Secure Composition of Insecure Components

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Abstract

Software systems are becoming heterogeneous: instead of a small number of large programs from well-established sources, a user's desktop may now consist of many smaller components that interact in intricate ways. Some components will be downloaded from the network from sources that are only partially trusted. A user would like to know that a number of security properties hold, e.g. that personal data is not leaked to the net, but it is typically infeasible to verify that such components are well-behaved. Instead, they must be executed in a secure environment, or *wrapper*, that provides fine-grain control of the allowable interactions between them, and between components and other system resources.

In this paper we study such wrappers, focusing on how they can be expressed in a way that enables their security properties to be stated and proved rigorously. We introduce a model programming language, the box-π calculus, that supports composition of software components and the enforcement of security policies. Several example wrappers are expressed using the calculus; we explore the delicate security properties they guarantee.

Contents

1 Introduction

Software systems are evolving. Increasingly, monolithic applications are being replaced with assemblages of software components coming from different sources. Instead of a small number of large programs from well-established suppliers, nowadays a user's desktop is made up of many smaller applications and software modules that interact in intricate ways to carry out a variety of information processing tasks. Moreover, whereas it used to be that a software base was fairly static and often controlled by a system administrator, it is now easy to download code from the network; technologies such as Java even allow an application program to be extended with new components while the program is running.

In such fluid operating environments, traditional security mechanisms and policies appear almost irrelevant. While passwords and access control mechanisms are adequate to protect the integrity of the computer system as whole, they utterly fail to address the issue of protecting the user from downloaded code being run from her account [IAJR97, GWTB96, NL98]. Approaches such as the Java sandbox that promise security by isolation are not satisfactory either: components can interact freely or not at all [VB99, Gon97]. What is needed is much finer-grained protection mechanisms that take into account the interconnection of software components and the specific security requirements of individual users.

We give a small motivating example (based on a true story) involving a fictional character, Karen, performing some financial computation. To manage her accounts she downloads a software package called *Quickest* from a company Q. Karen does not want any information about her to be leaked without her consent, so she would like to run Quickest in an environment that does not allow it access to the Internet (she has observed that it sometimes uploads information – presumably for marketing purposes – to Q). On the other hand she often needs stock quotes, for which she must allow net access. At present she runs two instances of Quickest, one on an isolated PC, with her financial records, and one connected, used to obtain stock quotes. She transfers data from the second to the first only on floppy disc, thereby manually ensuring that no information flows in the converse direction.

Karen would like to dispose of the isolated PC, using a software solution to prevent her personal data being leaked to the net. Now, Quickest is a large piece of commercial software that was not programmed by Karen. The source code is not available to her and its internal behaviour is complex and inaccessible; ensuring the desired properties by program analysis will not be feasible. Instead she must run the two copies of the package in secure software environments that allow control of the information flow between them and between each package and the net.

More generally, she will wish to run many packages, each trusted in different ways, and will want to be able to dynamically control the interactions between them and between these packages and other resources – the net, regions of the local disc, the terminal, audio and video capture devices etc. In some cases she will wish to log the data sent from one to another; in others she will wish to limit the allowed bandwidth (e.g. to disallow audio and video channels). In general her notion of what data is to be considered "sensitive" is likely to be context dependent. In a Web browser, she may choose to consider her e-mail address as a secret that should be protected from broadcast to junk mail lists, while the same e-mail will not be treated specially in her text editor.

While it is not feasible to analyse or modify large third-party software packages, it *is* possible to intercept the communications between a package and the other parts of the system, interposing code at the boundaries of the different software components [Jon99, FHL+96, BTS+98, GWTB96]. It is thus possible to monitor or control the operations that these components are able to invoke, and the data that is exchanged between them. We call a code fragment that encapsulates untrusted components a *security wrapper* or *wrapper* for short.

Clearly the task of writing wrappers should not be left solely to the end-user. Rather we envision wrappers as reusable software components, users should then only have to pick the most appropriate wrappers, customize them with some parameters and install them. All of this process should be dynamic: wrappers must be no harder to add to a running system than new applications. A user will require a clear description of the security properties that a wrapper guarantees. Moreover, wrappers should compose with a clear notion of which properties are preserved.

The goal of this work is to study such secure environments, focusing on how they can be expressed in a way that enables their security properties to be stated and proved rigorously. It appears that there is a wide range of rather delicate properties, making hard for designers to develop sufficiently clear intuitions without such rigour. Moreover the wrappers, although critical, may be rather small pieces of software, making it feasible to prove properties about them, or about mild idealisations.

To express and reason about wrappers we require a small programming language, with a well-defined semantics, that allows the composition of software components to be expressed straightforwardly and also supports the enforcement of security policies. Such a language, the box-π calculus, is introduced in §2. We begin with a simple example, a wrapper $\mathcal{W}_1$ written in the calculus. It encapsulates a single component and controls its interactions with the environment, limiting them to two channels *in* and *out*. $\mathcal{W}_1$ is written as a unary context:

$$\mathcal{W}_1[\_] \stackrel{def}{=} (\boldsymbol{\nu}\, a)(\begin{array}{l} a[\_] \\ \mid\, !\, in^{\uparrow} y.\overline{in}^{a} y \\ \mid\, !\, out^{a} y.\overline{out}^{\uparrow} y) \end{array}$$

This creates a box with a new name a, installing in parallel with it two forwarders – one that receives messages from the environment on channel *in* and sends them to the wrapped program, and one that receives messages from the wrapped program on channel *out* and sends them to the environment. An arbitrary program P (possibly malicious) can be wrapped to give $\mathcal{W}_1[P]$; the design of the calculus and of $\mathcal{W}_1$ ensures that no matter how P behaves the wrapped program $\mathcal{W}_1[P]$ can only interact with its environment on the two channels *in* and *out*. This could be achieved simply by forbidding *all* interaction between P and the outside world, a rather unsatisfactory wrapper — $\mathcal{W}_1$ is also *honest*, in that it faithfully forwards messages on *in* and *out*. These informal properties are made precise in Propositions 2 and 5 below. We also discuss the sense in which wrapping a *well-behaved* P has no effect on its behaviour. $\mathcal{W}_1$ is atypical in that it has no behaviour except the forwarding of legitimate messages – other reasonable unary wrappers may perform some kind of logging, or have a control interface for the wrapper. The honesty property that should hold for any reasonable wrapper is therefore somewhat delicate; to state it (and our other security properties) we make extensive use of a labelled transition semantics for the calculus.

The wrapper $\mathcal{W}_1$ controls interaction between a single component and its environment. Our second main example goes further towards solving Karen's problem, allowing control of the interaction between components. $\mathcal{W}_2$ (defined in §3) is a binary wrapper that encapsulates two components P and Q as $\mathcal{W}_2[P, Q]$, allowing each to interact with the environment in a limited way but also allowing information to flow from P to Q (but not vice versa) along a directed communication channel. Making this precise is the subject of §5.

Both $\mathcal{W}_1$ and $\mathcal{W}_2$ are chosen to be as simple as possible, in particular with fixed interfaces for components to interact with each other and with the environment. Generalising this to arbitrary interfaces and to wrappers taking any number of components should be straightforward but complicates the notation; other generalisations are discussed in the conclusion.

Overview We begin in the next section (§2) by introducing the calculus and giving its operational semantics. A number of wrappers are defined in §3, including one which logs traffic. The basic properties of honesty and well-behaviour are introduced in §4. Information flows between wrapped components are studied in §5, then we conclude in §6 with discussion of related and future work. This paper describes work in progress – Sections 4 and 5 contain a number of conjectures which are yet to be proved, but which we hope will stimulate discussion. This technical report is an extended version of a paper appearing in the Computer Security Foundations Workshop (CSFW-99).

2 A Boxed π Calculus

The language – known as the *box-π calculus* – that we use for studying encapsulation properties must allow interacting components to be composed. The components will typically be executing concurrently, introducing nondeterminism. It is therefore natural to base the language on a process calculus. The box-π calculus lies in a large design space of distributed calculi that build on the π-calculus of Milner, Parrow and Walker [MPW92]. Related calculi have been used by a number of authors, e.g. in [AFG98, Ama97, AP94, CG98, CG99, FGL$^+$96, HR98c, HR98b, RH98, Sew97, Sew98, SWP98a, SWP98b, VC98, VC99]. A brief overview of the design space can be found in [Sew99]; here we highlight the main design choices for box-π, deferring comparison with related work to §6.

The calculus is based on asynchronous message passing, with components interacting only by the exchange of unordered asynchronous messages. Box-π has an asynchronous π-calculus as a subcalculus – we build on a large body of work studying such calculi, notably [HT91, Bou92, ACS96]. They are known to be very expressive, supporting many programming idioms including functions and objects, and are Turing-complete; a box-π process may therefore perform arbitrary internal computation.

To π we must add primitives for constraining communication – in standard π-calculi, if one process can send a message to another then the only way to prevent information flowing in the reverse direction is to impose a type system, which (as observed above) is not appropriate here. We therefore add a boxing primitive. Boxes may be nested, giving hierarchical protection domains; communication across box boundaries is strictly limited. Underlying the calculus design is the principle that *each box should be able to control all interactions of its children, both with the outside world and with each other* [VC98]. Communication is therefore allowed only between a box and its parent, or within the process running in a particular box. In particular, two sibling boxes cannot interact without the assistance of their parent. To enable a box to interact with a particular child, boxes are named, analogously to π channel names. The security properties of our wrappers depend on the ability to create fresh box names.

Turning to the values that may be communicated, it is convenient to allow arbitrary tuples of names (or other tuples). Note that we do *not* allow communication of process terms. Moreover, no primitives for movement of boxes are provided. The calculus is therefore entirely first order, which is important for the tractable theory of behaviour (the labelled transition semantics) that we require to state and prove security properties. The calculus is also untyped – we wish to consider the wrapping of ill-understood, probably buggy and possibly malicious programs.

2.1 Syntax

The syntax of the calculus is as follows:

Names We take an infinite set $\mathcal{N}$ of *names*, ranged over by lower-case roman letters n, m, x, y, z etc. (except i, j, k, o, p, u, v). Both boxes and communication channels are named; names also play the role of variables, as in the π-calculus.

Values and Patterns Processes will interact by communicating values which are deconstructed by pattern-matching upon reception. Values u, v can be names or tuples, with patterns p correspondingly tuple-structured:

$$
\begin{array}{rcll}
u, v & ::= & x & \text{name} \\
 & & \langle v_1 \mathinner{..} v_k \rangle & \text{tuple } (k \geq 0) \\
p & ::= & \_ & \text{wildcard} \\
 & & x & \text{name pattern} \\
 & & (p_1 \mathinner{..} p_k) & \text{tuple pattern } (k \geq 0\text{, no repeated names})
\end{array}
$$

Processes The main syntactic category is that of *processes*, ranged over by P, Q. We introduce the primitives in three groups.

Boxes A box $n[P]$ has a name n, it can contain an arbitrary process P. Box names are not necessarily unique – the process $n[0] \mid n[0]$ consists of two distinct boxes named n, both containing an empty process, in parallel.

$$
\begin{array}{rcll}
P & ::= & n[P] & \text{box named } n \text{ containing } P \\
 & & P \mid P' & P \text{ and } P' \text{ in parallel} \\
 & & 0 & \text{the nil process} \\
 & & \ldots &
\end{array}
$$

Communication The standard asynchronous π-calculus communication primitives are $\overline{x}v$, indicating an output of value v on the channel named x, and $xp.P$, a process that will receive a value output on channel x, binding it to p in P. Here we refine these with a tag indicating the direction of the communication in the box hierarchy. An *input tag* ι can be either $\star$, for input within a box, $\uparrow$, for input from the parent box, or a name n, for input from a sub-box named n. An *output tag* o can be any of these, similarly. For technical reasons we must also allow an output tag to be $\overline{\uparrow}$, indicating an output received from the parent that has not yet interacted with an input, or $\overline{n}$, indicating an output received from child n that has not yet interacted. The communication primitives are then

$$
\begin{array}{rcll}
P & ::= & \ldots & \\
 & & \overline{x}^o v & \text{output } v \text{ on channel } x \text{ to } o \\
 & & x^\iota p.P & \text{input on channel } x \text{ from } \iota \\
 & & !\, x^\iota p.P & \text{replicated input} \\
 & & \ldots &
\end{array}
$$

The replicated input $!\, x^\iota p.P$ behaves essentially as infinitely many copies of $x^\iota p.P$ in parallel. This gives computational power, allowing e.g. recursion to be encoded simply, while keeping the theory simple. In $x^\iota p.P$ and $!\,x^\iota p.P$ the names occurring in the pattern p bind in P.

New name creation Both box and channel names can be created fresh, with the standard π-calculus $(\boldsymbol{\nu}\,x)P$ operator. This declares any free instances of x within P to be instances of a globally fresh name.

$$
\begin{array}{lll}
P & ::= & \ldots \\
 & & (\boldsymbol{\nu}\,x)P \qquad\qquad \text{new name creation}
\end{array}
$$

In $(\boldsymbol{\nu}\,x)P$ the x binds in P. We work up to alpha conversion of bound names throughout, writing the free name function, defined in the obvious way for values, tags and processes, as $\mathrm{fn}(\_)$.

2.2 Reduction

The simplest semantic definition of the calculus is a *reduction semantics*, a one-step reduction relation $P \to P'$ indicating that P can perform one step of internal computation to become P'. We first define the complement $\overline{\iota}$ of a tag ι in the obvious way, with $\overline{\star} = \star$ and $\overline{\overline{\iota}} = \iota$. We define a partial function $\{{}^{\_}/_{\_}\}$, taking a pattern and a value and giving, where it is defined, a partial function from names to values.

$$
\begin{array}{rcl}
\{{}^{v}/_{\_}\} & = & \{\} \\
\{{}^{v}/_{x}\} & = & \{x \mapsto v\} \\
\{{}^{\langle v_1\,..\,v_{k'}\rangle}/_{\langle p_1\,..\,p_k\rangle}\} & = & \{{}^{v_1}/_{p_1}\} \cup \ldots \cup \{{}^{v_k}/_{p_k}\} \text{ if these are defined and } k = k' \\
 & & \text{undefined, otherwise}
\end{array}
$$

The natural definition of the application of a substitution σ (from names to values) to a process term P, written σP, is also a partial operation, as the syntax does not allow arbitrary values in all the places where free names can occur. We write $\{{}^{v}/_{p}\}P$ for the result of applying the substitution $\{{}^{v}/_{p}\}$ to P. This may be undefined either because $\{{}^{v}/_{p}\}$ is undefined, or because $\{{}^{v}/_{p}\}$ is a substitution but the application of that substitution to P is undefined. Note that the result $\{{}^{y}/_{x}\}P$ of applying a name-for-name substitution is always defined. We define structural congruence $\equiv$ as the least congruence relation such that the axioms below hold. This allows the parts of a redex to be brought syntactically adjacent.

$$
\begin{array}{rclrcll}
P \mid 0 & \equiv & P & (\boldsymbol{\nu}\,x)(\boldsymbol{\nu}\,y)P & \equiv & (\boldsymbol{\nu}\,y)(\boldsymbol{\nu}\,x)P & \\
P \mid Q & \equiv & Q \mid P & (\boldsymbol{\nu}\,x)(P \mid Q) & \equiv & P \mid (\boldsymbol{\nu}\,x)Q & x \notin \mathrm{fn}(P) \\
(P \mid Q) \mid R & \equiv & P \mid (Q \mid R) & (\boldsymbol{\nu}\,x)n[P] & \equiv & n[(\boldsymbol{\nu}\,x)P] & x \neq n
\end{array}
$$

The reduction relation is now the least relation over processes satisfying the axioms and rules below. The (Red Comm) and (Red Repl) axioms are subject to the condition that $\{{}^{v}/_{p}\}P$ is well-defined.

$$
\begin{array}{ll}
n[\overline{x}^{\uparrow}v \mid Q] \to \overline{x}^{\overline{n}}v \mid n[Q] & \text{(Red Up)} \\
\overline{x}^{n}v \mid n[Q] \to n[\overline{x}^{\overline{\uparrow}}v \mid Q] & \text{(Red Down)} \\
\overline{x}^{\overline{\iota}}v \mid x^{\iota}p.P \to \{{}^{v}/_{p}\}P & \text{(Red Comm)} \\
\overline{x}^{\overline{\iota}}v \mid !\,x^{\iota}p.P \to !\,x^{\iota}p.P \mid \{{}^{v}/_{p}\}P & \text{(Red Repl)} \\
P \to Q \;\;\Rightarrow\;\; P \mid R \to Q \mid R & \text{(Red Par)} \\
P \to Q \;\;\Rightarrow\;\; (\boldsymbol{\nu}\,x)P \to (\boldsymbol{\nu}\,x)Q & \text{(Red Res)} \\
P \to Q \;\;\Rightarrow\;\; n[P] \to n[Q] & \text{(Red Box)} \\
P \equiv P' \to Q' \equiv Q \;\;\Rightarrow\;\; P \to Q & \text{(Red Struct)}
\end{array}
$$

The (Red Up) axiom allows an output to the parent of a box to cross the enclosing box boundary. Similarly, the (Red Down) axiom allows an output to a child box n to cross the boundary of n. The (Red Comm) axiom then allows synchronisation between a complementary output and input within the same box. The (Red Repl) axiom is similar, but preserves the replicated input in the resulting state.

Communications across box boundaries thus take two reduction steps, for example in the following upwards and downwards communications.

$$\begin{array}{rcl} n[\overline{x}^{\uparrow}v] \mid x^{n}p.P & \rightarrow & n[0] \mid \overline{x}^{\overline{n}}v \mid x^{n}p.P \\ & \rightarrow & n[0] \mid \{{}^{v}/_{p}\}P \\ \\ \overline{x}^{n}v \mid n[x^{\uparrow}p.P] & \rightarrow & n[\overline{x}^{\overline{\uparrow}}v \mid x^{\uparrow}p.P] \\ & \rightarrow & n[\{{}^{v}/_{p}\}P] \end{array}$$

This removes the need for 3-way synchronisations between a box, an output and an input (as in [VC98]), simplifying both the semantics and the implementation model.

2.3 Labelled Transitions

The reduction semantics defines only the internal computation of processes. The statements of our security properties must involve the interactions of processes with their environments, requiring more structure: a *labelled transition relation* characterising the potential inputs and outputs of a process. We give a labelled semantics for box-π in an explicitly-indexed early style, defined inductively on process structure by an SOS. The *labels* are

$$\begin{array}{rcll} \ell & ::= & \tau & \text{internal action} \\ & & \overline{x}^{o}v & \text{output action} \\ & & x^{\gamma}v & \text{input action} \end{array}$$

where γ ranges over all output tags except $\overline{\uparrow}$. The labelled transitions can be divided into those involved in moving messages across box boundaries and those involved in communications between outputs and inputs. The movement labels are

$\overline{x}^{n}v$ (sending to child n)	$x^{\overline{n}}v$ (box n receiving from its parent)
$\overline{x}^{\uparrow}v$ (sending to the parent)	

Say $\text{mv}(o)$ is true if o is of the form n or $\uparrow$. The communication labels are

$\overline{x}^{\star}v$ (local output)	$x^{\star}v$ (local input)
$\overline{x}^{\overline{n}}v$ (output received from child n)	$x^{n}v$ (input a message received from child n)
$\overline{x}^{\overline{\uparrow}}v$ (output received from parent)	$x^{\uparrow}v$ (input a message received from parent)

Labels will synchronise in the pairs given. The labelled transition relation has the form

$$A \vdash P \xrightarrow{\ell} Q$$

where A is a finite set of names and $\text{fn}(P) \subseteq A$; it should be read as 'in a state where the names A may be known to P and its environment, process P can do ℓ to become Q'. The relation is defined as the smallest relation satisfying the rules in Figure 1. We write A, x for $A \cup \{x\}$ where x is assumed not to be in A, and A, p for the union of A and the names occurring in the pattern p, where these are assumed disjoint. For the subcalculus without new-binding the labelled transition rules are straightforward —

$$\frac{}{\overline{x}^o v \xrightarrow{\overline{x}^o v} 0} \text{ (Out)} \qquad \frac{}{x^\iota p.P \xrightarrow{x^\iota v} \{^v/_p\}P} \text{ (In)}$$

$$\frac{}{!\, x^\iota p.P \xrightarrow{x^\iota v} !\, x^\iota p.P \mid \{^v/_p\}P} \text{ (Repl)}$$

$$\frac{}{n[P] \xrightarrow{x^{\overline{n}} v} n[\overline{x}^{\overline{\uparrow}} v \mid P]} \text{ (Box-2)} \qquad \frac{A \vdash P \xrightarrow{\overline{x}^\uparrow v} P'}{A \vdash n[P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x,v) - A)(\overline{x}^{\overline{n}} v \mid n[P'])} \text{ (Box-1)}$$

$$\frac{P \xrightarrow{\tau} P'}{n[P] \xrightarrow{\tau} n[P']} \text{ (Box-3)}$$

$$\frac{P \xrightarrow{\ell} P'}{P \mid Q \xrightarrow{\ell} P' \mid Q} \text{ (Par)} \qquad \frac{A \vdash P \xrightarrow{\overline{x}^{\overline{\gamma}} v} P' \quad A \vdash Q \xrightarrow{x^\gamma v} Q'}{A \vdash P \mid Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x,v) - A)(P' \mid Q')} \text{ (Comm)}$$

$$\frac{A, x \vdash P \xrightarrow{\ell} P'}{A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} (\boldsymbol{\nu}\, x)P'} \text{ (Res-1)} \qquad \frac{A, x \vdash P \xrightarrow{\overline{y}^o v} P'}{A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\overline{y}^o v} P'} \text{ (Res-2)}$$

$$\frac{P \xrightarrow{\ell} P' \quad P' \equiv P''}{P \xrightarrow{\ell} P''} \text{ (Struct Right)}$$

The (Res-1) rule is subject to $x \notin \mathrm{fn}(\ell)$, the (Res-2) rule is subject to $x \in \mathrm{fn}(v) - \mathrm{fn}(y, o)$ if $\neg\mathrm{mv}(o)$ and to $x \in \mathrm{fn}(y, v) - \mathrm{fn}(o)$ otherwise. The indexing $A \vdash$ has beeen elided in rules where it is not involved in any interesting way. In all rules with conclusion of the form $A \vdash P \xrightarrow{\ell} Q$ there is an implicit side condition $\mathrm{fn}(P) \subseteq A$. In the (In) and (Repl) axioms there is an implicit side condition that $\{^v/_p\}P$ is well-defined. Symmetric versions of (Par) and (Comm) are elided.

