4 East) Here (IfCellOccupied (CellYofX Me West) (IfNorth (CellYofX Me North) (CellYofX Agent4 South) West South) Here))) (IfEast (CellYofX Me East) (CellYofX Me Here) East (IfCellOccupied (CellYofX Agent4 South) West West))))) (IfEast (CellYofX Me South) (CellYofX Agent4 (IfCellOccupied (CellYofX Me Here) (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 Here) (IfCellOccupied (CellYofX Me North) West North) Here) North)) West (IfEast (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 North) (ADF0 (CellYofX Agent4 North) (CellYofX Agent4 West)) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 Here) Here East))) (CellYofX Me (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 North) North North)) South West)))
Fitness : Fitness(13122.0,35.483333333333334,43)
Complexity : 352

A.10 Experiment 4b - CRO Heterogenous (Best of Run 2)

ADF[0] : (IfNorth ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellOccupied ADF0Arg0 North North) (IfNorth ADF0Arg0 ADF0Arg0 East (IfCellsEqual ADF0Arg1 ADF0Arg0 (IfEast ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 North West) North) West))) North) (IfCellOccupied ADF0Arg0 (IfEast ADF0Arg1 ADF0Arg1 North South) East))
ADF[1] : (IfCellsEqual (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 (IfCellsEqual (CellYofX Me East) (CellYofX Me Here) North (IfCellsEqual (CellYofX Agent4 West) (CellYofX Agent4 East) South Here))) (CellYofX Me (IfCellOccupied (CellYofX Agent4 North) East Here)) South (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 (IfEast (CellYofX Agent4 North) (CellYofX Me South) East West)) (IfEast (CellYofX Me East) (CellYofX Agent4 South) (IfCellOccupied (CellYofX Agent4 West) East West) South) (ADF0 (CellYofX Agent4 East) (CellYofX Me South))))) (CellYofX Agent4 (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 South) (IfEast (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 North) East North)) (CellYofX Me (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 Here) East South)) (IfCellsEqual (CellYofX Me Here) (CellYofX Me South) Here Here) West) South)) West (ADF0 (CellYofX Me (IfNorth (CellYofX Agent4 West) (CellYofX Me (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 South) South West)) South (IfCellsEqual (CellYofX Agent4 East) (CellYofX Agent4 Here) (ADF0 (CellYofX Me (IfCellsEqual (CellYofX Agent4 West) (CellYofX Agent4 East) South West)) (CellYofX Me East)) Here))) (CellYofX Agent4 South)))
ADF[2] : (IfEast ADF0Arg0 ADF0Arg0 Here (IfEast ADF0Arg0 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg0 East (IfEast ADF0Arg0 ADF0Arg1 North (IfNorth ADF0Arg0 ADF0Arg0 North East))) (IfNorth ADF0Arg1 ADF0Arg0 East South)))
ADF[3] : (IfEast (CellYofX Me East) (CellYofX Agent4 East) West (IfEast (CellYofX Agent4 (IfNorth (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me Here) South East)) (CellYofX Me (IfCellOccupied (CellYofX Me North) East North)) (IfNorth (CellYofX Me Here) (CellYofX Agent4 Here) (IfEast (CellYofX Agent4 East) (CellYofX Me South) East South) East) (IfEast (CellYofX Agent4 South) (CellYofX Agent4 South) (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 North) Here Here) (IfCellsEqual (CellYofX Me Here) (CellYofX Agent4 East) South Here)))) (CellYofX Agent4 East) (IfCellOccupied (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) (IfCellsEqual (CellYofX Me South) (CellYofX Me North) East West) (IfCellOccupied (CellYofX Agent4 East) North East))) (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 East) (IfCellOccupied (CellYofX Agent4 North) South (IfEast (CellYofX Me North) (CellYofX Agent4 North) North East)) Here) (IfNorth (CellYofX Me (IfEast (CellYofX Me West) (CellYofX Agent4 Here) East Here)) (CellYofX Me Here) (IfNorth (CellYofX Me South) (CellYofX Agent4 South) (IfNorth (CellYofX Agent4 Here) (CellYofX Agent4 East) South North) West) (IfCellOccupied (CellYofX Agent4 East) East North))) (ADF0 (CellYofX Me East) (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) Here (ADF0 (CellYofX Me East) (CellYofX Agent4 North)))))))
ADF[4] : South
ADF[5] : (IfCellsEqual (CellYofX Agent4 (IfEast (CellYofX Agent4 (IfEast (CellYofX Me Here) (CellYofX Me Here) East Here)) (CellYofX Agent4 (IfCellOccupied (CellYofX Me East) East (IfCellsEqual (CellYofX Me South) (CellYofX Me West) South East))) Here East)) (CellYofX Me (IfCellOccupied (CellYofX Agent4 East) West (IfCellOccupied (CellYofX Agent4 (IfEast (CellYofX Me East) (CellYofX Me East) East Here)) West (ADF0 (CellYofX Agent4 East) (CellYofX Me West))))) West (IfNorth (CellYofX Me East) (CellYofX Agent4 East) (IfEast (CellYofX Me (IfNorth (CellYofX Me West) (CellYofX Me South) (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 East) North Here) (IfCellOccupied (CellYofX Me West) East East))) (CellYofX Agent4 (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 West)

