Discrete Maths Supervision 1
Ian Orton

The first part of the course is designed to teach you how to properly construct proofs when doing discrete maths. As such, the aim here isn’t simply to get to the result but to illustrate your understanding of what a rigorous proof is and how to properly construct one.

1 Jargon

Explain the difference between a proposition/statement and a predicate. [2 marks]

2 Proof patterns

The notes describe that we can prove an implication \( P \Rightarrow Q \) by assuming \( P \) and then proving \( Q \).

\( a \) Let \( P \) and \( Q \) be propositions, explain how you would prove the following:

\( i \) \( P \Leftrightarrow Q \) [1 mark]

\( ii \) \( P \land Q \) [1 mark]

\( iii \) \( P \lor Q \) [1 mark]

\( iv \) \( \neg P \) [1 mark]

\( b \) Now let \( P(x) \) be a predicate over some variable \( x \). Explain how you would prove the following:

\( i \) \( \forall x. P(x) \) [1 mark]

\( ii \) \( \exists x. P(x) \) [1 mark]

3 Using assumptions

The notes describe that we can use an implication \( P \Rightarrow Q \) in a proof by establishing \( P \) and then using Modus Ponens to deduce \( Q \).

\( a \) Let \( P \) and \( Q \) be propositions, explain how you could use the following assumptions in a proof:

\( i \) \( P \Leftrightarrow Q \) [1 mark]
(ii) \( P \land Q \) [1 mark]

(iii) \( P \lor Q \) [2 marks]

(iv) \( \neg P \) [2 marks]

(b) Now let \( P(x) \) be a predicate over some variable \( x \). Explain how you could use the following assumptions in a proof:

(i) \( \forall x. P(x) \) [2 marks]

(ii) \( \exists x. P(x) \) [2 marks]

4 Abstract proofs

Prove the following statements:

(a) \( P \lor \neg P \) [2 marks]

(b) \( (P \lor Q) \Rightarrow (\neg Q \Rightarrow P) \) [6 marks]

(c) \( \neg (P \lor Q) \Rightarrow (\neg P \land \neg Q) \) [6 marks]

(d) \([\forall x. P(x)) \lor (\forall y. Q(y))\] \( \Rightarrow (\forall z. P(z) \lor Q(z)) \) [8 marks]

(e) \([\forall x. P(x) \Rightarrow Q(x)) \land \neg Q(7)] \Rightarrow \exists y. \neg P(y) \) [8 marks]

5 Translating words to symbols

Express the following statements in symbols, i.e. using \( \land, \lor, \neg, \Rightarrow, \exists, \forall, \leq, < \):

(a) For all integers \( m \) and \( n \), either \( m \) is less than \( n \), or \( n \) is less than or equal to \( m \). [2 marks]

(b) For every integer \( x \), there exists an integer \( y \) such that \( x \) is less than \( y \). [2 marks]

(c) For all integers \( l, m \) and \( n \), if \( l \) is less than \( m \) and \( m \) is less than \( n \) then \( l \) is less than \( n \). [2 marks]

6 Real proofs

The previous questions were all about abstract propositions/predicates. Now apply the same proof principles to some real problems. From the exercise sheet, complete the odd numbered questions from Section 1 “On proofs (basic exercises)”.
The exercise sheet can be found here:


Optional: The more practise you get now the easier you’ll find the rest of the course later on, so if you have the time and the willpower then attempt the even numbered questions too.