1 On proofs, advanced

(a) Exercise 1 from section 2 on the examples sheet.

(b) Exercise 2 from section 2 on the examples sheet.

(c) Deduce the following principle of strong/course-of-values induction. Given a predicate $P(m)$ with $m$ ranging over the natural numbers we have:

$$P(0) \land (\forall n. P^\#(n) \Rightarrow P(n + 1)) \Rightarrow \forall m. P(m)$$

(d) Using course-of-values induction do the following two questions:

(i) Show that all integers $n > 1$ can be written as a product of primes. (The single product is allowed, i.e. 2 is written as the single product of 2).

(ii) Starting at 0, you may move up by 1 or 2. In how many distinct ways can you get to the number $n$? (e.g. for $n = 3$ the distinct ways are: (1,1,1), (2,1), (1,2)). [Hint: \url{https://en.wikipedia.org/wiki/Fibonacci_number#Closed-form_expression}]

(e) Exercise 3 from section 2 on the examples sheet.

2 On numbers, basic

All exercises from section 3 on the examples sheet.