Pythagoras and the Fleet Air Arm

M.J.Sewell
Department of Mathematics, University of Reading,
Whiteknights, Reading, RG6 6AX.
http://www.cl.cam.ac.uk/users/pes20/MJSewell/mjs.html
michael@sewells.org

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Abstract

This article contains the substance of a brief talk which I gave at The Royal Institution Primary Mathematics Organisers' Meeting on 17 June 2010. It will be published in the associated Autumn 2010 Newsletter.

During my first visit to the Fleet Air Arm Museum at Yeovilton in Somerset, in 2009, I found that in addition to the numerous historical aeroplanes there (including a Concorde - was it used to land on aircraft carriers at sea?), there was available an A5 leaflet, for visitors to take away, entitled "How to calculate the distance to the horizon". The recipe described there has three steps. Firstly, estimate the height h of your eye above the sea surface. Secondly, calculate $1.5h$ or $13h$, according to whether h is in feet or metres, respectively. Thirdly, take the square roots $\sqrt{1.5h}$ or $\sqrt{13h}$, and these will be the distance to the horizon in miles or kilometres respectively.

By implication the definition of horizon is where the sky seems to meet the sea surface. For example, if I am standing on the beach and my eyes are 6 feet above the sand, then the horizon is $\sqrt{1.5 \times 6} = 3$ miles away. If I am inland, and I can still see the sea, the same algebraic formula applies. So if I am on a hillside 600 feet above the beach, the horizon at sea is 30 miles away. But if I am inland and the horizon is inland, the formula becomes ambiguous because, as the definition in the leaflet says, "the true horizon is where the sky and Earth meet", and this

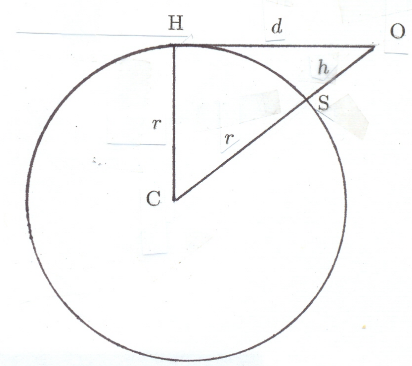

Figure 1: Great circle section of the Earth

no longer justifies spherical geometry.

So what is the basis for these formulae? As the leaflet says, it is "a version of Pythagoras' Theorem - remember him?" What version?

I had not seen this version before, so what is the proof it? I worked it out so that I could discuss it with my weekly Masterclass for ten-year-olds in a primary school. Treat the Earth as a sphere, so that a cross-section through the centre C is a circle. If the observer's eye is at O in the diagram and H is the horizon, the definition of horizon means that the angle CHO is a right angle. Let $r = CH$ be the radius of the Earth, and $d = HO$ be the distance to the visible horizon. Let $CO = r + h$ be the distance from the centre of the Earth to the observer. Of course, unless the observer is in an aeroplane or spaceship, O will be much nearer to H than the diagram suggests, but we need to have a legible diagram, and the geometry will be qualitatively the same whether O is in an aeroplane, or looking from a pier or beach.

Pythagoras' Theorem gives $d^2+r^2 = (r+h)^2$ and therefore $d^2 = 2rh+h^2$. But $h \ll r$, so we can approximate Pythagoras' Theorem by $d^2 = 2rh$ and therefore $d = \sqrt{2rh}$. Consulting an atlas tells us that the Earth's diameter is $2r = 7926$ miles so, remembering to use consistent units so that $h = 6$ feet $= 6/5280$ miles, we find that the distance to the horizon as seen from the beach is the square root of $(7926 \times 6)/5280$ or $\sqrt{1.5 \times 6} = 3$ miles.