South East)) (IfCellOccupied (CellYofX Me West) East (IfCellOccupied (CellYofX Agent4 West) West (IfNorth (CellYofX Agent4 North) (CellYofX Agent4 East) South East))) (IfNorth (CellYofX Me South) (CellYofX Agent4 West) (IfNorth (CellYofX Me East) (CellYofX Me South) South (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 North) East West)) (IfEast (CellYofX Agent4 Here) (CellYofX Me Here) East South))) North))
ADF[6] : (IfNorth ADF0Arg1 ADF0Arg1 South (IfNorth ADF0Arg1 ADF0Arg0 South North))
ADF[7] : (IfCellsEqual (CellYofX Me (ADF0 (CellYofX Agent4 North) (CellYofX Me North))) (CellYofX Me (IfCellsEqual (CellYofX Agent4 North) (CellYofX Agent4 (ADF0 (CellYofX Me West) (CellYofX Agent4 (IfCellsEqual (CellYofX Me North) (CellYofX Agent4 Here) (ADF0 (CellYofX Me North) (CellYofX Agent4 Here)) (IfNorth (CellYofX Agent4 East) (CellYofX Agent4 Here) East Here))))) North East)) West (IfEast (CellYofX Me (IfCellOccupied (CellYofX Agent4 North) (IfNorth (CellYofX Me North) (CellYofX Me East) (ADF0 (CellYofX Agent4 North) (CellYofX Me East)) East) North)) (CellYofX Agent4 (IfCellOccupied (CellYofX Me North) North (ADF0 (CellYofX Agent4 Here) (CellYofX Agent4 North)))) (IfCellOccupied (CellYofX Me (IfCellOccupied (CellYofX Agent4 West) South (IfCellOccupied (CellYofX Me North) South South))) (IfCellsEqual (CellYofX Agent4 North) (CellYofX Me North) (IfNorth (CellYofX Agent4 (IfNorth (CellYofX Agent4 East) (CellYofX Me Here) North Here)) (CellYofX Me North) (IfNorth (CellYofX Me North) (CellYofX Agent4 East) South (IfCellOccupied (CellYofX Me North) South East)) West) (ADF0 (CellYofX Me (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Me North) North South)) (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 East) West West)))) (ADF0 (CellYofX Agent4 North) (CellYofX Me North))) East))
Fitness : Fitness(15528.816666666668,30.216666666666665,57)
Complexity : 730

A.11 Experiment 8 - CRO Homogenous Full Evaluation (Best of Run 4)