Figure 1: Box-π Labelled Transitions

instances of the reduction rule (Red Up) correspond to uses of (Box-1), (Out), and (Par); instances of (Red Down) correspond to uses of (Comm), (Out), and (Box-2); instances of (Red Comm) correspond to uses of (Comm), (Out), and (In). The derivations of the corresponding τ-transitions can be found in the proof of Lemma 19. The addition of new-binding introduces several subtleties, some inherited from the π-calculus and some related to scope extrusion and intrusion across box boundaries. We discuss the latter briefly.

The (Red Down) rule involves synchronisation on the box name n but *not* on the channel name x — there are reductions such as

$$((\boldsymbol{\nu}\, x)\overline{x}^n z) \mid n[0] \quad \rightarrow \quad (\boldsymbol{\nu}\, x)n[\overline{x}^{\overline{\uparrow}} z]$$

in which a new-bound name enters a box boundary. To correctly match this with a τ-transition the side-condition for (Res-2) for labels with output tag n requires the bound name to occur either in channel or value position, and the (Comm) rule reintroduces the

x binder on the right hand side.

Similarly, the (Red Up) rule allows new-bound names in channel position to exit a box boundary, for example in

$$n[(\boldsymbol{\nu}\, x)\overline{x}^{\uparrow} z] \quad \to \quad (\boldsymbol{\nu}\, x)(\overline{x}^{\overline{n}} z \mid n[0])$$

The (Res-2) condition for output tag $\uparrow$ again requires the bound name to occur either in channel or value position, here the (Box-1) rule reintroduces the x binder on the right hand side.

Reductions generated by (Red Comm) involve synchronisation both on the tags and on the channel name. The (Res-2) condition for output tags $\star$, $\overline{\uparrow}$ and $\overline{n}$ is analogous to the standard π-calculus (Open) rule; requiring the bound name to occur in the value but not in the tag or channel. The (Comm) rule for these output tags is analogous to the standard π rule — in particular, here it is guaranteed that $x \in A$ (see Lemma 11).

Some auxiliary notation is useful. For a sequence of labels $\ell_1 \ldots \ell_k$ we write

$$A \vdash P_1 \xrightarrow{\ell_1} \ldots \xrightarrow{\ell_k} P_{k+1}$$

to mean $\exists P_2, \ldots, P_k \;.\; \forall i \in 1..k \;.\; A_i \vdash P_i \xrightarrow{\ell_i} P_{i+1}$, where $A_i = A \cup \bigcup_{j \in 1..i} \mathrm{fn}(\ell_j)$. If $\ell \neq \tau$ we write $A \vdash P \stackrel{\hat{\ell}}{\Longrightarrow} P'$ for $A \vdash P \xrightarrow{\tau}^{*} \xrightarrow{\ell} \xrightarrow{\tau}^{*} P'$; if $\ell = \tau$ then $A \vdash P \stackrel{\hat{\ell}}{\Longrightarrow} P'$ is defined as $A \vdash P \xrightarrow{\tau}^{*} P'$.

The two semantics coincide in the following sense.

Theorem 1 *If* $\mathrm{fn}(P) \subseteq A$ *then* $A \vdash P \xrightarrow{\tau} Q$ *iff* $P \to Q$.

This give confidence that the labelled semantics carries enough information. The proof is somewhat delicate — it can be found in Appendix A.

2.4 Bisimulation

The statements of some relationships between the behaviour of a wrapped and an unwrapped program require an operational equivalence relation. As box-π is asynchronous, an appropriate notion can be based on the *weak asynchronous bisimulation* of [ACS96]. Consider a family S of relations indexed by finite sets of names such that each S_A is a symmetric relation over $\{\, P \mid \mathrm{fn}(P) \subseteq A \,\}$. Say S is a *weak asynchronous bisimulation* if

- $P \; S_A \; Q$, $A \vdash P \xrightarrow{\ell} P'$ and ℓ is an output or τ transition imply $\exists Q' \;.\; A \vdash Q \stackrel{\hat{\ell}}{\Longrightarrow} Q' \wedge P' \; S_{A \cup \mathrm{fn}(\ell)} \; Q'$, and
- $P \; S_A \; Q$, $A \vdash P \xrightarrow{x^{\gamma} v} P'$ imply either $\exists Q' \;.\; A \vdash Q \stackrel{x^{\gamma} v}{\Longrightarrow} Q' \wedge P' \; S_{A \cup \mathrm{fn}(x^{\gamma} v)} \; Q'$ or $\exists Q' \;.\; A \vdash Q \Longrightarrow Q' \wedge P' \; S_{A \cup \mathrm{fn}(x^{\gamma} v)} \; (Q' \mid \overline{x}^{\gamma} v)$.

We write $\approx$ for the union of all weak asynchronous bisimulations. (This definition has not been thoroughly tested – in particular, it has not been proved to be a congruence.)

3 Security Wrappers

This section gives three example wrappers. The first encapsulates a single component, restricting its interactions with the outside world to communications obeying a certain protocol. The second is similar, but also writes a log of all such communications. The third wrapper encapsulates two components, allowing each to interact with the outside world in a limited way but also allowing information to flow from the first to the second (but not vice versa).

A wrapper design must be in the context of some fixed protocol which components should use for communication with their environment and with each other. For the first two wrappers we fix two channel names, *in* and *out*, for components to receive and send messages respectively. Moreover, we assume that components will always be executed within some box and should be communicating with the parent box. A trivial component that receives values v and then copies pairs $\langle v\, v\rangle$ to the output would be written as

$$\mathbf{!}\, in^{\uparrow} y . \overline{out}^{\uparrow} \langle y\, y\rangle$$

A malicious component might also write data to another illicit output channel available in the environment, e.g.

$$\mathbf{!}\, in^{\uparrow} y . (\overline{net}^{\uparrow} y \mid \overline{out}^{\uparrow} \langle y\, y\rangle)$$

or eavesdrop on communications between other parts of the system, e.g.

$$\mathbf{!}\, c^{\star} y . (\overline{net}^{\uparrow} c \mid \overline{c}^{\star} y)$$

We can express whether a component obeys the protocol in terms of the labelled transition semantics – say P is *well-behaved* for a unary wrapper iff whenever $A \vdash P \xrightarrow{l_1 .. l_k} Q$ then the l_j are of the form $in^{\uparrow} v$, $\overline{out}^{\uparrow} v$, or τ.

A Filtering Wrapper A filter is a wrapper that simply restricts the communication abilities of a process. We consider a static filter that allows interaction on two channels *in* and *out* only.

$$\begin{array}{lcll} \mathcal{W}_1[\_] & \stackrel{def}{=} & (\boldsymbol{\nu}\, a)(& a[\_] \\ & & & \mid \mathbf{!}\, in^{\uparrow} y . \overline{in}^{a} y \\ & & & \mid \mathbf{!}\, out^{a} y . \overline{out}^{\uparrow} y \ ) \end{array}$$

$\mathcal{W}_1$ executes its component within a freshly-named box, installing forwarders to move legitimate messages across the boundary. Note that this and further wrappers are non-binding contexts – equivalently, we assume wherever we apply $\mathcal{W}_1$ to a process P that the new-bound a does not occur free in P (in an implementation this could be ensured either probabilistically or with a linear-time scan of P). Irrespective of the behaviour of P, $\mathcal{W}_1[P]$ does obey the protocol – this can be stated clearly using the labelled transition semantics:

Proposition 2 *For any program P with $a \notin \mathrm{fn}(P)$, if $A \vdash \mathcal{W}_1[P] \xrightarrow{l_1 .. l_k} Q$ then the l_j are of the form $in^{\uparrow} v$, $\overline{out}^{\uparrow} v$, or τ.*

The proof is via an explicit characterisation of the states reachable by labelled transitions of $\mathcal{W}_1[P]$. A proof of this, and of the other properties of $\mathcal{W}_1$, can be found in the Appendices. We say a unary wrapper with this property is *pure*.

The Logging Wrapper The filter can be extended to maintain a log of all communications of a process, sending copies on a channel *log* to the environment:

$$\begin{array}{lcll} \mathcal{L}[\_] & \stackrel{def}{=} & (\boldsymbol{\nu}\, a)(& a[\_] \\ & & & \mid \mathbf{!}\, in^{\uparrow} y . (\overline{log}^{\uparrow} y \mid \overline{in}^{a} y) \\ & & & \mid \mathbf{!}\, out^{a} y . (\overline{log}^{\uparrow} y \mid \overline{out}^{\uparrow} y) \ ) \end{array}$$

A wrapped program $\mathcal{L}[P]$ again can interact only in limited ways.

Proposition 3 *For any program P with $a \notin \mathrm{fn}(P)$, if $A \vdash \mathcal{L}[P] \xrightarrow{l_1..l_n} Q$ then the l_j are of the form $in^{\uparrow} v$, $\overline{out}^{\uparrow} v$, $\overline{log}^{\uparrow} v$, or τ.*

A Pipeline Wrapper A pipeline wrapper allows a controlled flow of information between two components. We give a binary wrapper $\mathcal{W}_2$ that takes two processes. In an execution of $\mathcal{W}_2[Q_1, Q_2]$ the two wrapped processes Q_i can interact with the environment as before, on channels in_i and out_i. In addition, Q_1 can send messages to Q_2 on a channel *mid*. The pipeline implemented here is unordered.

$$\begin{array}{rcll}
\mathcal{W}_2[_1, _2] & \stackrel{def}{=} & (\boldsymbol{\nu}\, a_1, a_2)(& a_1[_1] \mid a_2[_2] \\
& & & \mid \,!\, {in_1}^{\uparrow} y. \overline{in_1}^{a_1} y \\
& & & \mid \,!\, {in_2}^{\uparrow} y. \overline{in_2}^{a_2} y \\
& & & \mid \,!\, {out_1}^{a_1} y. \overline{out_1}^{\uparrow} y \\
& & & \mid \,!\, {out_2}^{a_2} y. \overline{out_2}^{\uparrow} y \\
& & & \mid \,!\, mid^{a_1} y. \overline{mid}^{a_2} y \;)
\end{array}$$

As before $\mathcal{W}_2$ is a non-binding context – we assume, wherever we apply it to two processes P_1, P_2, that $\{a_1, a_2\} \cap \mathrm{fn}(P_1, P_2) = \emptyset$. Say a binary wrapper $\mathcal{C}$ is pure iff for any programs P_1, P_2, (satisfying the appropriate free name condition – for $\mathcal{W}_2$ that with $\{a_1, a_2\} \cap \mathrm{fn}(P_1, P_2) = \emptyset$), if $A \vdash \mathcal{C}[P_1, P_2] \xrightarrow{l_1..l_k} Q$ then the l_j are of the form ${in_i}^{\uparrow} v$, $\overline{out_i}^{\uparrow} v$, or τ.

Proposition 4 *$\mathcal{W}_2$ is pure.*

For an example of a blocked attempt by the second process to send a value to the first, suppose $P_2 = \overline{mid}^{\uparrow} v$. We have

$$\begin{array}{l}
\mathcal{W}_2[P_1, \overline{mid}^{\uparrow} v] = (\boldsymbol{\nu}\, a_1, a_2)\big(a_1[P_1] \mid a_2[\overline{mid}^{\uparrow} v] \mid R\big) \\
\qquad\qquad\qquad \to (\boldsymbol{\nu}\, a_1, a_2)\big(a_1[P_1] \mid a_2[0] \mid \overline{mid}^{a_2} v \mid R\big)
\end{array}$$

where R is the parallel composition of forwarders. The output $\overline{mid}^{a_2} v$ in the final state cannot interact further – not with the environment, as a_2 is restricted, and not with the forwarder $!\, mid^{a_1} y. \overline{mid}^{a_2} y$, as $a_1 \neq a_2$.

These wrappers all assume a rather simple fixed protocol. It would be straightforward to generalise to arbitrary sets of channels instead of *in*, *out* and *mid*. It would also be straightforward to allow n-ary wrappers, encapsulating many components and allowing information to flow only on a given preorder between them. Other generalisations are discussed in the conclusion.

4 Honesty and Composition

The properties of wrappers stated in the previous section are very weak. For example, the unary wrapper

$$\mathcal{C}[\_] \stackrel{def}{=} 0$$

is also pure, but is useless. In this section we identify the class of *honest* wrappers that are guaranteed to forward legitimate messages. This gives the authors of components a clear statement of (some of) the properties of the environment that can be relied upon.

An initial attempt might be to take $\mathcal{W}_1$ as a specification, defining a unary wrapper $\mathcal{C}$ to be honest iff for any program P the processes $\mathcal{C}[P]$ and $\mathcal{W}_1[P]$ are operationally equivalent. This is unsatisfactory – it rules out wrappers such as $\mathcal{L}$, and it does not give a very clear statement of the properties that may be assumed of an honest wrapper.

A better attempt might be to say that a unary wrapper $\mathcal{C}$ is honest iff for any well-behaved P the processes $\mathcal{C}[P]$ and P are operationally equivalent. This would be unsatisfactory in two ways. Firstly, some intuitively sound wrappers have additional interactions with the environment – e.g. the logging outputs of $\mathcal{L}$ – and so would not be considered honest by this definition. Secondly, this definition would not constrain the behaviour of wrappers for non-well-behaved P at all – if a component P attempted, in error, a single illicit communication then $\mathcal{C}[P]$ might behave arbitrarily.

To address these points we give explicit definitions of honesty, first for unary wrappers and then for binary, in the style of weak asynchronous bisimulation. Consider a family R indexed by finite sets of names such that each R_A is a relation over $\{\, P \mid \mathrm{fn}(P) \subseteq A \,\}$. Say R is an *h-bisimulation* if, whenever $C\ R_A\ Q$ then:

1. if $A \vdash C \xrightarrow{\ell} C'$ for $\ell = \overline{out}^{\uparrow} v, \tau$ then $A \vdash Q \stackrel{\hat{\ell}}{\Longrightarrow} Q' \wedge C'\ R_{A \cup \mathrm{fn}(\ell)}\ Q'$

2. if $A \vdash C \xrightarrow{in^{\uparrow} v} C'$ then either $A \vdash Q \stackrel{in^{\uparrow} v}{\Longrightarrow} Q'$ and $C'\ R_{A \cup \mathrm{fn}(in, v)}\ Q'$ or $A \vdash Q \Longrightarrow Q'$ and $C'\ R_{A \cup \mathrm{fn}(in, v)}\ Q' \mid \overline{in}^{\uparrow} v$

3. if $A \vdash C \xrightarrow{\ell} C'$ for any other label then $C'\ R_{A \cup \mathrm{fn}(\ell)}\ Q$

together with symmetric versions of clauses 1 and 2. Say a unary wrapper $\mathcal{C}$ is *honest* if for any program P (satisfying the appropriate free name condition) and any $A \supseteq \mathrm{fn}(\mathcal{C}[P])$ there is an h-bisimulation R with $\mathcal{C}[P]\ R_A\ P$.

Loosely, clauses 1, 2 and the symmetric versions ensure that legitimate communications and internal reductions must be weakly matched. Clause 3 ensures that if the wrapper performs some additional communication then this does not affect the state as seen by the wrapped process.

Proposition 5 *The unary wrappers $\mathcal{W}_1$ and $\mathcal{L}$ are honest.*

We give some examples of dishonest wrappers. Take

$$\mathcal{C}[\_] \stackrel{def}{=} (\boldsymbol{\nu}\, a) a[\_]$$

This is not honest – a transition $A \vdash P \xrightarrow{\overline{out}^{\uparrow} v} P'$ cannot be matched by $\mathcal{C}[P]$, violating the symmetric version of clause 1. Now consider

$$\mathcal{C}[\_] \stackrel{def}{=} \_$$

This wrapper is also dishonest as $\mathcal{C}[P]$ can perform actions not in the protocol that essentially affect the state of P. For example, take $P = x^{\star} y.\overline{out}^{\uparrow}\langle\rangle$. Suppose $\mathcal{C}[P]\ R_A\ P$ for an h-bisimulation R. We have $A \vdash \mathcal{C}[P] \xrightarrow{x^{\star}\langle\rangle} \overline{out}^{\uparrow}\langle\rangle$ so by clause 3 $\overline{out}^{\uparrow}\langle\rangle\ R_A\ P$, but then clause 1 cannot hold – the left hand side can perform an $\overline{out}^{\uparrow}\langle\rangle$ transition that cannot be matched be the right hand side.

Composition of Wrappers The protocol for communication between a component and a unary wrapper is designed so that wrappers may be nested. We conjecture that the composition of any honest unary wrappers is honest.

Conjecture 6 *If $\mathcal{C}_1$ and $\mathcal{C}_2$ are honest unary wrappers then $\mathcal{C}_1 \circ \mathcal{C}_2$ is honest.*

Analogous results for non-unary wrappers would require wrappers with more complex interfaces so that the input, output and mid channels could be connected correctly.

A desirable property of a pure wrapper is that it should not affect the behaviour of any well-behaved component — one might expect for any pure and honest $\mathcal{C}$ and well-behaved P that $\mathcal{C}[P] \approx_A P$ (where $A \supseteq \mathrm{fn}(\mathcal{C}[P])$). Unfortunately this does not hold, even for $\mathcal{W}_1$, as the wrapper can make input transitions that cannot be matched. One can check $\mathcal{W}_1[0] \not\approx_A 0$, yet 0 is well-behaved. In practice one would expect the environment of a wrapper to not be able to detect these inputs, but to make this precise would require an operational equivalence relativised to such 'well-behaved' environments.

A simpler property would be that multiple wrappings have no effect. We conjecture that $\mathcal{W}_1$ is idempotent, i.e. that $\mathcal{W}_1[\mathcal{W}_1[P]]$ and $\mathcal{W}_1[P]$ have the same behaviour (up to weak asynchronous bisimulation):

Conjecture 7 *For any program P with $a \notin \mathrm{fn}(P)$ and $A \supseteq \mathrm{fn}(\mathcal{W}_1[P])$ we have $\mathcal{W}_1[P] \approx_A \mathcal{W}_1[\mathcal{W}_1[P]]$.*

4.1 Honesty for Binary Wrappers

The definition of honesty for binary wrappers must take into account the *mid* communication. Consider a family R indexed by finite sets of names such that each R_A is a relation between terms and pairs of terms, all with free names contained in A. Say R is a *binary h-bisimulation* if, whenever $C \; R_A \; (Q_1, Q_2)$ the clauses below hold. The key difference with the unary definition is clause 7; the other clauses are routine, albeit notationally complex.