ADF[0] : (IfEast ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg0 (IfCellOccupied ADF0Arg0 West East) (IfCellOccupied ADF0Arg1 South North)) (IfNorth ADF0Arg0 ADF0Arg0 East (IfCellOccupied ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg0 ADF0Arg0 (IfNorth ADF0Arg0 ADF0Arg1 South East) East) (IfCellOccupied ADF0Arg1 (IfCellOccupied ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellOccupied ADF0Arg0 East (IfCellsEqual ADF0Arg0 ADF0Arg0 South West)) Here) North) (IfCellsEqual ADF0Arg0 ADF0Arg1 East Here))) North)))
ADF[1] : (IfEast (CellYofX Agent4 (IfCellOccupied (CellYofX Me Here) (IfNorth (CellYofX Agent4 East) (CellYofX Agent4 (IfEast (CellYofX Me South) (CellYofX Me South) South Here)) West (IfNorth (CellYofX Me North) (CellYofX Agent4 (IfEast (CellYofX Agent4 West) (CellYofX Me Here) South (IfCellOccupied (CellYofX Me North) East North))) West (ADF0 (CellYofX Agent4 South) (CellYofX Me South)))) Here)) (CellYofX Me North) East (IfEast (CellYofX Agent4 East) (CellYofX Me (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me West) (IfNorth (CellYofX Agent4 Here) (CellYofX Me (ADF0 (CellYofX Me South) (CellYofX Agent4 East))) West West) (IfCellsEqual (CellYofX Me Here) (CellYofX Me South) West (IfNorth (CellYofX Me (IfCellOccupied (CellYofX Me East) South North)) (CellYofX Agent4 North) (IfNorth (CellYofX Me South) (CellYofX Me South) Here North) (IfCellOccupied (CellYofX Me West) West West))))) (IfNorth (CellYofX Me (ADF0 (CellYofX Agent4 Here) (CellYofX Me South))) (CellYofX Agent4 West) (IfEast (CellYofX Agent4 East) (CellYofX Me (IfCellOccupied (CellYofX Me (IfCellOccupied (CellYofX Me East) South North)) East North)) South (IfCellsEqual (CellYofX Me (IfCellOccupied (CellYofX Me East) South North)) (CellYofX Agent4 East) (IfNorth (CellYofX Me Here) (CellYofX Agent4 East) South Here)

(IfCellsEqual (CellYofX Agent4 South) (CellYofX Me South) North West))) North) West))
Fitness : Fitness(14918.783333333333,32.11666666666667,53)
Complexity : 237

Appendix B

Best Evolved Communicative Agents

B.1 Experiment 2 - Best Agent of Run 4

ADF[0] : (If (LT RecvA (Add (SendA (Sub RecvB (SendB GetY))) (SendA (SendA GetX)))) (If (GE (SendA GetY) (SendB (SendB GetY))) (If (LT GetX GetX) (If (GE (Sub GetX (Sub GetX RecvA)) RecvB) (If (GE GetX GetY) (If (GE GetY RecvA) S E) E) (If (GE RecvB (SendB GetY)) SW (If (GE GetX GetX) S NW))) (If (GE RecvA (SendA (SendA GetX))) N (If (LT GetY RecvA) NW NW))) E) (If (GE RecvB GetY) NE (If (LT RecvA (SendA GetX)) SW SE)))
Fitness : Fitness(1.0,80,100,0.3912639405204461)
Complexity : 80

B.2 Experiment 2 - Best Agent of Run 2

ADF[0] : (If (GE (Sub GetX (SendB GetX)) (Sub RecvA (SendA (SendA GetY)))) (If (GE (SendB GetX) RecvB) SW SE) (If (GE (SendB GetX) RecvB) (If (LT RecvB GetX) NW (If (LT (SendA GetY) GetY) (If (LT GetX (SendA GetY)) (If (GE RecvA RecvB) (If (GE (SendB GetY) GetX) E SE) E) N) (If (GE GetX GetY) (If (GE RecvB RecvB) NE NE) NE))) (If (GE (Sub (SendB RecvB) GetX) (Sub RecvA (SendA GetY))) (If (GE (SendB GetX) RecvB) (If (LT GetX RecvB) SE N) NE) (If (GE (SendB GetX) RecvB) (If (LT RecvB GetX) NW (If (LT (SendA (SendA RecvA)) GetX) (If (LT GetX (SendA GetX)) (If (GE GetX RecvB) SE SE) (If (GE GetY GetY) SE NW)) (If (GE (Add RecvB RecvA) RecvB) (If (GE GetX RecvB) NE NE) N))) N))))
Fitness : Fitness(1.0,131,100,0.37187997875730217)
Complexity : 131

Appendix C

Best Socialised Agents

C.1 Experiment 5 - Best Socialised Communicating Agent of Run 4

ADF[0] : (If (GE GetY GetY) (If (GE RecvB (Sub GetX (Sub (SendA (SendA (SendA RecvA))) (SendB GetY)))) (If (GE RecvB (Sub RecvA (Sub (SendA GetX) (SendB GetY)))) (If (LT GetX RecvA) NE (If (GE (SendA GetX) GetX) (If (LT GetY (Add (Sub GetX GetX) RecvB)) NW (If (LT RecvB (SendB GetY)) E E)) E)) (If (LT GetY (Add (Sub RecvB GetX) RecvB)) (If (LT GetY GetY) NE E) (If (GE (SendA (SendA (SendB GetY))) (Sub (SendA GetX) GetY)) (If (LT GetY (Add (Sub RecvA RecvA) RecvB)) NW (If (LT (SendB GetX) (SendB GetY)) E E)) E))) (If (LT (SendA GetX) (Add (Sub GetX RecvB) GetY)) (If (GE (SendA (SendA RecvA)) (SendA GetX)) S SW) W)) S)
Fitness : Fitness(1.0,115,100,0.37347318109399896)
Complexity : 115

C.2 Experiment 6a - Best Socialised Homegenous Pursuit Agent of Run 5

ADF[0] : (IfCellOccupied ADF0Arg1 West (IfCellOccupied ADF0Arg1 (IfCellOccupied ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfCellOccupied ADF0Arg1 South South) (IfCellOccupied ADF0Arg1 Here South)) (IfNorth ADF0Arg1 ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 South South) East)) (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 (IfCellOccupied ADF0Arg1 (IfCellsEqual ADF0Arg0 ADF0Arg1 (IfCellOccupied ADF0Arg1 North South) (IfCellOccupied ADF0Arg1 West West)) (IfNorth ADF0Arg1 ADF0Arg0 (IfCellsEqual ADF0Arg1 ADF0Arg1 South West) North)) West) (IfEast ADF0Arg1 ADF0Arg0 East South)) (IfEast ADF0Arg1 ADF0Arg1 (IfEast ADF0Arg0 ADF0Arg1 North West) (IfCellsEqual ADF0Arg0 ADF0Arg1 South West)))))
ADF[1] : (IfEast (CellYofX Agent4 Here) (CellYofX Agent4 North) (IfEast (CellYofX Me East) (CellYofX Agent4 West) West (IfEast (CellYofX Agent4 Here) (CellYofX Me South) South (IfNorth (CellYofX Me East) (CellYofX Me Here) (IfEast (CellYofX Me East) (CellYofX Me South) South East) West))) (IfNorth (CellYofX Me (IfNorth (CellYofX Me South) (CellYofX Agent4 West) Here (ADF0 (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 North) North East)) (CellYofX Me East)))) (CellYofX Agent4 Here) (IfEast (CellYofX Me (IfCellOccupied (CellYofX Me North) (IfNorth (CellYofX Agent4 Here) (CellYofX Me North) West East) (IfNorth (CellYofX Agent4 South) (CellYofX Me Here) West Here))) (CellYofX Agent4 Here) West South) (IfEast (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 South) (CellYofX Agent4 East) (IfCellOccupied (CellYofX Agent4 South) North Here) North)) (CellYofX Me (IfEast (CellYofX Agent4 Here) (CellYofX Me South) South (IfNorth (CellYofX Me East)