1. if $A \vdash C \xrightarrow{\overline{out_i}\uparrow v} C'$ then $A \vdash Q_i \stackrel{\overline{out_i}\uparrow v}{\Longrightarrow} Q_i'$, $A \vdash Q_{3-i} \Longrightarrow Q_{3-i}'$ and $C' \; R_{A \cup \mathrm{fn}(v)} \; (Q_1', Q_2')$.
2. if $A \vdash C \xrightarrow{in_i \uparrow v} C'$ then $A \vdash Q_{3-i} \Longrightarrow Q_{3-i}'$ and either $A \vdash Q_i \stackrel{in_i\uparrow v}{\Longrightarrow} Q_i' \wedge C' \; R_{A \cup \mathrm{fn}(v)} \; (Q_1', Q_2')$ or $A \vdash Q_i \Longrightarrow Q_i'' \wedge C' \; R_{A\cup \mathrm{fn}(v)} \; (Q_1', Q_2')$, where $Q_i' = Q_i'' \mid \overline{in}^{\overline{\uparrow}} v$.
3. if $A \vdash C \xrightarrow{\tau} C'$ then $A \vdash Q_1 \Longrightarrow Q_1'$, $A \vdash Q_2 \Longrightarrow Q_2'$ and $C' \; R_A \; (Q_1', Q_2')$.
4. if $A \vdash C \xrightarrow{\ell} C'$ for any other label then $C' \; R_{A \cup \mathrm{fn}(\ell)} \; (Q_1, Q_2)$
5. if $A \vdash Q_i \xrightarrow{\ell} Q_i'$ for $\ell = \overline{out_i}^{\uparrow} v, \tau$ then $A \vdash C \stackrel{\hat{\ell}}{\Longrightarrow} C'$, and $C' \; R_{A \cup \mathrm{fn}(\ell)} \; (Q_1', Q_2')$, where $Q_{3-i}' = Q_{3-i}$.
6. if $A \vdash Q_i \xrightarrow{in_i\uparrow v} Q_i'$ then either $A \vdash C \stackrel{in_i\uparrow v}{\Longrightarrow} C' \wedge C' \; R_{A\cup\mathrm{fn}(v)} \; (Q_1', Q_2')$ or $A \vdash C \Longrightarrow C' \wedge C' \mid \overline{in}^{\overline{\uparrow}} v \; R_{A \cup \mathrm{fn}(v)} \; (Q_1', Q_2')$, where $Q_{3-i}' = Q_{3-i}$.
7. if $A \vdash Q_1 \xrightarrow{\overline{mid}\uparrow v} Q_1'$ then $A \vdash C \Longrightarrow C' \wedge C' \; R_{A \cup \mathrm{fn}(v)} \; (Q_1', Q_2 \mid \overline{mid}^{\overline{\uparrow}} v)$.

A binary wrapper $\mathcal{C}$ is honest if for all P_1, P_2 (satisfying the appropriate free name condition) and any $A \supseteq \mathrm{fn}(\mathcal{C}[P_1, P_2])$ there exists a binary h-bisimulation R with $\mathcal{C}[P_1, P_2] \; R_A \; (P_1, P_2)$.

Conjecture 8 *$\mathcal{W}_2$ is honest.*

5 Constrained Interaction Between Components

In our motivating example Karen required fine-grain control over the information flows between components – in the binary case, allowing unidirectional flow. By examining the code for $\mathcal{W}_2$ it is intuitively clear that it achieves this, preventing information flowing from Q to P within $\mathcal{W}_2[P, Q]$. When one comes to make this intuition precise, however, it becomes far from clear exactly what behavioural properties $\mathcal{W}_2$ guarantees that make it a satisfactory wrapper from the user's point of view (who should not have to examine the wrapper code). Honesty is one, but it does not prohibit bad flows. In this section we give a number of candidate properties, stating four precisely and the others informally. We conjecture that all are satisfied by $\mathcal{W}_2$ but that none are equivalent. None are entirely satisfactory; we hope to provoke discussion of exactly what guarantees should be desired by users and by component designers. For simplicity, only pure binary wrappers $\mathcal{C}$ are considered – recall that for a pure binary $\mathcal{C}$ the labelled transitions of $\mathcal{C}[P_1, P_2]$ will only be of the forms $in_i{}^{\uparrow}v$, $\overline{out_i}{}^{\uparrow}v$ and τ.

5.1 New-name directionality

As we are using a calculus with creation of new names, we can test a wrapper by supplying a new name to the second component, on in_2, and observing whether it can ever be output by the first component on out_1. Say $\mathcal{C}$ is *directional for new names* if whenever

$$A \vdash \mathcal{C}[P_1, P_2] \xrightarrow{\ell_1} \ldots \xrightarrow{\ell_j} \xrightarrow{in_2{}^{\uparrow}u} \xrightarrow{\ell'_1} \ldots \xrightarrow{\ell'_k} \xrightarrow{\overline{out_1}{}^{\uparrow}u'} P$$

with $x \in \text{fn}(u)$, but x is new, i.e. $x \notin A \cup \text{fn}(\ell_1 \ldots \ell_j)$, and x is not subsequently input to the first component, i.e.

$$x \notin \bigcup_{i \in 1..k \wedge \ell'_i = in_1{}^{\uparrow}v} \text{fn}(v)$$

then x is not output by the first component, i.e. $x \notin \text{fn}(u')$. This property does not prevent all information flow, however – a variant of $\mathcal{W}_2$ containing a reverse-forwarder that only forwards particular values, such as

$$\mathbf{!}\, mid^{a_2} y.\mathbf{if}\ y \in \{0, 1\}\ \mathbf{then}\ \overline{mid}^{a_1} y$$

could still satisfy it. (Here 0 and 1 are free names, which must therefore be in A.)

Note that a binary wrapper $\mathcal{C}$ is intended only to limit information flow *within* $\mathcal{C}[P_1, P_2]$. We do not wish to place any constraint on the environment of the wrapper, for example forbidding the environment to copy values received from out_2 to in_1. Such a restriction could only be imposed by draconian measures, e.g. by waiting for P_1 to terminate before starting P_2, that would not be acceptable to the desktop user. Many programs are essentially non-terminating; if they are executing concurrently then the user cannot be prevented from reading the output of one and copying it to the other. In many circumstances this should be explicitly supported by the desktop cut-and-paste, perhaps with a warning signal.

5.2 Permutation

Our second property formalises the intuition that if no observable behaviour due to P_1 depends on the behaviour of P_2 then in any trace it should be possible to move the actions associated with P_1 before all actions associated with P_2. Say $\mathcal{C}$ has the *permutation property* if whenever

$$A \vdash \mathcal{C}[P_1, P_2] \stackrel{\ell_1}{\Longrightarrow} \ldots \stackrel{\ell_k}{\Longrightarrow} P$$

with $\ell_i \neq \tau$ there exists a permutation π of $\{1, \ldots, k\}$ such that

$$A \vdash \mathcal{C}[P_1, P_2] \stackrel{\ell_{\pi(1)}}{\Longrightarrow} \ldots \stackrel{\ell_{\pi(k)}}{\Longrightarrow} P$$

and no in_1 or out_1 transition occurs after any in_2 or out_2 transition in $\ell_{\pi(1)} \ldots \ell_{\pi(k)}$. For an example wrapper without this property, consider

$$\begin{array}{rcl} \mathcal{C}[\_{1}, \_{2}] & \stackrel{def}{=} & (\boldsymbol{\nu}\, a_1, a_2)\big(\, a_1[\_{1}] \mid a_2[\_{2}] \\ & & \quad \mid\, !\, in_2{}^{\uparrow} y.(\overline{in_2}^{a_2} y \mid !\, in_1{}^{\uparrow} y.\overline{in_1}^{a_1} y) \\ & & \quad \mid\, !\, out_1{}^{a_1} y.\overline{out_1}^{\uparrow} y \\ & & \quad \mid\, !\, out_2{}^{a_2} y.\overline{out_2}^{\uparrow} y \\ & & \quad \mid\, !\, mid^{a_1} y.\overline{mid}^{a_2} y\big) \end{array}$$

Here the in_1 messages are not forwarded until at least one in_2 input is received from the environment. Nonetheless, in some sense there is still no information flow from the second component to the first.

The new-name directionality and permutation properties are expressed purely in terms of the externally observable behaviour of $\mathcal{C}[P, Q]$ (in fact, they are properties of its trace set, a very extensional semantics). Note, however, that the intuitive statement that information does not flow from Q to P depends on an understanding of the internal computation of P and Q that is not present in the reduction or labelled transition relations (given only that $\mathcal{C}[P, Q] \rightarrow^* R$ there is no way to associate subterms of R with an 'origin' in $\mathcal{C}$, P or Q). Our next two properties involve a more intensional semantics in which output and input processes are tagged with sets of colours. The semantics propagates colours in interaction steps, thereby tracking the dependencies of reductions.

5.3 Coloured Reductions

Take a set col of colours (disjoint from $\mathcal{N}$), and let c and d range over subsets of col. We define a coloured box-π calculus by annotating all outputs and inputs with sets of colours:

$$\begin{array}{rcl} P & ::= & \mathsf{c}{:}\overline{x}^{o} v \;\big|\; \mathsf{c}{:}x^{\iota} p.P \;\big|\; \mathsf{c}{:}!\, x^{\iota} p.P \;\big| \\ & & n[P] \;\big|\; 0 \;\big|\; P \mid P' \;\big|\; (\boldsymbol{\nu}\, x)P \end{array}$$

If P is a coloured term we write $|P|$ for the term of the original syntax obtained by erasing all annotations. Conversely, for a term P of the original syntax $\mathsf{c} \circ P$ denotes the term with every particle coloured by c. For a coloured P we write $\mathsf{c} \bullet P$ for the coloured term which is as P but with c unioned to every set of colours occurring in it. We write cd for the union $\mathsf{c} \cup \mathsf{d}$. The reduction relation now takes the form $P \rightarrow_{\mathsf{c}} Q$, where P and Q are coloured terms and c is a set of colours indicating what this reduction depends upon. It is defined as follows, in which structural congruence is defined by the same axioms as before.

$$
\begin{array}{ll}
n[\mathsf{c}\!:\!\overline{x}^{\uparrow}v \mid Q] \rightarrow_{\mathsf{c}} \mathsf{c}\!:\!\overline{x}^{n}v \mid n[Q] & \text{(C Red Up)}\\
\mathsf{c}\!:\!\overline{x}^{n}v \mid n[Q] \rightarrow_{\mathsf{c}} n[\mathsf{c}\!:\!\overline{x}^{\uparrow}v \mid Q] & \text{(C Red Down)}\\
\mathsf{c}\!:\!\overline{x}^{\iota}v \mid \mathsf{d}\!:\!x^{\iota}p.P \rightarrow_{\mathsf{cd}} \mathsf{cd}\bullet(\{^{v}/_{p}\}P) & \text{(C Red Comm)}\\
\mathsf{c}\!:\!\overline{x}^{\iota}v \mid \mathsf{d}\!:\,!\,x^{\iota}p.P \rightarrow_{\mathsf{cd}} \mathsf{d}\!:\,!\,x^{\iota}p.P \mid \mathsf{cd}\bullet(\{^{v}/_{p}\}P) & \text{(C Red Repl)}\\
P \rightarrow_{\mathsf{c}} Q \;\Rightarrow\; P \mid R \rightarrow_{\mathsf{c}} Q \mid R & \text{(C Red Par)}\\
P \rightarrow_{\mathsf{c}} Q \;\Rightarrow\; (\boldsymbol{\nu}\, x)P \rightarrow_{\mathsf{c}} (\boldsymbol{\nu}\, x)Q & \text{(C Red Res)}\\
P \rightarrow_{\mathsf{c}} Q \;\Rightarrow\; n[P] \rightarrow_{\mathsf{c}} n[Q] & \text{(C Red Box)}\\
P \equiv P' \rightarrow_{\mathsf{c}} Q' \equiv Q \;\Rightarrow\; P \rightarrow_{\mathsf{c}} Q & \text{(C Red Struct)}
\end{array}
$$

The coloured calculus has the same essential behaviour as the original calculus:

Proposition 9 *For any coloured P we have $|P| \rightarrow Q$ iff $\exists \mathsf{c}, P' \,.\, P \rightarrow_{\mathsf{c}} P' \wedge |P'| = Q$.*

Mediation We can now capture the intuition that all interaction between wrapped components should be mediated by the wrapper. We consider coloured reduction sequences of a wrapper $\mathcal{C}$ and two components P_1, P_2 from an initial state in which each is coloured differently. Let gr, bl and rd be distinct singleton subsets $\{\mathsf{green}\}$, $\{\mathsf{blue}\}$, $\{\mathsf{red}\}$ of col. Suppose

$$(\mathsf{gr} \circ \mathcal{C})\big[\mathsf{bl} \circ P_1,\, \mathsf{rd} \circ P_2\big] \mid \mathsf{bl} \circ I_1 \mid \mathsf{rd} \circ I_2 \rightarrow_{\mathsf{c}_1} \ldots \rightarrow_{\mathsf{c}_k} Q$$

where each I_i is a parallel composition of messages on in_i, i.e. of terms of the form $\overline{in_i}^{\uparrow}v$. Say $\mathcal{C}$ is *mediating* iff whenever $\mathsf{red} \in \mathsf{c}_j$ and $\mathsf{blue} \in \mathsf{c}_j$ then $\mathsf{green} \in \mathsf{c}_j$.

Colour flow The coloured semantics can also be used to express the property that no output on out_1 should depend on the second wrapped component. Say $\mathcal{C}$ has the *colour directionality* property if whenever there is a reduction sequence as above and $Q \equiv (\boldsymbol{\nu}\, A)(\mathsf{c}\!:\!\overline{out_1}^{\uparrow}v \mid Q')$ then $\mathsf{red} \notin \mathsf{c}$.

For an example wrapper that we conjecture has the permutation property but not the colour directionality property, consider a version of $\mathcal{W}_2$ that has an extra parallel component $out_2{}^{a_2}y.(\overline{out_2}^{\uparrow}y \mid out_1{}^{a_1}y.\overline{out_1}^{\uparrow}y)$. This establishes an additional one-shot forwarder for out_1 after forwarding a message on out_2.

These statements of mediation and coloured directionality share a defect: the use of a reduction semantics makes it awkward to consider inputs of values containing new names that have previously been output by the wrapped components. To address this one would need a coloured labelled transition semantics, allowing e.g. a refined colour directionality property to be stated as follows. Whenever

$$A \vdash (\mathsf{gr} \circ \mathcal{C})\big[\mathsf{bl} \circ P_1,\, \mathsf{rd} \circ P_2\big] \xrightarrow{\ell_1}_{\mathsf{c}_1} \ldots \xrightarrow{\ell_k}_{\mathsf{c}_k},$$

if the inputs are properly coloured (i.e. for each $i \in 1..k$ we have $\ell_i = in_1^{\uparrow}v \implies \mathsf{c}_i = \mathsf{blue}$ and $\ell_i = in_2^{\uparrow}v \implies \mathsf{c}_i = \mathsf{red}$), then for each $i \in 1..k$ the out_1 outputs should be properly coloured, i.e.

$$\ell_i = \overline{out_1}^{\uparrow}v \implies \mathsf{red} \notin \mathsf{c}_i$$

Causality A very strong directionality property that one might ask for – perhaps the strongest – would be that in an execution of $\mathcal{C}[P_1, P_2]$ no output on out_1 can be *causally dependent* on any action of P_2. Casual semantics for process calculi have been much studied, often under the name 'true concurrency semantics' – see [WN95] for an overview. It would be interesting to give a causal semantics to the box π calculus. There is a trade-off here, however – such a semantics would be rather complex; it would have to be understood in order to understand any property stated using it. The coloured reduction semantics can be considered as an more tractable approximation to real causality.

Another point is that a causal property is sometimes too strong – a usable wrapper may have to allow low-bandwidth communication in the reverse direction, perhaps not carrying any data values, to permit acknowledgement messages. A causal property would then not hold, while a modified colour flow property would.

6 Conclusion

The code base of modern systems is becoming increasingly diverse. Whereas previously a typical system would involve a small number of monolithic applications, obtained from trusted organisations, now users routinely download components from partially trusted or untrusted sources. Downloaded or mobile code fragments are commonly run under the user's authority to grant access to system resources and permit interaction with other software components. This presents obvious security risks for the secrecy and integrity of the user's data.

In this paper we have developed a theory of security wrappers. These are small programs that can regulate the interactions between untrusted software components, enforcing dynamic and flexible security policies. We have presented a minimal concurrent programming language for studying the problem, the box-π calculus, and proved a basic metatheoretic result: that a reduction and labelled transition semantics coincide. We have expressed a number of security wrappers in the calculus and begun an investigation of the security properties that wrappers should provide.

6.1 Related Work

There is an extensive literature on information flow properties of various kinds. Much of it is in the context of multi-level security, in which one has a fixed lattice of security levels and is concerned with properties which state that a component (expressed purely semantically, e.g. as a set of traces) respects the levels. The theory could be applied during the design of the components of a large multi-user system (with a relatively static security policy) by proving that the components obey particular properties. A concise introduction can be found in the survey of McLean [McL94]. The problem of designing and understanding wrappers appears to be rather different – we have focussed on the protection required by a single user executing a variety of partially-trusted components obtained from third parties. This requires flexible protection mechanisms – a static assignment of security levels would be inadequate – and cannot depend on static analysis of the components. Related work on dynamic enforcement of policies has been presented by Schneider [Sch98].

Other recent work has studied type systems that ensure security properties, e.g. the type systems of Volpano, Irvine and Smith [VIS96, VS98], the SLam calculus of Heintze and Riecke [HR98a], the systems allowing declassification of Myers and Liskov [ML98, Mye99], the type systems of Riely and Hennessy [HR98c, HR98b, RH98], and work on proof-carrying code [NL98]. If the producers of components that one uses all adopt such systems then they may become very effective. Until then, however, and until type systems can provide the flexible policies required, partially trusted code will in practice either be run dangerously or be wrapped.

In this paper we have made extensive use of techniques from process calculi and operational semantics. These are beginning to provide fruitful ways of studying problems in security and distributed systems, including the analysis of security protocols, for example in [AG97, Aba97, LR97], and more general secure language design, including work on the Ambient calculus [CG98, CG99], the Secure Join calculus [AFG98], the mobile agent calculi in [HR98c, HR98b, RH98, Sew97, Sew98, SWP98a, SWP98b], and the Seal calculus of [VC98, VC99]. These works have studied several different problems, using a variety of calculi designed for the purpose. Common to all is the use of a reduction or labelled-transition operational semantics, providing clear rigorous semantics to the rather high-level constructs involved. One distinguishing feature of the present work is that we do not consider any mobility primitives, allowing us to use a tractable early labelled transition system. This appears to be important for the statement of the delicate security properties of wrappers.

6.2 Future Directions

This paper opens up a number of directions that we would like to pursue. Most immediately, it gives several conjectures that should be proved or refuted, and we would like a better understanding of the properties of binary wrappers. There are then extensions for typing, to richer interfaces, and with mobility primitives.

Typing We are primarily interested in components for which it is infeasible to statically determine whether they are well-behaved. Nonetheless, for simple components one could conservatively ensure well-behaviour with a standard type system, most simply taking types

$$T \quad ::= \quad \mathbf{box} \;\big|\; \langle T_1..T_k \rangle \;\big|\; \updownarrow T$$

where $\updownarrow T$ is the type of channel names that can be used to communicate values of type T, together with the obvious inference rules. If P is well-typed with respect to a typing context $in : \updownarrow S$, $out : \updownarrow T$ for types S and T containing no instances of $\updownarrow$ then one would expect P to be well-behaved for unary wrappers.

Richer interfaces The wrappers of §3 allowed the encapsulated components to interact only on very simple interfaces. Ultimately, we would like to understand wrappers with more realistic interfaces. For example, in a mild extension of box-π one can express a wrapper that encapsulates k components, allows internal flow along an arbitrary preorder, and permits each component to open and close windows for character IO. Suppose $p_1, \ldots, p_k$ is a list of distinct names, and $\geq$ is a preorder over them giving the allowable information flow. Define a k-ary wrapper as follows.

$$\begin{array}{lcll} \mathcal{C}[\__1, \ldots, \__k] & \stackrel{def}{=} & (\boldsymbol{\nu}\, p_1, \ldots, p_k)(& p_1[\__1] \mid \ldots \mid p_k[\__k] \\ & & & \mid \mathbf{!}\, fwd^{(m)}(n\, z\, y).\mathbf{if}\ m \geq n\ \mathbf{then}\ \overline{z}^n y \text{ else } 0 \\ & & & \mid \text{BW}\textsc{indow}) \end{array}$$

where

$$
\begin{array}{lll}
\textsc{BWindow} & \stackrel{def}{=} & \mathbf{!}\, \mathit{openwindow}^{(m)}(s\,x). \\
& & \quad \overline{\mathit{openwindow}}^{\uparrow}\langle s\,x\rangle \\
& & \quad \mid x^{\uparrow}(\mathit{getc}\ \mathit{putc}\ \mathit{close}). \\
& & \qquad \overline{x}^{m}\langle \mathit{getc}\ \mathit{putc}\ \mathit{close}\rangle \\
& & \qquad \mid \mathbf{!}\, \mathit{getc}^{m} y.(\overline{\mathit{getc}}^{\uparrow} y \mid y^{\uparrow} c.\overline{y}^{m} c) \\
& & \qquad \mid \mathbf{!}\, \mathit{putc}^{m}(c\,y).(\overline{\mathit{putc}}^{\uparrow}\langle c\,y\rangle \mid y^{\uparrow}.\overline{y}^{m}) \\
& & \qquad \mid \mathbf{!}\, \mathit{close}^{m} y.(\overline{\mathit{close}}^{\uparrow} y \mid y^{\uparrow}.\overline{y}^{m})
\end{array}
$$

This uses an additional input tag – a process $x^{(n)}p.P$ will input from any child box, binding the name of the box to n in P. The BWINDOW part of $\mathcal{C}$ receives requests for a new window from the encapsulated components and forwards them to the OS. It then receives the interface for the new window from the OS, forwarding it down to the component and also setting up forwarders for the interface channels. Making the security properties of $\mathcal{C}$ precise is at present a challenging problem. One would like to extend $\mathcal{C}$ further by adding an interface allowing the user to dynamically add and remove pairs from $\geq$.

Covert channels It should be noted that none of the semantic models that we use for the box-π calculus make any commitment to the precise details of scheduling processes. The properties expressed using these semantics therefore cannot address timing-based covert channels such as those mentioned by Lampson [Lam73]. Certain other covert channels, in particular those involving system IO and disc access, could be addressed by expressing models of the IO and disc systems in the calculus, further enriching the wrapper interfaces.

Mobility The original motivation for this work involved downloadable or mobile code and mobile agents. To explicitly model the dynamic configuration of wrappers and applications the calculus must be extended with mobility primitives, while keeping both a tractable semantics and the principle that each box controls the interactions and movements of its contents [VC98].

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A Coincidence of the Two Semantics

This appendix contains the proof of equivalence of the labelled transition semantics and the reduction semantics. It is divided into three parts, the first giving basic properties of the labelled transition system, the second showing that any reduction can be matched by a τ-transition and the third showing the converse.

Basic Properties of the LTS

Lemma 10 *If $P \equiv Q$ then $\mathrm{fn}(P) = \mathrm{fn}(Q)$.*

Proof Routine induction on derivation of $P \equiv Q$. $\square$

Lemma 11 *If $A \vdash P \xrightarrow{\ell} Q$ then*

1. $\mathrm{fn}(P) \subseteq A$
2. $\mathrm{fn}(Q) \subseteq \mathrm{fn}(P, \ell)$
3. *if* $\ell = \overline{x}^o v$ *then* $\mathrm{fn}(\ell) \cap A \subseteq \mathrm{fn}(P)$
4. *if* $\ell = \overline{x}^o v$ *then* $\mathrm{fn}(o) \subseteq \mathrm{fn}(P)$
5. *if* $\ell = \overline{x}^o v$ *and* $\neg \mathrm{mv}(o)$ *then* $x \in \mathrm{fn}(P)$
6. *if* $\ell = x^\gamma v$ *then* $\mathrm{fn}(\gamma) \subseteq \mathrm{fn}(P)$.
7. *if* $\ell = x^\gamma v$ *and* $\gamma \neq \overline{n}$ *then* $x \in \mathrm{fn}(P)$.

Proof By induction on the derivation of $A \vdash P \xrightarrow{\ell} Q$. Part 1 is immediate in all cases by the implicit condition. For the other parts:

(Trans Out) By the condition $\mathrm{fn}(\overline{x}^o v) \subseteq A$.

(Trans In) For Part 2, $\mathrm{fn}(\{{}^v\!/_p\}P) \subseteq (\mathrm{fn}(P) - \mathrm{fn}(p)) \cup \mathrm{fn}(v) \subseteq \mathrm{fn}(x^\iota p.P) \cup \mathrm{fn}(x^\iota v)$. For Parts 6 and 7, $\mathrm{fn}(x, \iota) \subseteq \mathrm{fn}(x^\iota p.P)$. All other parts do not apply.

(Trans Repl) For Part 2, $\mathrm{fn}(\boldsymbol{!}\, x^\iota p.P \mid \{{}^v\!/_p\}P) \subseteq \mathrm{fn}(\boldsymbol{!}\, x^\iota p.P) \cup (\mathrm{fn}(P) - \mathrm{fn}(p)) \cup \mathrm{fn}(v) \subseteq \mathrm{fn}(\boldsymbol{!}\, x^\iota p.P) \cup \mathrm{fn}(x^\iota v)$. For Part 6 and 7, $\mathrm{fn}(x, \iota) \subseteq \mathrm{fn}(\boldsymbol{!}\, x^\iota p.P)$. All other parts do not apply.

(Trans Box-1) We have $\ell = \tau$. For Part 2:

$$
\begin{array}{lll}
 & \mathrm{fn}((\boldsymbol{\nu}\, \mathrm{fn}(x,v) - A)(\overline{x}^{\overline{n}} v \mid n[P'])) & \\
= & (\mathrm{fn}(\overline{x}^{\overline{n}} v) \cup \{n\} \cup \mathrm{fn}(P')) - (\mathrm{fn}(x,v) - A) & \text{(by definition of fn)} \\
\subseteq & (\mathrm{fn}(\overline{x}^{\overline{n}} v) \cup \{n\} \cup \mathrm{fn}(P) \cup \mathrm{fn}(\overline{x}^{\uparrow} v)) - (\mathrm{fn}(x,v) - A) & \text{(by ind. hyp., part 2)} \\
\subseteq & (\mathrm{fn}(\overline{x}^{\overline{n}} v) \cup \mathrm{fn}(n[P])) - (\mathrm{fn}(x,v) - A) & \\
\subseteq & \mathrm{fn}(n[P]) & \text{(by ind. hyp., part 3)} \\
= & \mathrm{fn}(n[P], \tau) &
\end{array}
$$

All other parts do not apply.

(Trans Box-2) We have $\ell = x^{\overline{n}} v$. For Part 2: $\mathrm{fn}(n[\overline{x}^{\uparrow} v \mid P]) = \mathrm{fn}(n[P]) \cup \mathrm{fn}(\overline{x}^{\uparrow} v) \subseteq \mathrm{fn}(n[P]) \cup \mathrm{fn}(x^{\overline{n}} v)$. For Part 6 note that $n \in \mathrm{fn}(n[P])$. All other parts do not apply.

(Trans Box-3) For Part 2, by the induction hypothesis $\mathrm{fn}(P') \subseteq \mathrm{fn}(P)$ so $\mathrm{fn}(n[P']) \subseteq \mathrm{fn}(n[P])$. All other parts do not apply.

(Trans Par) By the induction hypothesis.

(Trans Comm) Part 2 is by parts 2, 4 and 6 of the induction hypothesis. All other parts do not apply.

(Trans Res-1) By the induction hypothesis.

(Trans Res-2) For Part 2, by Part 2 of the induction hypothesis $\mathrm{fn}(P') \subseteq \mathrm{fn}(P) \cup \mathrm{fn}(\overline{y}^o v)$. As $x \in \mathrm{fn}(\overline{y}^o v)$ we have $\mathrm{fn}(P') \subseteq \mathrm{fn}((\boldsymbol{\nu}\, x)P) \cup \mathrm{fn}(\overline{y}^o v)$. For Part 3, by the induction hypothesis $\mathrm{fn}(\overline{y}^o v) \cap (A, x) \subseteq \mathrm{fn}(P)$ so $\mathrm{fn}(\overline{y}^o v) \cap A \subseteq \mathrm{fn}((\boldsymbol{\nu}\, x)P)$. For Part 4, by the induction hypothesis $\mathrm{fn}(o) \subseteq \mathrm{fn}(P)$ and by the side condition $x \neq o$ so $\mathrm{fn}(o) \subseteq \mathrm{fn}((\boldsymbol{\nu}\, x)P)$. For Part 5, if $\neg\mathrm{mv}(o)$ then by the induction hypothesis $y \in \mathrm{fn}(P)$ and by the side condition $x \neq y$ so $y \in \mathrm{fn}((\boldsymbol{\nu}\, x)P)$. All other parts do not apply.

(Trans Struct Right) By the induction hypothesis and Lemma 10.

□

Lemma 12 (Strengthening) *If $A, B \vdash P \xrightarrow{\ell} P'$ and $B \cap \mathrm{fn}(P, \ell) = \emptyset$ then $A \vdash P \xrightarrow{\ell} P'$.*

Proof Induction on derivations of transitions.

(Out), (In), (Repl), (Box-2) All immediate.

(Box-3),(Par),(Struct Right) Straightforward use of the induction hypothesis.

(Comm) We have a rule instance of the form

$$\frac{A, B \vdash P \xrightarrow{\overline{x}^{\gamma} v} P' \quad A, B \vdash Q \xrightarrow{x^{\gamma} v} Q'}{A, B \vdash P \mid Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - (A, B))(P' \mid Q')} \ (\mathrm{Comm})$$

By Lemma 11.3 $\mathrm{fn}(\overline{x}^{\gamma} v) \cap (A, B) \subseteq \mathrm{fn}(P)$ and by assumption $B \cap \mathrm{fn}(P) = \emptyset$ so $\mathrm{fn}(\overline{x}^{\gamma} v) \cap B = \emptyset$. By the induction hypothesis and (Comm) we then have $A \vdash P \mid Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - A)(P' \mid Q')$, but $\mathrm{fn}(x, v) - A = \mathrm{fn}(x, v) - (A, B)$, so $A \vdash P \mid Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - (A, B))(P' \mid Q')$ as required.

(Box-1) Similar to (Comm). In detail: we have a rule instance of the form

$$\frac{A, B \vdash P \xrightarrow{\overline{x}^{\uparrow} v} P'}{A, B \vdash n[P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - (A, B))(\overline{x}^{n} v \mid n[P'])} \ (\text{Box-1})$$

By Lemma 11.3 $\mathrm{fn}(\overline{x}^{\uparrow} v) \cap (A, B) \subseteq \mathrm{fn}(P)$ and by assumption $B \cap \mathrm{fn}(P) = \emptyset$ so $\mathrm{fn}(\overline{x}^{\uparrow} v) \cap B = \emptyset$. By the induction hypothesis and (Box-1) we then have $A \vdash n[P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - (A))(\overline{x}^{n} v \mid n[P'])$ but $\mathrm{fn}(x, v) - A = \mathrm{fn}(x, v) - (A, B)$, so $A \vdash n[P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - (A, B))(\overline{x}^{n} v \mid n[P'])$ as required.

(Res-1) We have a rule instance of the form

$$\frac{A, B, x \vdash P \xrightarrow{\ell} P'}{A, B \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} (\boldsymbol{\nu}\, x)P'} \quad \text{(Res-1)}$$

with $x \notin \text{fn}(\ell)$. By A, B, x well-formed we have $x \notin B$, so $B \cap \text{fn}((\boldsymbol{\nu}\, x)P) = \emptyset$ implies $B \cap \text{fn}(P) = \emptyset$. By the induction hypothesis $A, x \vdash P \xrightarrow{\ell} P'$ so by (Res-1) $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} (\boldsymbol{\nu}\, x)P'$.

(Res-2) Similar to (Res-1), noting that the sidecondition is a predicate on x and the label only.

□

Lemma 13 (Injective Substitution) *If $A \vdash P \xrightarrow{\ell} P'$, and $f : A \to B$ and $g : (\text{fn}(\ell) - A) \to (\mathcal{N} - B)$ are injective, then $B \vdash fP \xrightarrow{(f+g)\ell} (f+g)P'$.*

Proof Induction on derivations of transitions.

(Out),(Box-1) immediate.

(Box-3),(Par),(Struct Right) Straightforward uses of the induction hypothesis.

(In) Consider $A \vdash x^\iota p.P \xrightarrow{x^\iota v} \{^v/_p\}P$. We have $\text{fn}(x^\iota p.P) \subseteq A$ and $\{^v/_p\}P$ well defined. Take some $\hat{p}$ and $\hat{P}$ such that $x^\iota p.P = x^\iota \hat{p}.\hat{P}$ and $\text{n}(\hat{p}) \cap (A \cup B \cup (\text{fn}(\ell) - A) \cup \text{ran}(g)) = \emptyset$, then $f(x^\iota p.P) = f(x^\iota \hat{p}.\hat{P}) = f(x)^{f(\iota)}\hat{p}.f(\hat{P})$ and $\text{fn}(f(x)^{f(\iota)}\hat{p}.f(\hat{P})) \subseteq B$.

We have $\{^v/_{\hat{p}}\}\hat{P}$ defined, hence $\{^v/_{\hat{p}}\}(f\hat{P})$ is defined (as $\text{n}(\hat{p}) \cap (\text{dom}(f) \cup \text{ran}(f)) = \emptyset$), hence $\{^{(f+g)v}/_{\hat{p}}\}(f\hat{P})$ is defined (as $(f+g)v$ and v are the same shape).

By (In) $B \vdash f(x)^{f(\iota)}\hat{p}.f(\hat{P}) \xrightarrow{f(x)^{f(\iota)}(f+g)v} \{^{(f+g)v}/_{\hat{p}}\}f\hat{P}$.

Now $\text{fn}(\hat{P}) \subseteq A \cup \text{n}(\hat{p})$ so $\text{fn}(\hat{P}) \cap \text{dom}(g) = \emptyset$, so $f\hat{P} = (f+g)\hat{P}$. Hence $\{^{(f+g)v}/_{\hat{p}}\}f\hat{P} = \{^{(f+g)v}/_{\hat{p}}\}(f+g)\hat{P} = (f+g)(\{^v/_{\hat{p}}\}\hat{P}) = (f+g)(\{^v/_p\}P)$, so $B \vdash f(x^\iota p.P) \xrightarrow{(f+g)x^\iota v} (f+g)(\{^v/_p\}P)$.

(Repl) Similar to (In), using in addition that $f(!\, x^\iota p.P) = (f+g)(!\, x^\iota p.P)$.

(Comm) $\text{fn}(\tau) = \emptyset$, so we have $f : A \to B$ and $g : \emptyset \to (\mathcal{N} - B)$. Take some $\hat{g} : (\text{fn}(\overline{x}^{\overline{\gamma}} v) - A) \to (\mathcal{N} - B)$ injective. By the induction hypothesis and (Comm) we have

$$\frac{B \vdash fP \xrightarrow{(f+\hat{g})(\overline{x}^{\overline{\gamma}} v)} (f+\hat{g})P' \quad B \vdash fQ \xrightarrow{(f+\hat{g})(x^{\gamma} v)} (f+\hat{g})Q'}{B \vdash f(P \mid Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \text{fn}((f+\hat{g})x, (f+\hat{g})v) - B)((f+\hat{g})(P' \mid Q'))} \quad \text{(Comm)}$$

Now by Lemma 11.(4,1) $\text{fn}(\overline{\gamma}) \subseteq A$, so $\text{dom}(\hat{g}) = \text{fn}(x, v) - A$ and $\text{ran}(\hat{g}) = \text{fn}((f+\hat{g})x, (f+\hat{g})v) - B$, so $B \vdash f(P \mid Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \text{ran}(\hat{g}))((f+\hat{g})(P' \mid Q'))$. We have $f((\boldsymbol{\nu}\, \text{dom}(\hat{g}))(P' \mid Q')) = ((\boldsymbol{\nu}\, \text{ran}(\hat{g}))(f+\hat{g})(P' \mid Q'))$, so $B \vdash f(P \mid Q) \xrightarrow{\tau} f((\boldsymbol{\nu}\, \text{fn}(x, v) - A)(P' \mid Q'))$.

(Box-1) Again similar to (Comm). $\mathrm{fn}(\tau)=\emptyset$, so we have $f: A \to B$ and $g: \emptyset \to (\mathcal{N}-B)$. Take some $\hat{g}:(\mathrm{fn}(\overline{x}^{\uparrow}v)-A)\to(\mathcal{N}-B)$ injective. By the induction hypothesis and (Box-1) we have

$$\frac{B \vdash fP \stackrel{(f+\hat{g})(\overline{x}^{\uparrow}v)}{\longrightarrow} (f+\hat{g})P'}{B \vdash f(n)[fP] \stackrel{\tau}{\longrightarrow} (\boldsymbol{\nu}\,\mathrm{fn}((f+\hat{g})x,(f+\hat{g})v)-B)((f+\hat{g})(\overline{x}^{n}v \mid n[P']))}\ \text{(Box-1)}$$

using $f(n)=(f+\hat{g})(n)$. It follows that $B \vdash f(n[P]) \stackrel{\tau}{\longrightarrow} f((\boldsymbol{\nu}\,\mathrm{fn}(x,v)-A)(\overline{x}^{n}v \mid n[P']))$.

(Res-1) Take some $\hat{x} \notin B \cup \mathrm{ran}(g)$ and define $\hat{f}:(A,x)\to(B,\hat{x})$ by

$$\begin{array}{rcl} \hat{f}(x) & = & \hat{x} \\ \hat{f}(z) & = & f(z), \text{ for } z \in A. \end{array}$$

By the induction hypothesis $B,\hat{x} \vdash \hat{f}P \stackrel{(\hat{f}+g)\ell}{\longrightarrow} (\hat{f}+g)P'$. By (Res-1) $B \vdash (\boldsymbol{\nu}\,\hat{x})\hat{f}P \stackrel{(\hat{f}+g)\ell}{\longrightarrow} (\boldsymbol{\nu}\,\hat{x})(\hat{f}+g)P'$, so $B \vdash f((\boldsymbol{\nu}\,x)P) \stackrel{(f+g)\ell}{\longrightarrow} (f+g)(\boldsymbol{\nu}\,x)P'$.

(Res-2) Define $\hat{f}:(A,x)\to(B,g(x))$ and $\hat{g}$ as $f+(x\mapsto g(x))$ and $g \downharpoonleft (\mathrm{fn}(\overline{y}^{o}v)-(A,x))$ respectively. By the induction hypothesis $B,g(x) \vdash \hat{f}P \stackrel{(\hat{f}+\hat{g})\overline{y}^{o}v}{\longrightarrow} (\hat{f}+\hat{g})P'$, so by (Res-2) $B \vdash (\boldsymbol{\nu}\,g(x))\hat{f}P \stackrel{(\hat{f}+\hat{g})\overline{y}^{o}v}{\longrightarrow} (\hat{f}+\hat{g})P'$, so as $f+g=\hat{f}+\hat{g}$ we have $B \vdash f((\boldsymbol{\nu}\,x)P) \stackrel{(f+g)\overline{y}^{o}v}{\longrightarrow} (f+g)P'$.

$\square$

Lemma 14 (Weakening and Strengthening) *$(A \vdash P \stackrel{\ell}{\longrightarrow} P' \wedge x \notin A \cup \mathrm{fn}(\ell))$ iff $(A,x \vdash P \stackrel{\ell}{\longrightarrow} P' \wedge x \notin \mathrm{fn}(P,\ell))$.*

Proof The right-to-left implication follows from the well-formedness of A,x and from Lemma 12. The left-to-right implication follows from the condition $\mathrm{fn}(P)\subseteq A$ in the definition of the transition rules and from Lemma 13, taking f to be the inclusion from A to A,x and g the identity on $\mathrm{fn}(\ell)-A$. $\square$

Lemma 15 (Shifting)

1. *$(A \vdash P \stackrel{z^{\iota}v}{\longrightarrow} P' \wedge x \in \mathrm{fn}(v)-A)$ iff $(A,x \vdash P \stackrel{z^{\iota}v}{\longrightarrow} P' \wedge x \in \mathrm{fn}(v)-\mathrm{fn}(P))$.*
2. *$(A \vdash P \stackrel{\overline{z}^{n}v}{\longrightarrow} P' \wedge x \in \mathrm{fn}(z,v)-A)$ iff $(A,x \vdash P \stackrel{\overline{z}^{n}v}{\longrightarrow} P' \wedge x \in \mathrm{fn}(z,v)-\mathrm{fn}(P))$*

Proof Each part is by two inductions on derivations of transitions. For the first:

(Out),(Box-1),(Box-2),(Box-3),(Comm),(Res-2) vacuous.

(Par),(Struct Right) Straightforward uses of the induction hypothesis.

(In),(Repl) Straightforward.

(Res-1) Consider

$$\frac{A,y \vdash P \xrightarrow{z^\iota v} P'}{A \vdash (\boldsymbol{\nu}\, y)P \xrightarrow{z^\iota v} (\boldsymbol{\nu}\, y)P'} \text{(Res-1)} \qquad \frac{A,x,y \vdash P \xrightarrow{z^\iota v} P'}{A,x \vdash (\boldsymbol{\nu}\, y)P \xrightarrow{z^\iota v} (\boldsymbol{\nu}\, y)P'} \text{(Res-1)}$$

$y \notin \text{fn}(z^\iota v)$
$x \in \text{fn}(v) - A$

$y \notin \text{fn}(z^\iota v)$
$x \in \text{fn}(v) - \text{fn}((\boldsymbol{\nu}\, y)P))$

For the left-to-right implication, note that $x \in \text{fn}(v) - (A,y)$, so by the induction hypothesis $A,y,x \vdash P \xrightarrow{z^\iota v} P'$ and $x \in \text{fn}(v) - \text{fn}(P)$. For the right-to-left implication, note that as A,x,y is well-formed we have $x \in \text{fn}(v) - \text{fn}(P)$, so by the induction hypothesis $A,y \vdash P \xrightarrow{z^\iota v} P'$ and $x \in \text{fn}(v) - (A,y)$.