(CellYofX Me Here) (IfEast (CellYofX Me East) (CellYofX Me South) East East) West))) (IfCellsEqual (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 Here) (IfEast (CellYofX Me South) (CellYofX Agent4 Here) Here West) South)) (CellYofX Me North) (IfCellsEqual (CellYofX Me (IfCellOccupied (CellYofX Agent4 North) East Here)) (CellYofX Agent4 North) (IfNorth (CellYofX Me Here) (CellYofX Me South) East Here) North) East) North)))
Fitness : Fitness(12971.516666666666,35.916666666666664,41)
Complexity : 261

C.3 Experiment 6b - Best Socialised Heterogeneous Pursuit Agent of Run 5

ADF[0] : (IfCellOccupied ADF0Arg0 East South)
ADF[1] : (IfNorth (CellYofX Agent4 (IfCellOccupied (CellYofX Me (IfEast (CellYofX Agent4 Here) (CellYofX Me South) (IfNorth (CellYofX Me North) (CellYofX Me South) Here Here) North)) North (IfCellOccupied (CellYofX Agent4 (IfNorth (CellYofX Agent4 West) (CellYofX Me Here) South Here)) Here (IfCellOccupied (CellYofX Me North) West East)))) (CellYofX Me South) (IfEast (CellYofX Me East) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 West) East Here)) (IfNorth (CellYofX Agent4 Here) (CellYofX Me East) North West) West) (ADF0 (CellYofX Me (IfNorth (CellYofX Agent4 (IfEast (CellYofX Agent4 South) (CellYofX Me North) North Here)) (CellYofX Agent4 (IfNorth (CellYofX Me Here) (CellYofX Agent4 North) East West)) (IfNorth (CellYofX Me North) (CellYofX Agent4 West) Here (IfNorth (CellYofX Me West) (CellYofX Me North) South (IfCellOccupied (CellYofX Me North) (IfCellOccupied (CellYofX Me South) East Here) East))) South)) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 South) West North)) (CellYofX Agent4 (IfCellsEqual (CellYofX Agent4 Here) (CellYofX Agent4 Here) East Here)) (IfCellOccupied (CellYofX Me South) South Here) (IfCellsEqual (CellYofX Agent4 South) (CellYofX Me Here) (IfCellsEqual (CellYofX Me South) (CellYofX Agent4 West) Here East) East)))))
ADF[2] : (IfEast ADF0Arg0 ADF0Arg1 (IfCellOccupied ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 East (IfCellOccupied ADF0Arg1 (IfEast ADF0Arg1 ADF0Arg1 East South) (IfNorth ADF0Arg1 ADF0Arg0 (IfNorth ADF0Arg1 ADF0Arg1 South East) (IfCellOccupied ADF0Arg1 (IfCellOccupied ADF0Arg0 East East) South)))) (IfNorth ADF0Arg0 ADF0Arg1 (IfNorth ADF0Arg1 ADF0Arg0 East South) (IfCellOccupied ADF0Arg1 (IfCellOccupied ADF0Arg1 South East) East))) West)
ADF[3] : (IfNorth (CellYofX Me Here) (CellYofX Agent4 West) (ADF0 (CellYofX Agent4 East) (CellYofX Me (IfEast (CellYofX Agent4 South) (CellYofX Me East) North (IfCellsEqual (CellYofX Agent4 (IfCellOccupied (CellYofX Agent4 Here) West Here)) (CellYofX Me South) (IfCellsEqual (CellYofX Me East) (CellYofX Agent4 East) (IfCellOccupied (CellYofX Agent4 East) South South) East) Here)))) North)
Fitness : Fitness(15272.0,30.866666666666667,55)
Complexity : 270

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