For the second part:

(Out),(In),(Repl),(Box-1),(Box-3),(Comm),(Res-2) vacuous.

(Par),(Struct Right) Straightforward uses of the induction hypothesis.

(Box-2) Straightforward.

(Res-1) Similar to the (Res-1) case of the first part.

□

As we are working up to alpha conversion a little care is required when analysing transitions. We need the following lemma (of which only the input and restriction cases are at all interesting).

Lemma 16

1. $A \vdash \overline{x}^o v \xrightarrow{\ell} Q$ *iff* $\text{fn}(\overline{x}^o v) \subseteq A$, $\ell = \overline{x}^o v$ *and* $Q \equiv 0$.

2. $A \vdash x^\iota p.P \xrightarrow{\ell} Q$ *iff there exists* v *such that* $\text{fn}(x^\iota p.P) \subseteq A$, $\ell = x^\iota v$, $\{^v/_p\}P$ *is defined and* $Q \equiv \{^v/_p\}P$.

3. $A \vdash \mathbf{!}\, x^\iota p.P \xrightarrow{\ell} Q$ *iff there exists* v *such that* $\text{fn}(\mathbf{!}\, x^\iota p.P) \subseteq A$, $\ell = x^\iota v$, $\{^v/_p\}P$ *is defined and* $Q \equiv \mathbf{!}\, x^\iota p.P \mid \{^v/_p\}P$.

4. $A \vdash n[P] \xrightarrow{\ell} Q$ *iff one of the following hold.*

   (a) *there exist* x, v, *and* $\hat{P}$ *such that* $n \in A$, $\ell = \tau$, $A \vdash P \xrightarrow{\overline{x}^\uparrow v} \hat{P}$, *and* $Q \equiv (\boldsymbol{\nu}\, \text{fn}(x,v) - A)(\overline{x}^{\overline{n}} v \mid n[\hat{P}])$.

   (b) *there exist* x *and* v *such that* $\text{fn}(n[P]) \subseteq A$, $\ell = x^{\overline{n}} v$ *and* $Q \equiv n[\overline{x}^{\overline{\uparrow}} v \mid P]$.

   (c) *there exists* $\hat{P}$ *such that* $n \in A$, $\ell = \tau$, $A \vdash P \xrightarrow{\tau} \hat{P}$, *and* $Q \equiv n[\hat{P}]$.

5. $A \vdash P \mid Q \xrightarrow{\ell} R$ *iff either*

   (a) *there exists* $\hat{P}$ *such that* $\text{fn}(Q) \subseteq A$, $A \vdash P \xrightarrow{\ell} \hat{P}$ *and* $R \equiv \hat{P} \mid Q$.

   (b) *there exists* x, γ, v, $\hat{P}$ *and* $\hat{Q}$ *such that* $\ell = \tau$, $A \vdash P \xrightarrow{\overline{x}^{\overline{\gamma}} v} \hat{P}$, $A \vdash Q \xrightarrow{x^\gamma v} \hat{Q}$, *and* $R \equiv (\boldsymbol{\nu}\, \text{fn}(x,v) - A)(\hat{P} \mid \hat{Q})$.

or symmetric cases.

6. $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$ *iff either*

   (a) *there exists* $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(P) - x)$ *and* $\hat{Q}$ *such that* $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\ell} \hat{Q}$ *and* $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$.

   (b) *there exists* y*,* o*,* v*,* $\hat{Q}$ *and* $\hat{x} \notin A \cup \mathrm{fn}(y, o) \cup (\mathrm{fn}(P) - x)$ *such that* $\ell = \overline{y}^o v$*,* $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$*,* $\hat{x} \in \mathrm{fn}(v)$*,* $\neg \mathrm{mv}(o)$ *and* $Q \equiv \hat{Q}$.

   (c) *there exists* y*,* o*,* v*,* $\hat{Q}$ *and* $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(P) - x)$ *such that* $\ell = \overline{y}^o v$*,* $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$*,* $\hat{x} \in \mathrm{fn}(y, v)$*,* $\mathrm{mv}(o)$ *and* $Q \equiv \hat{Q}$.

Proof The right-to-left implications are all shown using a single transition rule together with (Trans Struct Right). The left-to-right implications are shown by induction on derivations of transitions. Only the input, replicated input and restriction cases are at all interesting; we give just the restriction case.

Case 6a, ($\Leftarrow$) By Lemma 11, $\mathrm{fn}(\{\hat{x}/_x\}P) \subseteq A, \hat{x}$, so we have $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P) \subseteq A$. By (Trans Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P \xrightarrow{\ell} (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By $\hat{x} \notin \mathrm{fn}(P) - x$ we have $(\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P = (\boldsymbol{\nu}\, x)P$. By (Trans Struct Right), $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$.

Case 6b, ($\Leftarrow$) Again by Proposition 11, $\mathrm{fn}(\{\hat{x}/_x\}P) \subseteq A, \hat{x}$, so we have $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P) \subseteq A$. By (Trans Res-2-nmv), $A \vdash (\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$. Again by $\hat{x} \notin \mathrm{fn}(P) - x$, we have $(\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P = (\boldsymbol{\nu}\, x)P$ so by (Trans Struct Right) $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$.

Case 6c, ($\Leftarrow$) Again by Proposition 11 $\mathrm{fn}(\{\hat{x}/_x\}P) \subseteq A, \hat{x}$, so $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P) \subseteq A$. By (Trans Res-2-mv) $A \vdash (\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P \xrightarrow{\overline{\hat{x}}^{\uparrow} v} \hat{Q}$. Again by $\hat{x} \notin \mathrm{fn}(P) - x$ we have $(\boldsymbol{\nu}\, \hat{x})\{\hat{x}/_x\}P = (\boldsymbol{\nu}\, x)P$ so by (Trans Struct Right) $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$.

Case 6, ($\Rightarrow$) Let $\Phi(A, R, \ell, Q) \overset{def}{\Leftrightarrow} R = (\boldsymbol{\nu}\, x)P \implies (a) \vee (b) \vee (c)$. We show Φ is closed under the rules defining labelled transitions.

(Trans Res-1) An instance of (Trans Res-1) with conclusion $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$ must be of the form

$$\frac{A, \hat{x} \vdash \hat{P} \xrightarrow{\ell} \hat{Q}}{A \vdash (\boldsymbol{\nu}\, \hat{x})\hat{P} \xrightarrow{\ell} (\boldsymbol{\nu}\, \hat{x})\hat{Q}} \;\; \hat{x} \notin \mathrm{fn}(\ell) \text{ (Trans Res-1)}$$

for some $\hat{x}$, $\hat{P}$, $\hat{Q}$ with $(\boldsymbol{\nu}\, \hat{x})\hat{P} = (\boldsymbol{\nu}\, x)P$, $(\boldsymbol{\nu}\, \hat{x})\hat{Q} = Q$ and $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\hat{P}) \subseteq A$. By $A, \hat{x}$ defined and $\hat{x} \notin \mathrm{fn}(\ell)$ we have $\hat{x} \notin A \cup \mathrm{fn}(\ell)$. By $(\boldsymbol{\nu}\, \hat{x})\hat{P} = (\boldsymbol{\nu}\, x)P$ we have $\hat{x} \notin \mathrm{fn}(P) - x$ and $\hat{P} = \{\hat{x}/_x\}P$, so $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\ell} \hat{Q}$. By reflexivity of $\equiv$, we have $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. So clause 6a holds.

(Trans Res-2-nmv) An instance of (Trans Res-2-nmv) with the conclusion $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$ must be of the form

$$\frac{A, \hat{x} \vdash \hat{P} \xrightarrow{\overline{y}^o v} Q}{A \vdash (\boldsymbol{\nu}\, \hat{x})\hat{P} \xrightarrow{\overline{y}^o v} Q} \;\; \neg \mathrm{mv}(o) \wedge \hat{x} \in \mathrm{fn}(v) - \mathrm{fn}(y, o) \text{ (Trans Res-2-nmv)}$$

for some $\hat{x}$, $\hat{P}$, y, o, v with $(\boldsymbol{\nu}\, \hat{x})\hat{P} = (\boldsymbol{\nu}\, x)P$, $\overline{y}^o v = \ell$ and $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\hat{P}) \subseteq A$. As before $\hat{x} \notin A \cup (\mathrm{fn}(P) - x)$ and $\hat{P} = \{\hat{x}/_x\}P$, so taking $\hat{Q} = Q$ clause 6b holds.

(Trans Res-2-mv) An instance of (Trans Res-2-mv) with the conclusion $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$ must be of the form

$$\frac{A, \hat{x} \vdash \hat{P} \xrightarrow{\overline{y}^o v} Q}{A \vdash (\boldsymbol{\nu}\, \hat{x})\hat{P} \xrightarrow{\overline{y}^o v} Q} \ \mathrm{mv}(o) \wedge \hat{x} \in \mathrm{fn}(y, v) - \mathrm{fn}(o) \ \text{(Trans Res-2-mv)}$$

for some $\hat{x}$, $\hat{P}$, y, o, v with $(\boldsymbol{\nu}\, \hat{x})\hat{P} = (\boldsymbol{\nu}\, x)P$, $\overline{y}^o v = \ell$ and $\mathrm{fn}((\boldsymbol{\nu}\, \hat{x})\hat{P}) \subseteq A$. As before $\hat{x} \notin A \cup (\mathrm{fn}(P) - x)$ and $\hat{P} = \{\hat{x}/_x\}P$, so taking $\hat{Q} = Q$ clause 6c holds.

(Trans Struct Right) An instance of (Trans Struct Right) with conclusion $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$ must be of the form

$$\frac{A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q' \quad Q' \equiv Q}{A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q} \ \text{(Trans Struct Right)}$$

for some Q' with $\mathrm{fn}((\boldsymbol{\nu}\, x)P) \subseteq A$. By $\Phi(A, (\boldsymbol{\nu}\, x)P, \ell, Q')$ either

Case 6a there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(P) - x)$ and $\hat{Q}$ such that $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\ell} \hat{Q}$ and $Q' \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By $\equiv$ an equivalence we have $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$, so clause 6a holds.

Case 6b there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(y, o) \cup (\mathrm{fn}(P) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(o)$ and $Q' \equiv \hat{Q}$. By $\equiv$ an equivalence we have $Q \equiv \hat{Q}$, so clause 6b holds.

Case 6c there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(P) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{\hat{x}/_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(y, v)$, $\mathrm{mv}(o)$ and $Q' \equiv \hat{Q}$. By $\equiv$ an equivalence we have $Q \equiv \hat{Q}$, so clause 6c holds.

The cases for all other rules are vacuous.

□

Reductions Imply Transitions

Take the *size* of a derivation of a structural congruence to be number of instances of inference rules contained in it.

Lemma 17 *If $P' \equiv P$ and $\{v/_p\}P$ is defined then $\{v/_p\}P'$ is defined and $\{v/_p\}P' \equiv \{v/_p\}P$. Moreover, for any derivation of $P' \equiv P$ there is a derivation of the same size of $\{v/_p\}P' \equiv \{v/_p\}P$.*

Proof Obvious. □

Proposition 18 *If $P' \equiv P$ then $A \vdash P' \xrightarrow{\ell} Q$ iff $A \vdash P \xrightarrow{\ell} Q$.*

Proof Induction on the size of derivation of $P' \equiv P$. In symmetric cases we show only the right-to-left direction of the conclusion.

(Struct Cong Refl) By the reflexivity of iff.

(Struct Cong Sym) By the symmetry of iff.

(Struct Cong Tran) By the induction hypothesis and transitivity of iff.

(Struct Cong Input) Consider $P' \equiv P$ and $A \vdash x^\iota p.P \xrightarrow{\ell} Q$. By Lemma 16.2, there exists v such that $\mathrm{fn}(x^\iota p.P) \subseteq A$, $\ell = x^\iota v$, $\{^v/_p\}P$ is defined and $Q \equiv \{^v/_p\}P$. Using Lemma 10, $\mathrm{fn}(x^\iota p.P') = \mathrm{fn}(x^\iota p.P)$. By Lemma 17, $\{^v/_p\}P'$ is defined and $\{^v/_p\}P' \equiv \{^v/_p\}P$, so $Q \equiv \{^v/_p\}P'$. Finally by Lemma 16.2, $A \vdash x^\iota p.P' \xrightarrow{\ell} Q$.

(Struct Cong Repl) Consider $P' \equiv P$ and $A \vdash \mathbf{!}\, x^\iota p.P \xrightarrow{\ell} Q$. By Lemma 16.3 there exists v such that $\mathrm{fn}(\mathbf{!}\, x^\iota p.P) \subseteq A, \ell = x^\iota v$, $\{^v/_p\}P$ is defined and $Q \equiv \mathbf{!}\, x^\iota p.P \mid \{^v/_p\}P$. Using Lemma 10, $\mathrm{fn}(\mathbf{!}\, x^\iota p.P') = \mathrm{fn}(\mathbf{!}\, x^\iota p.P)$. By Lemma 17, $\{^v/_p\}P'$ is defined and $\{^v/_p\}P' \equiv \{^v/_p\}P$, so $Q \equiv \mathbf{!}\, x^\iota p.P' \mid \{^v/_p\}P'$. Finally by Lemma 16.3, $A \vdash x^\iota p.P' \xrightarrow{\ell} Q$.

(Struct Cong Box) Consider $P' \equiv P$ and $A \vdash n[P] \xrightarrow{\ell} Q$. By Lemma 16.4 one of the following hold:

Case 16.4a there exist x, v, and $\hat{P}$ such that $n \in A$, $\ell = \tau$, $A \vdash P \xrightarrow{\overline{x}^\uparrow v} \hat{P}$, and $Q \equiv (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - A)(\overline{x}^{\overline{n}} v \mid n[\hat{P}])$. By the inductive hypothesis $A \vdash P' \xrightarrow{\overline{x}^\uparrow v} \hat{P}$. By Lemma 16.4 $A \vdash n[P'] \xrightarrow{\ell} Q$

Case 16.4b there exist x and v such that $\mathrm{fn}(n[P]) \subseteq A$, $\ell = x^{\overline{n}} v$ and $Q \equiv n[\overline{x}^\uparrow v \mid P]$. Using Lemma 10, $\mathrm{fn}(n[P']) = \mathrm{fn}(n[P])$. Clearly $n[\overline{x}^\uparrow v \mid P] \equiv n[\overline{x}^\uparrow v \mid P']$, so $Q \equiv n[\overline{x}^\uparrow v \mid P']$. Finally by Lemma 16.4, $A \vdash n[P'] \xrightarrow{\ell} Q$.

Case 16.4c there exists $\hat{P}$ such that $n \in A$, $\ell = \tau$, $A \vdash P \xrightarrow{\tau} \hat{P}$, and $Q \equiv n[\hat{P}]$. By the inductive hypothesis $A \vdash P' \xrightarrow{\ell} \hat{P}$, so by Lemma 16.4, $A \vdash n[P'] \xrightarrow{\ell} Q$.

(Struct Cong Par) Consider $P' \equiv P$, $Q' \equiv Q$ and $A \vdash P \mid Q \xrightarrow{\ell} R$. By Lemma 16.5 one of the following holds.

Case 16.5a there exists $\hat{P}$ such that $\mathrm{fn}(Q) \subseteq A$, $A \vdash P \xrightarrow{\ell} \hat{P}$ and $R \equiv \hat{P} \mid Q$. By Lemma 10, $\mathrm{fn}(Q') = \mathrm{fn}(Q)$. By the inductive hypothesis $A \vdash P' \xrightarrow{\ell} \hat{P}$ and clearly $\hat{P} \mid Q \equiv \hat{P} \mid Q'$, so by Lemma 16.5, $A \vdash P' \mid Q' \xrightarrow{\ell} R$.

Case 16.5b there exists x, γ, v, $\hat{P}$ and $\hat{Q}$ such that $\ell = \tau$, $A \vdash P \xrightarrow{\overline{x}^{\overline{\gamma}} v} \hat{P}$, $A \vdash Q \xrightarrow{x^\gamma v} \hat{Q}$, and $R \equiv (\boldsymbol{\nu}\, \mathrm{fn}(x, v) - A)(\hat{P} \mid \hat{Q})$. By the induction hypothesis $A \vdash P' \xrightarrow{\overline{x}^{\overline{\gamma}} v} \hat{P}$ and $A \vdash Q' \xrightarrow{x^\gamma v} \hat{Q}$. By Lemma 16.5, $A \vdash P' \mid Q' \xrightarrow{\ell} R$.

or symmetric cases.

(Struct Cong Res) Consider $P' \equiv P$ and $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\ell} Q$. By Lemma 16.6 one of the following holds.

Case 16.6a there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(P) - x)$ and $\hat{Q}$ such that $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\ell} \hat{Q}$ and $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By Lemma 17 $\{^{\hat{x}}/_x\}P' \equiv \{^{\hat{x}}/_x\}P$ (with a derivation of the same size). By the induction hypothesis $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P' \xrightarrow{\ell} \hat{Q}$. By Lemma 16.6 $A \vdash (\boldsymbol{\nu}\, x)P' \xrightarrow{\ell} Q$.

Case 16.6b there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(y,o) \cup (\mathrm{fn}(P) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(o)$ and $Q \equiv \hat{Q}$. By Lemma 17 $\{{}^{\hat{x}}\!/\!{}_x\}P' \equiv \{{}^{\hat{x}}\!/\!{}_x\}P$, with a derivation of the same size. By the induction hypothesis $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P' \xrightarrow{\overline{y}^o v} \hat{Q}$. By Lemma 10 $\mathrm{fn}(P') = \mathrm{fn}(P)$, so $\hat{x} \notin A \cup \mathrm{fn}(y,o) \cup (\mathrm{fn}(P') - x)$. By Lemma 16.6, $A \vdash (\boldsymbol{\nu}\, x)P' \xrightarrow{\ell} Q$.

Case 16.6c there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(P) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(y, v)$, $\mathrm{mv}(o)$ and $Q \equiv \hat{Q}$. By Lemma 17 $\{{}^{\hat{x}}\!/\!{}_x\}P' \equiv \{{}^{\hat{x}}\!/\!{}_x\}P$ (with a derivation of the same size). By the induction hypothesis $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P' \xrightarrow{\overline{y}^o v} \hat{Q}$. By Lemma 10 $\mathrm{fn}(P') = \mathrm{fn}(P)$, so $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(P') - x)$. By Lemma 16.6 $A \vdash (\boldsymbol{\nu}\, x)P' \xrightarrow{\ell} Q$.

(Struct Par Nil), (Struct Par Comm), (Struct Par Assoc), (Struct Res Res) These should be straightforward. We check the other two axioms in detail.

(Struct Res Par) $(\boldsymbol{\nu}\, x)(P \mid Q) \equiv P \mid (\boldsymbol{\nu}\, x)Q$ where $x \notin \mathrm{fn}(P)$. In the following, we use the fact $\{{}^{\hat{x}}\!/\!{}_x\}P = P$ since $x \notin \mathrm{fn}(P)$, and the fact that $(\boldsymbol{\nu}\, x)Q = (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q$ when $\hat{x} \notin \mathrm{fn}(Q) - x$. The proofs in the first part will yield results of the form $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} R'$ with $R' \equiv R$, thus we get $A \vdash P \mid (\boldsymbol{\nu}\, x)Q \xrightarrow{\ell} R$ by an application of (Trans Struct Right).

Consider $A \vdash (\boldsymbol{\nu}\, x)(P \mid Q) \xrightarrow{\ell} R$. By Lemma 16.6 this holds iff one of the following holds:

Case 16.6a (Trans Res-1) there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}((P \mid Q)) - x)$ and $\hat{R}$ such that $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}(P \mid Q) \xrightarrow{\ell} \hat{R}$ and $R \equiv (\boldsymbol{\nu}\, \hat{x})\hat{R}$. By Lemma 16.5 this transition holds iff one of the following holds:

Case 16.5a (Trans Par)[Left] there exists $\hat{P}$ such that $\mathrm{fn}(\{{}^{\hat{x}}\!/\!{}_x\}Q) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P \xrightarrow{\ell} \hat{P}$ and $\hat{R} \equiv \hat{P} \mid \{{}^{\hat{x}}\!/\!{}_x\}Q$. It follows that $A, \hat{x} \vdash P \xrightarrow{\ell} \hat{P}$. By Lemma 14, $A \vdash P \xrightarrow{\ell} \hat{P}$. By (Trans Par), we get $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} \hat{P} \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q$. By Lemma 11 $\hat{x} \notin \mathrm{fn}(\hat{P})$. By (Trans Struct Right), we obtain $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} (\boldsymbol{\nu}\, \hat{x})(\hat{P} \mid \{{}^{\hat{x}}\!/\!{}_x\}Q)$.

Case 16.5a′ (Trans Par)[Right] there exists $\hat{Q}$ such that $\mathrm{fn}(\{{}^{\hat{x}}\!/\!{}_x\}P) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} \hat{Q}$ and $\hat{R} \equiv \hat{Q} \mid \{{}^{\hat{x}}\!/\!{}_x\}P$. By (Trans Res-1) and the fact that $\hat{x} \notin \mathrm{fn}(\ell)$, we get $A \vdash (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By (Trans Par)[Right], we get $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} P \mid (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By the fact that $\hat{x} \notin \mathrm{fn}(P)$ and (Trans Struct Right), we get $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\ell} (\boldsymbol{\nu}\, \hat{x})(P \mid \hat{Q})$.

Case 16.5b (Trans Comm) there exists z, γ, v, $\hat{P}$ and $\hat{Q}$ such that $\ell = \tau$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}P \xrightarrow{\overline{z}^\gamma v} \hat{P}$, $A, \hat{x} \vdash \{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{z^\gamma v} \hat{Q}$, and $\hat{R} \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{Q})$. By $\hat{x} \notin \mathrm{fn}(\{{}^{\hat{x}}\!/\!{}_x\}P)$ and Lemma 11.3, $\hat{x} \notin \mathrm{fn}(\overline{z}^\gamma v)$. By $\hat{x} \notin \mathrm{fn}(z^\gamma v)$ and (Trans Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{z^\gamma v} (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By the fact that $\hat{x} \notin \mathrm{fn}(P, \overline{z}^\gamma v)$ and Lemma 14, we get $A \vdash P \xrightarrow{\overline{z}^\gamma v} \hat{P}$. By (Trans Comm), $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{{}^{\hat{x}}\!/\!{}_x\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\hat{P} \mid (\boldsymbol{\nu}\, \hat{x})\hat{Q})$. By Lemma 11.2 $\hat{x} \notin \mathrm{fn}(\hat{P})$, so we may calculate $(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\hat{P} \mid (\boldsymbol{\nu}\, \hat{x})\hat{Q}) \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\boldsymbol{\nu}\, \hat{x})(\hat{P} \mid \hat{Q}) \equiv R$.

Case 16.5b′ (Trans Comm) there exists z, γ, v, $\hat{Q}$ and $\hat{P}$ such that $\ell = \tau$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{z}^{\gamma} v} \hat{Q}$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}P \xrightarrow{z^{\gamma} v} \hat{P}$, and $\hat{R} \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\hat{Q} \mid \hat{P})$. There are some cases to consider:

Case $\gamma = \iota$ By Lemma 11.(6,7) $\hat{x} \notin \mathrm{fn}(z, \gamma)$.

Case $\hat{x} \notin \mathrm{fn}(v)$ By Lemma 14 $A \vdash P \xrightarrow{z^{\gamma} v} \hat{P}$. By (Res-1) $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{z}^{\gamma} v} (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By (Comm) we have $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid (\boldsymbol{\nu}\, \hat{x})\hat{Q})$. By Lemma 11.2 $\hat{x} \notin \mathrm{fn}(\hat{P})$, so $(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid (\boldsymbol{\nu}\, \hat{x})\hat{Q}) \equiv (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\hat{P} \mid \hat{Q})$.

Case $\hat{x} \in \mathrm{fn}(v)$ By Lemma 15.1 $A \vdash P \xrightarrow{z^{\gamma} v} \hat{P}$. By (Res-2) $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{z}^{\gamma} v} \hat{Q}$. By (Comm) we have

$$A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid \hat{Q}).$$

Clearly $(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid \hat{Q}) \equiv (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\hat{P} \mid \hat{Q})$.

Case $\gamma = \overline{n}$ By Lemma 11.6 $\hat{x} \notin \mathrm{fn}(\gamma)$.

Case $\hat{x} \notin \mathrm{fn}(z, v)$ Exactly as the $\hat{x} \notin \mathrm{fn}(v)$ case above.

Case $\hat{x} \in \mathrm{fn}(z, v)$ By Lemma 15.2 $A \vdash P \xrightarrow{z^{\gamma} v} \hat{P}$. By (Res-2) $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{z}^{\gamma} v} \hat{Q}$. By (Comm) we have

$$A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid \hat{Q}).$$

Clearly $(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\hat{P} \mid \hat{Q}) \equiv (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\hat{P} \mid \hat{Q})$.

Case 16.6b (Trans Res-2-nmv) there exists y, o, v, $\hat{R}$ and $\hat{x} \notin A \cup \mathrm{fn}(y, o) \cup (\mathrm{fn}((P \mid Q)) - x)$ such that $\ell = \overline{y}^{o} v$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}(P \mid Q) \xrightarrow{\overline{y}^{o} v} \hat{R}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(o)$ and $R \equiv \hat{R}$. By Lemma 16.5 either one of the following holds:

Case 16.5a there exists $\hat{P}$ such that $\mathrm{fn}(\{^{\hat{x}}/_{x}\}Q) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}P \xrightarrow{\overline{y}^{o} v} \hat{P}$ and $\hat{R} \equiv \hat{P} \mid \{^{\hat{x}}/_{x}\}Q$. This leads to a contradiction, as by Lemma 11 $x \in \mathrm{fn}(P)$.

Case 16.5a′ there exists $\hat{Q}$ such that $\mathrm{fn}(\{^{\hat{x}}/_{x}\}P) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} \hat{Q}$ and $\hat{R} \equiv \hat{Q} \mid \{^{\hat{x}}/_{x}\}P$. We apply (Trans Res-2-nmv) to get $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} \hat{Q}$. By $\hat{x} \notin \mathrm{fn}(P)$, we can apply (Trans Par)[Right] to obtain $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} P \mid \hat{Q} \equiv R$.

Case 16.6c (Trans Res-2-mv) there exists y, o, v, $\hat{R}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}((P \mid Q)) - x)$ such that $\ell = \overline{y}^{o} v$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}(P \mid Q) \xrightarrow{\overline{y}^{o} v} \hat{R}$, $\hat{x} \in \mathrm{fn}(y, v)$, $\mathrm{mv}(o)$ and $R \equiv \hat{R}$. By Lemma 16.5 either one of the following holds:

Case 16.5a there exists $\hat{P}$ such that $\mathrm{fn}(\{^{\hat{x}}/_{x}\}Q) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}P \xrightarrow{\overline{y}^{o} v} \hat{P}$ and $\hat{R} \equiv \hat{P} \mid \{^{\hat{x}}/_{x}\}Q$. This leads to a contradiction, as by Lemma 11 $x \in \mathrm{fn}(P)$.

Case 16.5a′ there exists $\hat{Q}$ such that $\mathrm{fn}(\{^{\hat{x}}/_{x}\}P) \subseteq A, \hat{x}$, $A, \hat{x} \vdash \{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} \hat{Q}$ and $\hat{R} \equiv \hat{Q} \mid \{^{\hat{x}}/_{x}\}P$. By (Trans Res-2-mv) and the facts that $\hat{x} \in \mathrm{fn}(y, v) - \mathrm{fn}(o)$ and $\mathrm{mv}(o)$, we get $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} \hat{Q}$. By (Trans Par)[Right], we get $A \vdash P \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_{x}\}Q \xrightarrow{\overline{y}^{o} v} P \mid \hat{Q}$.

Now consider $A \vdash P \mid (\boldsymbol{\nu}\, x)Q \xrightarrow{\ell} R$. By Lemma 16.5 this transition holds iff one of the following holds.

Case 16.5a (Trans Par)[Left] there exists $\hat{P}$ such that $\mathrm{fn}((\boldsymbol{\nu}\,x)Q) \subseteq A$, $A \vdash P \xrightarrow{\ell} \hat{P}$ and $R \equiv \hat{P} \mid (\boldsymbol{\nu}\,x)Q$. Take $\hat{x}$ such that $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(P,Q) - x)$. By $\hat{x} \notin A \cup \mathrm{fn}(\ell)$ and Lemma 14, $A, \hat{x} \vdash P \xrightarrow{\ell} \hat{P}$. By (Trans Par), $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\ell} \hat{P} \mid \{\hat{x}/x\}Q$. By $\hat{x} \notin \ell$ and (Trans Res-1), $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\ell} (\boldsymbol{\nu}\,\hat{x})(\hat{P} \mid \{\hat{x}/x\}Q)$. Since $\hat{x} \notin \mathrm{fn}(\hat{P})$, $(\boldsymbol{\nu}\,\hat{x})(\hat{P} \mid \{\hat{x}/x\}Q) \equiv R$.

Case 16.5a′ (Trans Par)[Right] there exists $\hat{Q}$ such that $\mathrm{fn}(P) \subseteq A$, $A \vdash (\boldsymbol{\nu}\,x)Q \xrightarrow{\ell} \hat{Q}$ and $R \equiv \hat{Q} \mid P$. By Lemma 16.6 this transition holds iff one of the following holds.

Case 16.6a (Trans Res-1) there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(Q) - x)$ and $\hat{\hat{Q}}$ such that $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{\ell} \hat{\hat{Q}}$ and $\hat{Q} \equiv (\boldsymbol{\nu}\,\hat{x})\hat{\hat{Q}}$. By (Trans Par)[Right], we have $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\ell} P \mid \hat{\hat{Q}}$. By $\hat{x} \notin \mathrm{fn}(\ell)$ and (Trans Res-1), we get $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\ell} (\boldsymbol{\nu}\,\hat{x})(P \mid \hat{\hat{Q}})$. By $\hat{x} \notin \mathrm{fn}(P)$ and (Trans Struct Right), we obtain $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\ell} P \mid (\boldsymbol{\nu}\,\hat{x})\hat{\hat{Q}}$.

Case 16.6b (Trans Res-2-nmv) there exists y, o, v, $\hat{\hat{Q}}$ and $\hat{x} \notin A \cup \mathrm{fn}(y,o) \cup (\mathrm{fn}(Q) - x)$ such that $\ell = \overline{y}^{o}v$, $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{\overline{y}^{o}v} \hat{\hat{Q}}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(o)$ and $\hat{Q} \equiv \hat{\hat{Q}}$. By (Trans Par)[Right], we have $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\ell} P \mid \hat{\hat{Q}}$. By $\neg\mathrm{mv}(o)$, $\hat{x} \in \mathrm{fn}(v) - \mathrm{fn}(y,o)$ and (Trans Res-2-nmv), we get $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\ell} P \mid \hat{\hat{Q}}$.

Case 16.6c (Trans Res-2-mv) there exists y, o, v, $\hat{\hat{Q}}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(Q) - x)$ such that $\ell = \overline{y}^{o}v$, $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{\overline{y}^{o}v} \hat{\hat{Q}}$, $\hat{x} \in \mathrm{fn}(y,v)$, $\mathrm{mv}(o)$ and $\hat{Q} \equiv \hat{\hat{Q}}$. By (Trans Par)[Right], we have $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\ell} P \mid \hat{\hat{Q}}$. By $\mathrm{mv}(o)$, $\hat{x} \in \mathrm{fn}(y,v) - \mathrm{fn}(o)$ and (Trans Res-2-mv), we get $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\ell} P \mid \hat{\hat{Q}}$.

Case 16.5b (Trans Comm) there exists z, γ, v, $\hat{P}$ and $\hat{Q}$ such that $\ell = \tau$, $A \vdash P \xrightarrow{\overline{z}^{\overline{\gamma}}v} \hat{P}$, $A \vdash (\boldsymbol{\nu}\,x)Q \xrightarrow{z^{\gamma}v} \hat{Q}$, and $R \equiv (\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{Q})$. By Lemma 16.6 there exists $\hat{x} \notin A \cup \mathrm{fn}(z^{\gamma}v) \cup (\mathrm{fn}(Q) - x)$ and $\hat{\hat{Q}}$ such that $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{z^{\gamma}v} \hat{\hat{Q}}$ and $\hat{Q} \equiv (\boldsymbol{\nu}\,\hat{x})\hat{\hat{Q}}$. By Lemma 14 and $\hat{x} \notin A \cup \mathrm{fn}(\overline{z}^{\overline{\gamma}}v)$, we get $A, \hat{x} \vdash P \xrightarrow{\overline{z}^{\overline{\gamma}}v} \hat{P}$. By (Trans Comm), $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$. By (Tran Res-1) and $\hat{x} \notin \mathrm{fn}(z,v)$, we obtain $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\,\hat{x})(\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{\hat{Q}})$, hence $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{Q})$.

Case 16.5b′ (Trans Comm) there exists z, γ, v, $\hat{Q}$ and $\hat{P}$ such that $\ell = \tau$, $A \vdash (\boldsymbol{\nu}\,x)Q \xrightarrow{\overline{z}^{\overline{\gamma}}v} \hat{Q}$, $A \vdash P \xrightarrow{z^{\gamma}v} \hat{P}$, and $R \equiv (\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A)(\hat{Q} \mid \hat{P})$. By Lemma 16.6 the $(\boldsymbol{\nu}\,x)Q$ transition holds iff one of the following holds.

Case 16.6a (Trans Res-1) there exists $\hat{x} \notin A \cup \mathrm{fn}(\overline{z}^{\overline{\gamma}}v) \cup (\mathrm{fn}(Q) - x)$ and $\hat{\hat{Q}}$ such that $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{\overline{z}^{\overline{\gamma}}v} \hat{\hat{Q}}$ and $\hat{Q} \equiv (\boldsymbol{\nu}\,\hat{x})\hat{\hat{Q}}$. By Lemma 14 and $\hat{x} \notin A \cup \mathrm{fn}(z^{\gamma}v)$ we have $A, \hat{x} \vdash P \xrightarrow{z^{\gamma}v} \hat{P}$. By (Trans Comm), $A, \hat{x} \vdash P \mid \{\hat{x}/x\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$. By (Tran Res-1) and $\hat{x} \notin \mathrm{fn}(z,v)$, we obtain $A \vdash (\boldsymbol{\nu}\,\hat{x})(P \mid \{\hat{x}/x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\,\hat{x})(\boldsymbol{\nu}\,\mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{\hat{Q}})$.

Case 16.6b (Trans Res-2-nmv) there exists z, $\overline{\gamma}$, v, $\hat{\hat{Q}}$ and $\hat{x} \notin A \cup \mathrm{fn}(z, \overline{\gamma}) \cup (\mathrm{fn}(Q) - x)$ such that $\overline{z}^{\overline{\gamma}}v = \overline{z}^{\overline{\gamma}}v$, $A, \hat{x} \vdash \{\hat{x}/x\}Q \xrightarrow{\overline{z}^{\overline{\gamma}}v} \hat{\hat{Q}}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(\overline{\gamma})$

and $\hat{Q} \equiv \hat{\hat{Q}}$. By Lemma 15.1 and $\gamma \neq \overline{n}$, $A, \hat{x} \vdash P \xrightarrow{z^\gamma v} \hat{P}$. By (Trans Comm), $A, \hat{x} \vdash P \mid \{^{\hat{x}}/_x\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$. By (Tran Res-1) we obtain $A \vdash (\boldsymbol{\nu}\, \hat{x})(P \mid \{^{\hat{x}}/_x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$, hence as $\hat{x} \in \mathrm{fn}(v)$ $A \vdash (\boldsymbol{\nu}\, \hat{x})(P \mid \{^{\hat{x}}/_x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{\hat{Q}})$.

Case 16.6c (Trans Res-2-mv) there exists z, $\overline{\gamma}$, v, $\hat{\hat{Q}}$ and $\hat{x} \notin A \cup \mathrm{fn}(\overline{\gamma}) \cup (\mathrm{fn}(Q) - x)$ such that $\overline{z^{\gamma}}v = \overline{z}^{\overline{\gamma}}v$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}Q \xrightarrow{\overline{z}^{\overline{\gamma}} v} \hat{\hat{Q}}$, $\hat{x} \in \mathrm{fn}(z,v)$, $\mathrm{mv}(\overline{\gamma})$ and $\hat{Q} \equiv \hat{\hat{Q}}$. By $\mathrm{mv}(\overline{\gamma})$ we have $\gamma = \overline{n}$ for some n. By Lemma 15.2 $A, \hat{x} \vdash P \xrightarrow{z^\gamma v} \hat{P}$. By (Trans Comm), $A, \hat{x} \vdash P \mid \{^{\hat{x}}/_x\}Q \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$. By (Tran Res-1) $A \vdash (\boldsymbol{\nu}\, \hat{x})(P \mid \{^{\hat{x}}/_x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\hat{P} \mid \hat{\hat{Q}})$, hence as $\hat{x} \in \mathrm{fn}(z,v)$ $A \vdash (\boldsymbol{\nu}\, \hat{x})(P \mid \{^{\hat{x}}/_x\}Q) \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\hat{P} \mid \hat{\hat{Q}})$.

(Struct Res Box) $(\boldsymbol{\nu}\, x)n[P] \equiv n[(\boldsymbol{\nu}\, x)P]$ where $x \neq n$. Consider $A \vdash (\boldsymbol{\nu}\, x)n[P] \xrightarrow{\ell} Q$. By Lemma 16.6 this holds iff one of the following holds.

Case 16.6a (Trans Res-1) there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}(n[P]) - x)$ and $\hat{Q}$ such that $A, \hat{x} \vdash \{^{\hat{x}}/_x\}n[P] \xrightarrow{\ell} \hat{Q}$ and $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By $x \neq n$ we have $\{^{\hat{x}}/_x\}n = n$, so we have $A, \hat{x} \vdash n[\{^{\hat{x}}/_x\}P] \xrightarrow{\ell} \hat{Q}$. By Lemma 16.4 this transition exists iff one of the following hold:

Case 16.4a (Trans Box-1) there exist z, v, and $\hat{P}$ such that $n \in A, \hat{x}$, $\ell = \tau$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} \hat{P}$, and $\hat{Q} \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\overline{z}^{\overline{n}}v \mid n[\hat{P}])$. There are two cases to consider:

Case $\hat{x} \notin \mathrm{fn}(\overline{z}^{\uparrow}v)$ By (Trans Res-1) and the fact that $\hat{x} \notin \mathrm{fn}(\overline{z}^{\uparrow}v)$, we obtain $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} (\boldsymbol{\nu}\, \hat{x})\hat{P}$. By (Trans Box-1), we obtain $A \vdash n[(\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^{\overline{n}}v \mid n[(\boldsymbol{\nu}\, \hat{x})\hat{P}])$. Since $\hat{x} \notin \mathrm{fn}(\overline{z}^{\overline{n}}v)$ we have $(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^{\overline{n}}v \mid n[(\boldsymbol{\nu}\, \hat{x})\hat{P}]) \equiv (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\overline{z}^{\overline{n}}v \mid n[\hat{P}])$.

Case $\hat{x} \in \mathrm{fn}(\overline{z}^{\uparrow}v)$ By (Trans Res-2), $\mathrm{mv}(\uparrow)$, and $\hat{x} \in \mathrm{fn}(z,v) - \mathrm{fn}(\uparrow)$, we obtain $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} \hat{P}$. By (Trans Box-1), we obtain $A \vdash n[(\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^{\overline{n}}v \mid n[\hat{P}])$. Since $\hat{x} \in \mathrm{fn}(z,v) - A$, we get $(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^{\overline{n}}v \mid n[\hat{P}]) \equiv (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A, \hat{x})(\overline{z}^{\overline{n}}v \mid n[\hat{P}])$.

Case 16.4b (Trans Box-2) there exist z and v such that $\mathrm{fn}(n[\{^{\hat{x}}/_x\}P]) \subseteq A, \hat{x}$, $\ell = z^{\overline{n}}v$ and $\hat{Q} \equiv n[\overline{z}^{\uparrow}v \mid \{^{\hat{x}}/_x\}P]$. By (Trans Box-2), $A \vdash n[(\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P] \xrightarrow{z^{\overline{n}} v} n[\overline{z}^{\uparrow}v \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P]$. Since $\hat{x} \notin \mathrm{fn}(z^{\overline{n}}v)$, we have $n[\overline{z}^{\uparrow}v \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P] \equiv (\boldsymbol{\nu}\, \hat{x})n[\overline{z}^{\uparrow}v \mid \{^{\hat{x}}/_x\}P]$.

Case 16.4c (Trans Box-3) there exists $\hat{P}$ such that $n \in A, \hat{x}$, $\ell = \tau$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\tau} \hat{P}$, and $\hat{Q} \equiv n[\hat{P}]$. By (Trans Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})\hat{P}$. By (Trans Box-3), (Trans Struct Right), and $\hat{x} \neq n$, $A \vdash n[(\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})n[\hat{P}]$.

Case 16.6b (Trans Res-2-nmv) there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(y,o) \cup (\mathrm{fn}(n[P]) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}n[P] \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(o)$ and $Q \equiv \hat{Q}$. This leads to a contradiction as no such term has any output transitions.

Case 16.6c (Trans Res-2-mv) there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}(n[P]) - x)$ such that $\ell = \overline{y}^o v$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}n[P] \xrightarrow{\overline{y}^o v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(y, v)$, $\mathrm{mv}(o)$ and $Q \equiv \hat{Q}$. This leads to a contradiction as no such term has any output transitions.

Now consider $A \vdash n[(\boldsymbol{\nu}\, x)P] \xrightarrow{\ell} Q$. By Lemma 16.4 this holds iff one of the following hold:

Case 16.4a (Trans Box-1) there exist z, v, and $\hat{P}$ such that $n \in A$, $\ell = \tau$, $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\overline{z}^{\uparrow} v} \hat{P}$, and $Q \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\overline{z}^{\overline{n}} v \mid n[\hat{P}])$. By Lemma 16.6 this transition holds iff one of the following holds:

Case 16.6a (Trans Res-1) there exists $\hat{x} \notin A \cup \mathrm{fn}(\overline{z}^{\uparrow} v) \cup (\mathrm{fn}(P) - x)$ and $\hat{Q}$ such that $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} \hat{Q}$ and $\hat{P} \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$. By (Trans Box-1), we have $A, \hat{x} \vdash n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}])$. By (Trans Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}])$. Since $\hat{x} \notin \mathrm{fn}(\overline{z}^{\uparrow} v)$ and $\hat{x} \neq n$, we obtain $(\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}]) \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\overline{z}^{\overline{n}} v \mid n[(\boldsymbol{\nu}\, \hat{x})\hat{Q}])$.

Case 16.6b (Trans Res-2-nmv) there exists z, $\uparrow$, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(z, \uparrow) \cup (\mathrm{fn}(P) - x)$ such that $\overline{z}^{\uparrow} v = \overline{z}^{\uparrow} v$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(v)$, $\neg\mathrm{mv}(\uparrow)$ and $\hat{P} \equiv \hat{Q}$. This cannot hold, as $\mathrm{mv}(\uparrow)$.

Case 16.6c (Trans Res-2-mv) there exists z, $\uparrow$, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(\uparrow) \cup (\mathrm{fn}(P) - x)$ such that $\overline{z}^{\uparrow} v = \overline{z}^{\uparrow} v$, $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\overline{z}^{\uparrow} v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(z, v)$, $\mathrm{mv}(\uparrow)$ and $\hat{P} \equiv \hat{Q}$. By (Trans Box-1), $A, \hat{x} \vdash n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}])$. By (Tran Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}])$. Since $\hat{x} \in \mathrm{fn}(z, v) - A$ we obtain $(\boldsymbol{\nu}\, \hat{x})(\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A, \hat{x})(\overline{z}^{\overline{n}} v \mid n[\hat{Q}]) \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z, v) - A)(\overline{z}^{\overline{n}} v \mid n[\hat{Q}])$.

Case 16.4b (Trans Box-2) there exist z and v such that $\mathrm{fn}(n[(\boldsymbol{\nu}\, x)P]) \subseteq A$, $\ell = z^{\overline{n}} v$ and $Q \equiv n[\overline{z}^{\overline{\uparrow}} v \mid (\boldsymbol{\nu}\, x)P]$. Take $\hat{x} \notin A \cup \mathrm{fn}(z^{\overline{n}} v)$, then by (Tran Box-2), we obtain $A, \hat{x} \vdash n[\{^{\hat{x}}/_x\}P] \xrightarrow{z^{\overline{n}} v} n[\overline{z}^{\overline{\uparrow}} v \mid \{^{\hat{x}}/_x\}P]$. By (Trans Res-1), we get $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{z^{\overline{n}} v} (\boldsymbol{\nu}\, \hat{x})n[\overline{z}^{\overline{\uparrow}} v \mid \{^{\hat{x}}/_x\}P]$. By (Trans Struct Right) and $\hat{x} \notin \mathrm{fn}(n, \overline{z}^{\overline{\uparrow}} v)$, we obtain $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{z^{\overline{n}} v} n[\overline{z}^{\overline{\uparrow}} v \mid (\boldsymbol{\nu}\, \hat{x})\{^{\hat{x}}/_x\}P]$.

Case 16.4c (Trans Box-3) there exists $\hat{\hat{Q}}$ such that $n \in A$, $\ell = \tau$, $A \vdash (\boldsymbol{\nu}\, x)P \xrightarrow{\tau} \hat{\hat{Q}}$, and $Q \equiv n[\hat{\hat{Q}}]$. By Lemma 16.6 there exists $\hat{x} \notin A \cup (\mathrm{fn}(P) - x)$ and $\hat{P}$ such that $A, \hat{x} \vdash \{^{\hat{x}}/_x\}P \xrightarrow{\tau} \hat{P}$ and $\hat{\hat{Q}} \equiv (\boldsymbol{\nu}\, \hat{x})\hat{P}$. By (Trans Box-3), $A, \hat{x} \vdash n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} n[\hat{P}]$. By (Trans Res-1), $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} (\boldsymbol{\nu}\, \hat{x})n[\hat{P}]$. By (Trans Struct Right) and $x \neq n$, we obtain $A \vdash (\boldsymbol{\nu}\, \hat{x})n[\{^{\hat{x}}/_x\}P] \xrightarrow{\tau} n[(\boldsymbol{\nu}\, \hat{x})\hat{P}]$.

□

Lemma 19 *If* $\mathrm{fn}(P) \subseteq A$ *and* $P \to Q$ *then* $A \vdash P \xrightarrow{\tau} Q$.

Proof Induction on derivations of $P \to Q$. For the base cases we construct derivations of τ transitions:

(Red Up)

$$
\dfrac{\dfrac{\dfrac{}{A \vdash \overline{x}^{\uparrow} v \xrightarrow{\overline{x}^{\uparrow} v} 0}\ \text{(Trans Out)}}{A \vdash \overline{x}^{\uparrow} v \mid Q \xrightarrow{\overline{x}^{\uparrow} v} 0 \mid Q}\ \text{(Trans Par)}}{A \vdash n[\overline{x}^{\uparrow} v \mid Q] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(x,v) - A)(\overline{x}^{\overline{n}} v \mid n[0 \mid Q])}\ \text{(Trans Box-1)}
$$

By the premise $\mathrm{fn}(n[\overline{x}^{\uparrow} v \mid Q]) \subseteq A$ we have $\mathrm{fn}(x,v) \subseteq A$, so using (Trans Struct Right) we have $A \vdash n[\overline{x}^{\uparrow} v \mid Q] \xrightarrow{\tau} \overline{x}^{\overline{n}} v \mid n[Q]$, the right hand side of which is exactly the right hand side of (Red Up).

(Red Down)

$$
\dfrac{\dfrac{}{A \vdash \overline{x}^{n} v \xrightarrow{\overline{x}^{n} v} 0}\ \text{(Trans Out)} \qquad \dfrac{x \in A}{A \vdash n[Q] \xrightarrow{x^{\overline{n}} v} n[\overline{x}^{\overline{\uparrow}} v \mid Q]}\ \text{(Trans Box-2)}}{A \vdash \overline{x}^{n} v \mid n[Q] \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(v) - A)(0 \mid n[\overline{x}^{\overline{\uparrow}} v \mid Q])}\ \text{(Trans Comm)}
$$

By the premise $\mathrm{fn}(\overline{x}^{n} v \mid n[Q]) \subseteq A$ we have $x \in A$ and also $\mathrm{fn}(v) \subseteq A$, so using (Trans Struct Right) we have $A \vdash \overline{x}^{n} v \mid n[Q] \xrightarrow{\tau} n[\overline{x}^{\overline{\uparrow}} v \mid Q]$, the right hand side of which is exactly the right hand side of (Red Down).

(Red Comm)

$$
\dfrac{\dfrac{}{A \vdash \overline{x}^{\overline{\iota}} v \xrightarrow{\overline{x}^{\overline{\iota}} v} 0}\ \text{(Trans Out)} \qquad \dfrac{}{A \vdash x^{\iota} p.P \xrightarrow{x^{\iota} v} \{^{v}/_{p}\}P}\ \text{(Trans In)}}{A \vdash \overline{x}^{\overline{\iota}} v \mid x^{\iota} p.P \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(v) - A)(0 \mid \{^{v}/_{p}\}P)}\ \text{(Trans Comm)}
$$

The side condition $\{^{v}/_{p}\}P$ defined for (Trans In) is ensured by the same condition for (Red Comm). By the premise $\mathrm{fn}(\overline{x}^{\overline{\iota}} v \mid x^{\iota} p.P) \subseteq A$ we have $\mathrm{fn}(v) \subseteq A$, so using (Trans Struct Right) we have $A \vdash \overline{x}^{\overline{\iota}} v \mid x^{\iota} p.P \xrightarrow{\tau} \{^{v}/_{p}\}P$, the right hand side of which is exactly the right hand side of (Red Comm).

(Red Repl)

$$
\dfrac{\dfrac{}{A \vdash \overline{x}^{\overline{\iota}} v \xrightarrow{\overline{x}^{\overline{\iota}} v} 0}\ \text{(Trans Out)} \qquad \dfrac{}{\mathbf{!}\, x^{\iota} p.P \xrightarrow{x^{\iota} v} \mathbf{!}\, x^{\iota} p.P \mid \{^{v}/_{p}\}P}\ \text{(Trans Repl)}}{A \vdash \overline{x}^{\overline{\iota}} v \mid \mathbf{!}\, x^{\iota} p.P \xrightarrow{\tau} (\boldsymbol{\nu}\, \mathrm{fn}(v) - A)(0 \mid (\mathbf{!}\, x^{\iota} p.P \mid \{^{v}/_{p}\}P))}\ \text{(Trans Comm)}
$$

The side condition $\{^{v}/_{p}\}P$ defined for (Trans Repl) is ensured by the same condition for (Red Repl). By the premise $\mathrm{fn}(\overline{x}^{\overline{\iota}} v \mid \mathbf{!}\, x^{\iota} p.P) \subseteq A$ we have $\mathrm{fn}(v) \subseteq A$, so using (Trans Struct Right) we have $A \vdash \overline{x}^{\overline{\iota}} v \mid \mathbf{!}\, x^{\iota} p.P \xrightarrow{\tau} \mathbf{!}\, x^{\iota} p.P \mid \{^{v}/_{p}\}P$, the right hand side of which is exactly the right hand side of (Red Repl).

(Red Par), (Red Res) and (Red Box) require straightforward uses of induction hypothesis, using (Trans Par), (Trans Res-1) and (Trans Box-3).

(Red Struct) By Lemma 10, $\mathrm{fn}(P') \subseteq A$. By the inductive hypothesis, $A \vdash P' \xrightarrow{\tau} Q'$. By Proposition 18, $A \vdash P \xrightarrow{\tau} Q'$. By (Tran-Struct-Right), $A \vdash P \xrightarrow{\tau} Q$.

□

Transitions Imply Reductions

Lemma 20 *If* $A \vdash P \xrightarrow{\overline{z}^o v} P'$ *then* $P \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid P')$

Proof Induction on derivation of $A \vdash P \xrightarrow{\overline{z}^o v} P'$.

(Trans Out) Obvious.

(Trans Par) By the induction hypothesis, $P \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid P')$, so

$$\begin{array}{rcl} P \mid Q & \equiv & ((\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid P')) \mid Q \\ & \equiv & (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid P' \mid Q) \text{ (as by } \mathrm{fn}(P \mid Q) \subseteq A \text{ we have } \mathrm{fn}(Q) \subseteq A) \end{array}$$

(Trans Res-1) By the induction hypothesis $P \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - (A,x))(\overline{z}^o v \mid P')$, so

$$\begin{array}{rcl} (\boldsymbol{\nu}\, x)P & \equiv & (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - (A,x))(\overline{z}^o v \mid P') \\ & \equiv & (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid (\boldsymbol{\nu}\, x)P') \text{ (as } x \notin \mathrm{fn}(\overline{z}^o v)) \end{array}$$

(Trans Res-2-nmv) By the induction hypothesis $P \equiv (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - (A,x))(\overline{z}^o v \mid P')$, so

$$\begin{array}{rcl} (\boldsymbol{\nu}\, x)P & \equiv & (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, \mathrm{fn}(z,v) - (A,x))(\overline{z}^o v \mid P') \\ & \equiv & (\boldsymbol{\nu}\, \mathrm{fn}(z,v) - A)(\overline{z}^o v \mid P') \text{ (as } x \in \mathrm{fn}(v) - \mathrm{fn}(z,o)) \end{array}$$

(Trans Res-2-mv) By the induction hypothesis $P \equiv (\boldsymbol{\nu}\, \mathrm{fn}(x,v) - (A,x))(\overline{x}^o v \mid P')$, so

$$\begin{array}{rcl} (\boldsymbol{\nu}\, x)P & \equiv & (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, \mathrm{fn}(x,v) - (A,x))(\overline{x}^o v \mid P') \\ & \equiv & (\boldsymbol{\nu}\, \mathrm{fn}(x,v) - A)(\overline{x}^o v \mid P') \end{array}$$

(Trans Struct-Right) By the induction hypothesis.

All other cases are vacuous.

□

Lemma 21 *If* $A \vdash Q \xrightarrow{x^\iota v} Q'$ *then there exist* B, p, Q_1 *and* Q_2 *such that* $B \cap (A \cup \mathrm{fn}(x^\iota v)) = \{\}$ *and either* $Q \equiv (\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid Q_2)$ *and* $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid Q_2)$ *or* $Q \equiv (\boldsymbol{\nu}\, B)(\mathbf{!}\, x^\iota p.Q_1 \mid Q_2)$ *and* $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid \mathbf{!}\, x^\iota p.Q_1 \mid Q_2)$.

Proof Induction on derivation of $A \vdash Q \xrightarrow{x^\iota v} Q'$.

(Trans In), (Trans Repl) Obvious.

(Trans Par) Consider $A \vdash Q \mid P \xrightarrow{x^\iota v} Q' \mid P$. By the induction hypothesis there exist B, p, Q_1 and Q_2 such that $B \cap (A \cup \mathrm{fn}(x^\iota v)) = \{\}$ and either $Q \equiv (\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid Q_2)$ or $Q \equiv (\boldsymbol{\nu}\, B)(\mathbf{!}\, x^\iota p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid \mathbf{!}\, x^\iota p.Q_1 \mid Q_2)$.

Consider the first disjunct (the second is similar). Taking $\hat{Q}_2 = Q_2 \mid P$ we have

$$\begin{array}{rcl} Q \mid P & \equiv & (\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid Q_2) \mid P \\ & \equiv & (\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid \hat{Q}_2) \\ Q' \mid P & \equiv & (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid Q_2) \mid P \\ & \equiv & (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid \hat{Q}_2) \end{array}$$

(Trans Res-1) Consider $A \vdash (\boldsymbol{\nu}\, z)Q \xrightarrow{x^\iota v} (\boldsymbol{\nu}\, z)Q'$ with $z \notin A \cup \mathrm{fn}(x^\iota v)$. By the induction hypothesis there exist B, p, Q_1 and Q_2 such that $B \cap (A, z \cup \mathrm{fn}(x^\iota v)) = \{\}$ and either $Q \equiv (\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid Q_2)$ or $Q \equiv (\boldsymbol{\nu}\, B)(!\, x^\iota p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid !\, x^\iota p.Q_1 \mid Q_2)$.

Consider the first disjunct (the second is similar). Taking $\hat{B} = B, z$ we have

$$\begin{array}{rcl} (\boldsymbol{\nu}\, z)Q & \equiv & (\boldsymbol{\nu}\, z)(\boldsymbol{\nu}\, B)(x^\iota p.Q_1 \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\, \hat{B})(x^\iota p.Q_1 \mid Q_2) \\ (\boldsymbol{\nu}\, z)Q' & \equiv & (\boldsymbol{\nu}\, z)(\boldsymbol{\nu}\, B)(\{^v/_p\}Q_1 \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\, \hat{B})(\{^v/_p\}Q_1 \mid Q_2) \end{array}$$

(Trans Struct Right) By the induction hypothesis.

All other cases are vacuous.

□

Lemma 22 *If $A \vdash Q \xrightarrow{x^{\overline{n}} v} Q'$ then there exist B, Q_1 and Q_2 such that $B \cap (A \cup \mathrm{fn}(x^{\overline{n}} v)) = \{\}$, $Q \equiv (\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2)$.*

Proof Induction on derivation of $A \vdash Q \xrightarrow{x^{\overline{n}} v} Q'$.

(Trans Box-2) Obvious.

(Trans Par) Consider $A \vdash Q \mid P \xrightarrow{x^{\overline{n}} v} Q' \mid P$. By the induction hypothesis there exist B, Q_1 and Q_2 such that $B \cap (A \cup \mathrm{fn}(x^{\overline{n}} v)) = \{\}$, $Q \equiv (\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2)$.

Take $\hat{Q}_2 = Q_2 \mid P$. We have

$$\begin{array}{rcl} Q \mid P & \equiv & (\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2) \mid P \\ & \equiv & (\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2 \mid P) \text{ (as } \mathrm{fn}(P) \subseteq A) \\ & \equiv & (\boldsymbol{\nu}\, B)(n[Q_1] \mid \hat{Q}_2) \\ Q' \mid P & \equiv & (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2) \mid P \\ & \equiv & (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2 \mid P) \text{ (as } \mathrm{fn}(P) \subseteq A) \\ & \equiv & (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid \hat{Q}_2) \end{array}$$

(Trans Res-1) Consider $A \vdash (\boldsymbol{\nu}\, z)Q \xrightarrow{x^{\overline{n}} v} (\boldsymbol{\nu}\, z)Q'$ with $z \notin A \cup \mathrm{fn}(x^{\overline{n}} v)$. By the induction hypothesis there exist B, Q_1 and Q_2 such that $B \cap (A, z \cup \mathrm{fn}(x^{\overline{n}} v)) = \{\}$, $Q \equiv (\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2)$.

Let $\hat{B} = B, z$. We have

$$\begin{array}{rcl} (\boldsymbol{\nu}\, z)Q & \equiv & (\boldsymbol{\nu}\, z)(\boldsymbol{\nu}\, B)(n[Q_1] \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\, \hat{B})(n[Q_1] \mid Q_2) \\ (\boldsymbol{\nu}\, z)Q' & \equiv & (\boldsymbol{\nu}\, z)(\boldsymbol{\nu}\, B)(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\, \hat{B})(n[(\overline{x}^\uparrow v \mid Q_1)] \mid Q_2) \end{array}$$

(Trans Struct Right) By the induction hypothesis.

All other cases are vacuous.

$\square$

Lemma 23 *If $A \vdash P \xrightarrow{\tau} Q$ then $P \to Q$.*

Proof Induction on derivations of $A \vdash P \xrightarrow{\tau} Q$

(Trans Box-1) By Lemma 20 $P \equiv (\boldsymbol{\nu}\,\mathrm{fn}(x,v) - A)(\overline{x}^{\uparrow}v \mid P')$, so

$$\begin{array}{rcl} n[P] & \equiv & n[(\boldsymbol{\nu}\,\mathrm{fn}(x,v) - A)(\overline{x}^{\uparrow}v \mid P')] \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(x,v) - A)(n[\overline{x}^{\uparrow}v \mid P']) \text{ (by } \mathrm{fn}(n[P]) \subseteq A \text{ we have } n \in A) \\ & \to & (\boldsymbol{\nu}\,\mathrm{fn}(x,v) - A)(\overline{x}^{\overline{n}}v \mid n[P']) \text{ (by (Red Up))} \end{array}$$

(Trans Box-3) By the induction hypothesis and (Red Box).

(Trans Par) By the induction hypothesis and (Red Par).

(Trans Comm) By Lemma 20 $P \equiv (\boldsymbol{\nu}\,\mathrm{fn}(x,v) - A)(\overline{x}^{\overline{\gamma}}v \mid P')$. By Lemma 11 $x \in A$ so $P \equiv (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\overline{x}^{\overline{\gamma}}v \mid P')$.

Case $\gamma = \iota$. By Lemma 21 there exist B, p, Q_1 and Q_2 such that $B \cap (A \cup \mathrm{fn}(x^{\iota}v)) = \{\}$ and either $Q \equiv (\boldsymbol{\nu}\,B)(x^{\iota}p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\,B)(\{{}^{v}\!/_{p}\}Q_1 \mid Q_2)$ or $Q \equiv (\boldsymbol{\nu}\,B)(!\,x^{\iota}p.Q_1 \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\,B)(\{{}^{v}\!/_{p}\}Q_1 \mid !\,x^{\iota}p.Q_1 \mid Q_2)$. Consider the first disjunct. We have

$$\begin{array}{rcl} P \mid Q & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\overline{x}^{\overline{\gamma}}v \mid P') \mid (\boldsymbol{\nu}\,B)(x^{\iota}p.Q_1 \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\overline{x}^{\overline{\gamma}}v \mid P' \mid (\boldsymbol{\nu}\,B)(x^{\iota}p.Q_1 \mid Q_2)) \text{ (as } \mathrm{fn}(Q) \subseteq A) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\boldsymbol{\nu}\,B)(\overline{x}^{\overline{\gamma}}v \mid P' \mid x^{\iota}p.Q_1 \mid Q_2) \text{ (as } (A \cup \mathrm{fn}(v)) \cap B = \{\}) \\ & \to & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\boldsymbol{\nu}\,B)(\{{}^{v}\!/_{p}\}Q_1 \mid P' \mid Q_2) \text{ (by Red Comm)} \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(P' \mid (\boldsymbol{\nu}\,B)(\{{}^{v}\!/_{p}\}Q_1 \mid Q_2)) \text{ (as } (A \cup \mathrm{fn}(v)) \cap B = \{\}) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(P' \mid Q') \end{array}$$

The second disjunct is similar.

Case $\gamma = \overline{n}$. By Lemma 22 there exist B, Q_1 and Q_2 such that $B \cap (A \cup \mathrm{fn}(x^{\overline{n}}v)) = \{\}$, $Q \equiv (\boldsymbol{\nu}\,B)(n[Q_1] \mid Q_2)$ and $Q' \equiv (\boldsymbol{\nu}\,B)(n[(\overline{x}^{\overline{\uparrow}}v \mid Q_1)] \mid Q_2)$. We have

$$\begin{array}{rcl} P \mid Q & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\overline{x}^{n}v \mid P') \mid (\boldsymbol{\nu}\,B)(n[Q_1] \mid Q_2) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\overline{x}^{n}v \mid P' \mid (\boldsymbol{\nu}\,B)(n[Q_1] \mid Q_2)) \text{ (as } \mathrm{fn}(Q) \subseteq A) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\boldsymbol{\nu}\,B)(\overline{x}^{n}v \mid P' \mid n[Q_1] \mid Q_2) \text{ (as } (A \cup \mathrm{fn}(v)) \cap B = \{\}) \\ & \to & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(\boldsymbol{\nu}\,B)(P' \mid n[\overline{x}^{\overline{\uparrow}}v \mid Q_1] \mid Q_2) \text{ (by Red Down)} \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(P' \mid (\boldsymbol{\nu}\,B)(n[(\overline{x}^{\overline{\uparrow}}v \mid Q_1)] \mid Q_2)) \text{ (as } (A \cup \mathrm{fn}(v)) \cap B = \{\}) \\ & \equiv & (\boldsymbol{\nu}\,\mathrm{fn}(v) - A)(P' \mid Q') \end{array}$$

(Trans Res-1) By the induction hypothesis and (Red Res).

(Trans Struct Right) By the induction hypothesis and (Red Struct).

All other cases are vacuous.

$\square$

Proof (of Theorem 1) We must show that if $\mathrm{fn}(P) \subseteq A$ then $A \vdash P \xrightarrow{\tau} Q$ iff $P \to Q$. This is immediate from Lemmas 19 and 23 above. $\square$

B Other Proofs

We first give another transition-analysis lemma. This allows us to rename extruded names in a label instead of in the source process term.

Lemma 24 *If $A \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{\ell} Q$, $\ell = \overline{y}^{\uparrow}v$, and A, N and M are pairwise disjoint finite sets of names then there exists a partition N_1, N_2 of N, a process P', and*

$$h : (\mathrm{fn}(\ell) - A) \to (\mathcal{N} - (A, N_2, M))$$

injective such that

$$\begin{array}{l} A, N \vdash P \xrightarrow{(1_A + h)\ell} P' \\ A \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{(1_A + h)\ell} (\boldsymbol{\nu}\, N_2)P' \equiv (1_A + h)Q \\ N_2 = N - \mathrm{fn}((1_A + h)\ell) \end{array}$$

Proof Induction on N. For $N = \emptyset$ we have $A \vdash P \xrightarrow{\ell} Q$. Take any $h : (\mathrm{fn}(\ell) - A) \to (\mathcal{N} - (A, M))$ injective. By Lemma 13 $A \vdash 1_A P \xrightarrow{(1_A + h)\ell} (1_A + h)Q$. Now consider $A \vdash (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, N)P \xrightarrow{\ell} Q$ with A, (N, x), and M pairwise disjoint. By Lemma 16.6 one of the following cases hold.

Case 6a there exists $\hat{x} \notin A \cup \mathrm{fn}(\ell) \cup (\mathrm{fn}((\boldsymbol{\nu}\, N)P) - x)$ and $\hat{Q}$ such that $A, \hat{x} \vdash \{\hat{x}/x\}(\boldsymbol{\nu}\, N)P \xrightarrow{\ell} \hat{Q}$ and $Q \equiv (\boldsymbol{\nu}\, \hat{x})\hat{Q}$.

Take some

$$\begin{array}{l} f : A, \hat{x} \to A, x \\ g : (\mathrm{fn}(\ell) - A, \hat{x}) \to \mathcal{N} - (A, x, M) \end{array}$$

injective with f the identity on A. By Lemma 13

$$A, x \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{(f+g)\ell} (f + g)\hat{Q}$$

By the induction hypothesis there exists a partition N_1', N_2' of N, a process P', and

$$h' : (\mathrm{fn}((f + g)\ell) - (A, x)) \to (\mathcal{N} - (A, x, N_2', M))$$

injective such that

$$\begin{array}{l} A, x, N \vdash P \xrightarrow{(1_{A,x} + h')(f+g)\ell} P' \\ A, x \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{(1_{A,x} + h')(f+g)\ell} (\boldsymbol{\nu}\, N_2')P' \equiv (1_{A,x} + h')(f + g)\hat{Q} \end{array}$$

Now $\hat{x} \notin \mathrm{fn}\ell$, so $x \notin \mathrm{fn}((f + g)\ell)$, so $x \notin \mathrm{fn}((1_{A,x} + h')(f + g)\ell)$, so by (Res-1)

$$A \vdash (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, N)P \xrightarrow{(1_{A,x} + h')(f+g)\ell} (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, N_2')P' \equiv (\boldsymbol{\nu}\, x)(1_{A,x} + h')(f + g)\hat{Q}$$

Take $N_1 = N_1'$, $N_2 = N_2', x$ and $h = h'g$.

Case 6c there exists y, o, v, $\hat{Q}$ and $\hat{x} \notin A \cup \mathrm{fn}(o) \cup (\mathrm{fn}((\boldsymbol{\nu}\, N)P) - x)$ such that $\ell = \overline{y}^{o}v$, $A, \hat{x} \vdash \{\hat{x}/x\}(\boldsymbol{\nu}\, N)P \xrightarrow{\overline{y}^{o}v} \hat{Q}$, $\hat{x} \in \mathrm{fn}(y, v)$, $\mathrm{mv}(o)$ and $Q \equiv \hat{Q}$.

Similarly, take some

$$\begin{array}{l} f : A, \hat{x} \to A, x \\ g : (\mathrm{fn}(\ell) - A, \hat{x}) \to \mathcal{N} - (A, x, M) \end{array}$$

injective with f the identity on A. By Lemma 13

$$A, x \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{(f+g)\ell} (f+g)\hat{Q}$$

By the induction hypothesis there exists a partition N_1', N_2' of N, a process P', and

$$h' : (\mathrm{fn}((f+g)\ell) - (A, x)) \to (\mathcal{N} - (A, x, N_2', M))$$

injective such that

$$\begin{array}{l} A, x, N \vdash P \xrightarrow{(1_{A,x}+h')(f+g)\ell} P' \\ A, x \vdash (\boldsymbol{\nu}\, N)P \xrightarrow{(1_{A,x}+h')(f+g)\ell} (\boldsymbol{\nu}\, N_2')P' \equiv (1_{A,x} + h')(f+g)\hat{Q} \end{array}$$

Now here $\hat{x} \in \mathrm{fn}\ell$, so $x \in \mathrm{fn}((f+g)\ell)$, so $x \in \mathrm{fn}((1_{A,x}+h')(f+g)\ell)$, so by (Res-2)

$$A \vdash (\boldsymbol{\nu}\, x)(\boldsymbol{\nu}\, N)P \xrightarrow{(1_{A,x}+h')(f+g)\ell} (\boldsymbol{\nu}\, N_2')P' \equiv (1_{A,x} + h')(f+g)\hat{Q}$$

Take $N_1 = N_1', x$, $N_2 = N_2'$ and $h = \{x/\hat{x}\} + h'g$.

□

Explicit Characterisation

The simple security properties are proved using an explicit characterisation of the states and labelled transitions of $\mathcal{W}_1[P]$. If N is a finite set of names, a is a name and $\mathcal{A}$ and Q are processes define

$$\begin{array}{rcll} [\![a; N; \mathcal{A}; Q]\!] & \stackrel{def}{=} & (\boldsymbol{\nu}\, N \cup \{a\})(& \mathcal{A} \\ & & & \mid a[Q] \\ & & & \mid \,!\, in^{\uparrow} y.\overline{in}^{a} y \\ & & & \mid \,!\, out^{a} y.\overline{out}^{\uparrow} y\ ) \end{array}$$

Say the 4-tuple a, N, $\mathcal{A}$, Q is *good* if N, $\{a\}$, and $\{in, out\}$ are pairwise disjoint, $\mathcal{A}$ is a parallel composition of outputs of the forms

$$\overline{out}^{\overline{a}} v,\ \overline{out}^{\uparrow} v,\ \overline{in}^{a} v,\ \overline{x}^{\overline{a}} v \text{ where } x \notin \{out, a\}$$

with $a \notin \mathrm{fn}(v)$ in each case, and Q is a process with $a \notin \mathrm{fn}(Q)$. Say a process P is good if $P \equiv [\![a; N; \mathcal{A}; Q]\!]$ for some good a, N, $\mathcal{A}$, Q.

Lemma 25 *If $a \notin \mathrm{fn}(P)$ then $\mathcal{W}_1[P] \equiv [\![a; \emptyset; 0; P]\!]$, hence $\mathcal{W}_1[P]$ is good.*

Proof Straightforward. □

We define a transition relation $A \vdash P \xrightarrow{\ell} Q$ as the least satisfying the following rules.

$$\begin{array}{llrll}
t_1 & & A \vdash [\![a; N; \mathcal{A}; Q]\!] & \xrightarrow{in^{\uparrow} v} [\![a; N; \mathcal{A} \mid \overline{in}^{a} v; Q]\!] & \mathrm{fn}(v) \cap (N \cup \{a\}) = \emptyset \\
t_2 & & A \vdash [\![a; N; \mathcal{A} \mid \overline{in}^{a} v; Q]\!] & \xrightarrow{\tau} [\![a; N; \mathcal{A}; Q \mid \overline{in}^{\uparrow} v]\!] & \\
t_4 & A, N, a \vdash Q \xrightarrow{\overline{out}^{\uparrow} v} Q' & A \vdash [\![a; N; \mathcal{A}; Q]\!] & \xrightarrow{\tau} [\![a; N, \mathrm{fn}(v) - (A, N, a); \mathcal{A} \mid \overline{out}^{\overline{a}} v; Q']\!] & \\
t_5 & A, N, a \vdash Q \xrightarrow{\overline{x}^{\uparrow} v} Q' & A \vdash [\![a; N; \mathcal{A}; Q]\!] & \xrightarrow{\tau} [\![a; N, \mathrm{fn}(x, v) - (A, N, a); \mathcal{A} \mid \overline{x}^{\overline{a}} v; Q']\!] & \\
t_6 & & A \vdash [\![a; N; \mathcal{A} \mid \overline{out}^{\overline{a}} v; Q]\!] & \xrightarrow{\tau} [\![a; N; \mathcal{A} \mid \overline{out}^{\uparrow} v; Q]\!] & \\
t_7 & & A \vdash [\![a; N; \mathcal{A} \mid \overline{out}^{\uparrow} v; Q]\!] & \xrightarrow{\overline{out}^{\uparrow} v} [\![a; N - \mathrm{fn}(v); \mathcal{A}; Q]\!] & \\
t_8 & A, N, a \vdash Q \xrightarrow{\tau} Q' & A \vdash [\![a; N; \mathcal{A}; Q]\!] & \xrightarrow{\tau} [\![a; N; \mathcal{A}; Q']\!] &
\end{array}$$

$$\frac{A \vdash P \xrightharpoonup{\ell} P' \quad P' \equiv P''}{A \vdash P \xrightharpoonup{\ell} P''}$$

For rule t_5, we have a side condition that $x \neq \mathit{out}$. For all rules we have a sidecondition that the 4-tuple in the left hand side of the conclusion is good. For all rules we have a sidecondition that the free names of the process on the left hand side of the conclusion are contained in A.

Lemma 26 *If $A \vdash P \xrightharpoonup{\ell} P'$ then P' is good.*

Proof By inspection of the transition axioms, checking that the 4-tuple on the right hand side is good in each case, and noting that the definition of P good is preserved by structural congruence. For t_4 by the condition $\mathrm{fn}([\![a; N; \mathcal{A}; Q]\!]) \subseteq A$ we have $\{\mathit{in}, \mathit{out}\} \subseteq A$ so $\{\mathit{in}, \mathit{out}\} \cap (\mathrm{fn}(v) - (A, N, a)) = \emptyset$. By Lemma 11.3 $a \notin \mathrm{fn}(v)$ By Lemma 11.2 $a \notin \mathrm{fn}(Q')$. For t_5 by the condition $\mathrm{fn}([\![a; N; \mathcal{A}; Q]\!]) \subseteq A$ we have $\{\mathit{in}, \mathit{out}\} \subseteq A$ so $\{\mathit{in}, \mathit{out}\} \cap (\mathrm{fn}(x, v) - (A, N, a)) = \emptyset$. By Lemma 11.3 $a \notin \mathrm{fn}(x, v)$ By Lemma 11.2 $a \notin \mathrm{fn}(Q')$. For t_8 by Lemma 11.2 $a \notin \mathrm{fn}(Q')$. The other cases are straightforward. $\square$

Lemma 27 *For all good P we have $A \vdash P \xrightarrow{\ell} P'$ iff $A \vdash P \xrightharpoonup{\ell} P'$.*

Proof We first show that $A \vdash P \xrightharpoonup{\ell} P'$ implies $A \vdash P \xrightarrow{\ell} P'$, by induction on derivations of the former. The converse direction is by a case analysis of the possible transition derivations. $\square$

Purity

Proof (of Proposition 2) We show by induction on k that Q is good and that the conclusion holds. The $k = 0$ case is by Lemma 25. The inductive step uses Lemmas 26 and 27. $\square$

Proof (of Proposition 3) Similar to that of Proposition 2; we omit the details. $\square$

Proof (of Proposition 4) Similar to that of Proposition 2; we omit the details. $\square$

Honesty

Proof (of Proposition 5) We check that the unary wrapper $\mathcal{W}_1$ is honest (the proof for $\mathcal{L}$ should be similar). If N is a finite set of names, a is a name and $\mathcal{A}$ and Q are processes define

$$\begin{array}{rcl}
\langle\!\langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle\!\rangle & \stackrel{def}{=} & Q \\
& & \mid \{\!|\ \overline{out}^{\uparrow} v \mid \overline{out}^{\overline{a}} v \in \mathcal{A}\ |\!\} \\
& & \mid \{\!|\ \overline{out}^{\uparrow} v \mid \overline{out}^{\uparrow} v \in \mathcal{A}\ |\!\} \\
& & \mid \{\!|\ \overline{x}^{\uparrow} v \mid \overline{x}^{\overline{a}} v \in \mathcal{A} \wedge x \neq \mathit{out}\ |\!\} \\
& & \mid \{\!|\ \overline{in}^{\uparrow} v \mid \overline{in}^{a} v \in \mathcal{A}\ |\!\} \\
\langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle & \stackrel{def}{=} & (\boldsymbol{\nu}\, N) \langle\!\langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle\!\rangle
\end{array}$$

Note that if $a; N; \mathcal{A}; Q$ is good then $a \notin \mathrm{fn}(\langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle)$. Now take the family of relations below.

$$R_A \quad = \quad \equiv \circ \{ [\![a; N; \mathcal{A}; Q]\!], \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle \mid a; N; \mathcal{A}; Q \text{ good and } \mathrm{fn}([\![a; N; \mathcal{A}; Q]\!]) \subseteq A \} \circ \equiv$$

We must check that for any P with $a \notin \mathrm{fn}(P)$ and $A \supseteq \mathrm{fn}(\mathcal{W}_1[P])$ we have $\mathcal{W}_1[P]$ R_A P and that R is an h-bisimulation. The former follows from Lemma 25 and the fact $\langle\!\langle a; \emptyset; 0; P \rangle\!\rangle \equiv P$. For the latter there are a number of cases to check, as below. We give only the most interesting in detail.

Consider C R_A D. We know there exist good $a; N; \mathcal{A}; Q$ such that $C \equiv [\![a; N; \mathcal{A}; Q]\!]$, $D \equiv \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle$, and $\mathrm{fn}(C) \subseteq A$. Without loss of generality suppose A and N, a are disjoint. Note that by Proposition 18 if $A \vdash C \xrightarrow{\ell} C'$ then $A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\ell} C'$, and similarly for transitions of D.

Clause 1′ Suppose $A \vdash \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle \xrightarrow{\overline{out}^{\uparrow} v} U$.

By Lemma 24 there exists a partition N_1, N_2 of N, a process U', and

$$h : (\mathrm{fn}(v) - A) \to (\mathcal{N} - (A, N_2, a))$$

injective such that

$$\begin{array}{l} A, N \vdash \langle\!\langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle\!\rangle \xrightarrow{\overline{out}^{\uparrow} v'} U' \\ A \vdash \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle \xrightarrow{\overline{out}^{\uparrow} v'} (\boldsymbol{\nu}\, N_2) U' \equiv (1_A + h) U \\ N_2 = N - \mathrm{fn}(v') \end{array}$$

where $v' = (1_A + h)v$. There are three cases.

(a) due to $A, N \vdash Q \xrightarrow{\overline{out}^{\uparrow} v'} Q'$ with $U' \equiv \langle\!\langle\!\langle a; N; \mathcal{A}; Q' \rangle\!\rangle\!\rangle$.

By Lemma 14 $A, N, a \vdash Q \xrightarrow{\overline{out}^{\uparrow} v'} Q'$.

By t4,t6,t7 and Lemmas 26,27

$$A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\tau} \xrightarrow{\tau} \xrightarrow{\overline{out}^{\uparrow} v'} [\![a; N - \mathrm{fn}(v'); \mathcal{A}; Q']\!]$$

By Lemma 13

$$A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\tau} \xrightarrow{\tau} \xrightarrow{\overline{out}^{\uparrow} v} (1_A + h^{-1}) [\![a; N - \mathrm{fn}(v'); \mathcal{A}; Q']\!]$$

Now $a; N - \mathrm{fn}(v'); \mathcal{A}; Q'$ is good, hence

$$[\![a; N - \mathrm{fn}(v'); \mathcal{A}; Q']\!] \; R_{A \cup \mathrm{fn}(v')} \; \langle\!\langle a; N - \mathrm{fn}(v'); \mathcal{A}; Q' \rangle\!\rangle,$$

and R is closed under injective renamings that preserve $\{in, out\}$, so

$$(1_A + h^{-1}) [\![a; N - \mathrm{fn}(v'); \mathcal{A}; Q']\!] \; R_{A \cup \mathrm{fn}(v)} \; (1_A + h^{-1}) \langle\!\langle a; N - \mathrm{fn}(v'); \mathcal{A}; Q' \rangle\!\rangle \equiv U$$

(b) due to an $\overline{out}^{a} v \in \mathcal{A}$. Match using t6,t7.

(c) due to an $\overline{out}^{\uparrow} v \in \mathcal{A}$. Match using t7.

Suppose $A \vdash \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle \xrightarrow{\tau}$

(a) due to $A, N \vdash Q \xrightarrow{\tau} Q'$. Match using t8.

(b) due to $A, N \vdash Q \xrightarrow{in^{\uparrow} v} Q'$ and $\overline{in}^{a} v \in \mathcal{A}$. Match using t2,t8.

Clause 2′ Suppose $A \vdash \langle\!\langle a; N; \mathcal{A}; Q \rangle\!\rangle \xrightarrow{in^{\uparrow} v}$. This must be due to $A, N \vdash Q \xrightarrow{in^{\uparrow} v} Q'$. Match using t1,t2,t8.

Clause 1 Suppose $A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\overline{out}^{\uparrow} v}$. This must be by t7; it can be matched directly.

Suppose $A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\tau}$. This must be by one of the following rules.

- t2 Match with zero τ steps.
- t4 Using Lemma 20 the output particle is present in Q. The transition can then be matched with zero τ steps.
- t5 Similar to t4.
- t6 Match with zero τ steps.
- t8 Match with one τ step.

Clause 2 Suppose $A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{in^{\uparrow} v}$. This must be by t1. It can be matched with zero τ steps, using the second part of Clause 2 of the definition of h-bisimulation.

Clause 3 Suppose $A \vdash [\![a; N; \mathcal{A}; Q]\!] \xrightarrow{\ell}$ for another label ℓ. Vacuous.

□

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